DEVELOPMENT OF A NEW CORRELATION FOR BUBBLE-POINT OIL VISCOSITY

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الخلاصـــة :

تم تطوير معادلة تنبؤ وضعية للزوجة الزيت عند نقطة التفقع اعتماداً على توفر معلومات لزيوت كندية وشرق أوسطية. وتقترح هذه المعادلة علاقة بسيطة بين كل من لزوجة الزيت وكثافته عند نقطة التفقع. وقد بيّنت تحاليلُ الخطأ تفوّقَ هذه المعادلة الجديدة على سابقاتها عند اختبارها بنفس المعلومات.

ABSTRACT

An empirical correlation for oil viscosity at the bubble point was developed based on Canadian and Middle Eastern oil data. The correlation postulates a simple relationship between oil viscosity and density at the bubble point. Error analysis shows that the new correlation is superior to others when tested on the same data.

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DEVELOPMENT OF A NEW CORRELATION FOR BUBBLE-POINT OIL VISCOSITY

1. INTRODUCTION

Viscosity is an intensive property of a Newtonian fluid which reflects its resistance to flow. In the petroleum industry, the production, processing and transportation of oil involves numerous flow processes such as: flow through porous rock, wells, and surface pipes, and vessels. Accurate viscosity data is necessary for optimum reservoir management, and for sound design of facilities.

Like most liquid properties, the viscosity of oil is affected by ambient conditions of temperature and pressure. Since oil is principally a mixture of hydrocarbons possessing various thermodynamic properties, the viscosity is especially sensitive to pressure because pressure dictates the fraction of low molecular weight components that can escape from the liquid phase as a gas under equilibrium conditions. Therefore, oil is frequently regarded as a liquid containing a finite quantity of gas dissolved under pressure. The pressure below which the oil releases its first bubble of gas (at reservoir temperature) is the bubble-point pressure, BPP. Gas evolution continues as the pressure is decreased below the BPP until atmospheric pressure is attained where the oil, labled dead oil, is completely void of gas. Because gas evolution entails changing the composition of the liquid phase, the variation of oil viscosity has been differentiated into two regimes of pressure (Figure 1).

1.1. Pressures Above the BPP

When the environmental conditions are such that the oil is above the BPP, it is known as an undersaturated oil because gas can be added without evolution of gas as bubbles in the oil. The viscosity of an undersaturated oil increases as the pressure is increased due to an increase of oil density as the oil is compressed.

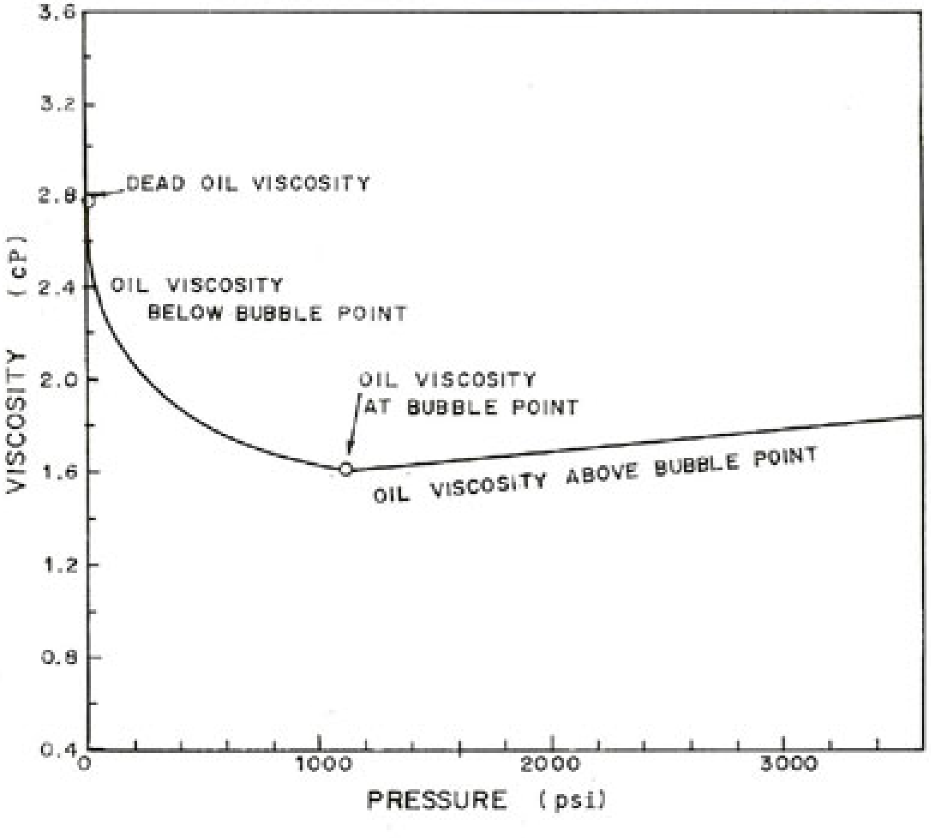

Figure 1. Viscosity Variation with Pressure for a Typical Gas-Rich Light Oil at Reservoir Temperature.

1.2. Pressures Below the BPP

An oil below its BPP is known as a saturated oil and its viscosity increases as the pressure is decreased since the loss of lighter components results in an increase of the oil density. The end point viscosity in this regime is the dead oil viscosity, μ_{od}, at atmospheric pressure. The viscosity also increases as the dead oil is cooled to atmospheric temperature. At the BPP (Figure 1), the viscosity is at its minimum and is referred to as the bubble-point viscosity, μ_{ob}.

Usually, the increase of viscosity as a function of pressure above the BPP is minor compared to the increase below the BPP. Viscosity data for saturated conditions, including the bubble point, is the most difficult to obtain because a saturated oil loses gas easily making it difficult to manage experimentally. Accurate data at the bubble point is difficult to obtain because it is a borderline point. Therefore, the probability of experimental error is highest when testing oil at μ_{ob}.

The viscosity of a gas-rich oil at a given temperature and pressure can be estimated in theory, if a detailed composition of the oil is known by applying approximate mixing rules to the viscosities of the pure components at that temperature and pressure. This task, however, is virtually impossible to accomplish due to the great number of components which make up crude oil and natural gas; and also because it is extremely rare to have available the compositional analysis of an oil at different conditions of temperature and pressure. Therefore, empirical correlations utilizing readily available data are more practical. These data usually comprise the following parameters: (1) dead oil density relative to water (γ_o), (2) the average density of the liberated gas relative to air (γ_g), (3) the solution gas/oil ratio at the BPP (R_s) which is the ratio of the total volume of liberated gas, measured at standard conditions, to the volume of dead oil that would result if the oil is brought down to atmospheric conditions, and (4) the reservoir temperature. It is on these parameters that most empirical viscosity correlations are based.

2. VISCOSITY CORRELATIONS

It is very difficult to derive a single correlation that predicts the oil viscosity at any pressure and temperature due to the changes of crude oil viscosity as the oil is taken from atmospheric conditions to undersaturation conditions. The historical approach has been to predict μ_{od} based on γ_o and T. Then with R_s the viscosity for any saturation condition is predicted up to the BPP. As shown by Al-Marhoun [1], BPP can be predicted if γ_g is also known. Above the BPP, only P and μ_{ob} are required for estimation of μ_o.

The most widely used correlations published in the literature were reviewed by Sutton and Farshad [2], and are listed in Table 1. Most of the correlations were based on data sets generated from a single geographical area. Due to the natural regional differences of oil characteristics (the most important being the oil composition), marked differences in viscosity can occur for oils having identical properties. Hence, a correlation might work very well for oils of a certain group but fail noticeably for others. Middle Eastern oil data was not utilized in any of those correlations; therefore, large errors are expected when they are applied to those oils. Since μ_{ob} is the subject of this paper, only published correlations related to this viscosity will be reviewed.

The correlations by Chew and Connally [3] and by Beggs and Robinson [4] (Table 1) both propose a power-function relationship between μ_{ob} and μ_{od} with the constants as functions of R_s. This necessitates de-

Table 1. Viscosity Correlations in the Literature.

Dead Oil Viscosity

Beal [7]: $\mu_{od} = [0.32 + (1.8 \times 10^7)/°\mathrm{API}^{4.53}][360/(T + 200)]^a$
where $a = \mathrm{antilog}(0.43 + 8.33/°\mathrm{API})$

Beggs and Robinson [4]: $\mu_{od} = 10^X - 1$
where $X = YT^{-1.163}$
$Y = 10^z$
$z = 3.0324 - 0.02023°\mathrm{API}$

Glaso [8]: $\mu_{od} = (3.141 \times 10^{10})T^{-3.444}(\log °\mathrm{API})^a$
where $a = 10.313(\log T) - 36.447$

Oil Viscosity Below and At The Bubble Point

Chew and Connally [3]: $\mu_{ob} = a(\mu_{od})^b$
where $a = 0.20 + 0.80\ \mathrm{antilog}(-0.00081R_s)$
$b = 0.43 + 0.57\ \mathrm{antilog}(-0.00072R_s)$

Beggs and Robinson [4]: $\mu_{ob} = a(\mu_{od})^b$
where $a = 10.715\ (R_s + 100)^{-0.515}$
$b = 5.44\ (R_s + 150)^{-0.338}$

Oil Viscosity Above Bubble Point

Beal [7]: $\mu_a = \mu_{ob} + 0.001\ (p - P_b)(0.024\mu_{ob}^{1.6} + 0.038\mu_{ob}^{0.56})$

Vazquez and Beggs [9]: $\mu_a = \mu_{ob}(P/P_b)^m$
where $m = 2.6P^{1.187}\ \mathrm{antilog}[(-3.9 \times 10^{-5})P - 5.0]$

termination, or estimation, of μ_{od} before hand. In addition, the two correlations can be used to estimate μ_o below the BPP if the gas/oil ratio at that pressure is used.

The correlation by Chew and Connally was based on data for 457 crude oils from the western hemisphere. The ranges of oil properties within these data are listed in Table 2. No error analyses were reported with the correlation.

Beggs and Robinson's correlation was based on 2533 bubble-point viscosities for 600 crude oil samples with property ranges also listed in Table 2. Similarly, no error analyses were reported.

The principal source of error of these two correlations is that the effect of γ_g is totally ignored. One cannot simply discount the lubricating effect of the small gas molecules on the friction between the large liquid molecules, especially when μ_g has always been found indispensible in BPP correlations.

Another possible source of error of these correlations is the complex mathematical expressions that do not include the natural variation of viscosity with other oil properties. In this study, a new μ_{ob} correlation has been developed which is based on a relationship between μ_{ob} and the relative density of the oil at the bubble point. The study utilized bubble-point data compiled for 62 Middle Eastern reservoirs which was augmented by a Canadian data base [5]. The data set contains a total of 459 points. The ranges of several parameters within the data are listed in Table 2 also.

Table 2. Date Ranges.

1. *Chew and Connally*	
Bubble-point pressure, psia	132 to 5645
Temperature, °F	72 to 292
Solution GOR, SCF/STB	51 to 3544
Dead-oil viscosity, cP	0.377 to 50
2. *Beggs and Robinson*	
Pressure, psia	15 to 5265
Temperature, °F	70 to 295
Tank-oil gravity, °API	16 to 58
Solution GOR, SCF/STB	20 to 2070
3. *This Study*	
Bubble-point viscosity, cP	0.105 to 17.65
Temperature, °F	74 to 240
Solution GOR, SCF/STB	21 to 3001
Gas relative density	0.525 to 1.588
Tank-oil gravity, °API	21 to 49
Bubble-point relative density	0.493 to 0.897

3. VISCOSITY CORRELATION

The correlation was based on the association of μ_{ob} and the oil relative density at the bubble point, γ_{ob}, which is computed as follows:

$$\gamma_{ob} = \frac{\gamma_o + 2.177 \times 10^{-4}\gamma_g R_s}{B_{ob}} \tag{1}$$

The formation-volume-factor at the bubble point, B_{ob}, was estimated using Al-Marhoun's correlation [1]:

$$B_{ob} = 0.497069 + 0.862963 \times 10^{-3}(T + 460)$$

$$+ 0.182594 \times 10^{-2}F + 0.318099 \times 10^{-5}F^2 \tag{2}$$

where:

$$F = R_s^{0.742390}\gamma_g^{0.323294}\gamma_o^{-1.202040} \tag{3}$$

While oil density has been correlated to oil compressibility [6], apparently no attempt has been made to correlate the density to other PVT properties.

The experimental μ_{ob} data is plotted in Figure 2 as a function of γ_{ob}. Employing non-linear regression analysis, the following equation was found to be the best fit to the data plotted in Figure 2:

$$\ln \mu_{ob} = a_0 + a_1\gamma_{ob}^4 \tag{4}$$

where

$$a_0 = -2.652294 \tag{5}$$

$$a_1 = 8.484462 \tag{6}$$

4. ERROR ANALYSIS

Two criteria were used to compare the performance of this correlation with those of Chew and Connally and Beggs and Robinson:

(*a*) The average absolute percent relative error:

$$\bar{E}_a = \frac{1}{m}\sum_{i=1}^{m}|E_r| \tag{7}$$

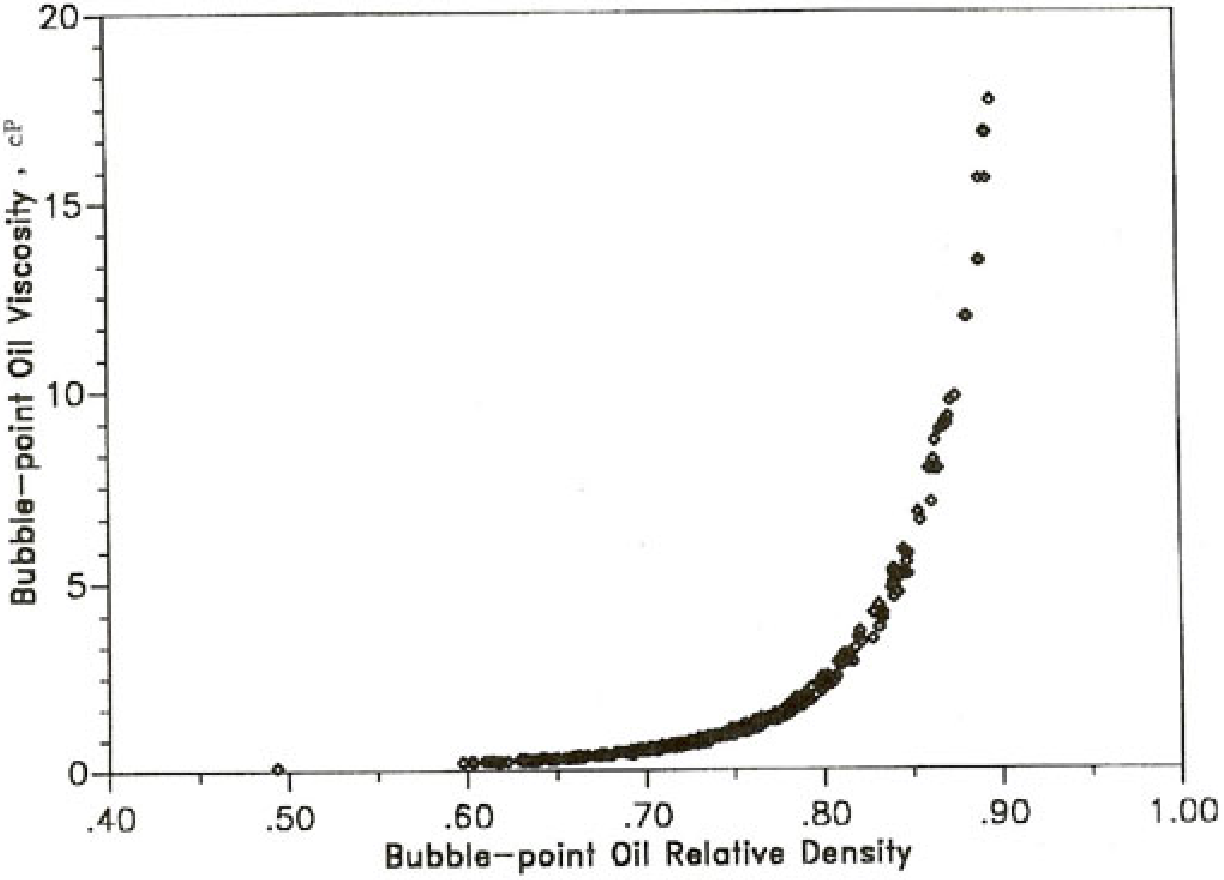

Figure 2. Experimental Viscosity Data versus Computed Relative Density.

Table 3. Statistical Accuracy of Correlations.

Correlation	$\overline{E}_r$	$\overline{E}_a$	Min E_a	Max E_a	R	s
Chew and Connally	−4.02	7.56	0.01	34.33	0.9871	8.9897
Beggs and Robinson	21.40	21.95	0.19	42.69	0.9310	10.2559
This Study	−0.17	4.91	0.03	11.31	0.9979	5.7608

where

$$E_r = \frac{\mu_{\text{exp}} - \mu_{\text{est}}}{\mu_{\text{exp}}} \times 100. \tag{8}$$

(*b*) The standard deviation

$$s = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (E_r - \overline{E}_r)^2} \tag{9}$$

where

$$\overline{E}_r = \frac{1}{m} \sum_{i=1}^{m} E_r \tag{10}$$

In some cases the average percent relative error, $\overline{E}_r$, which considers the real value of the error rather than the absolute, is used. This procedure, however, yields deceptively low values.

Table 3 lists the results of the error analyses for the various correlations using the data set. The column headings in Table 3 are defined in the nomenclature.

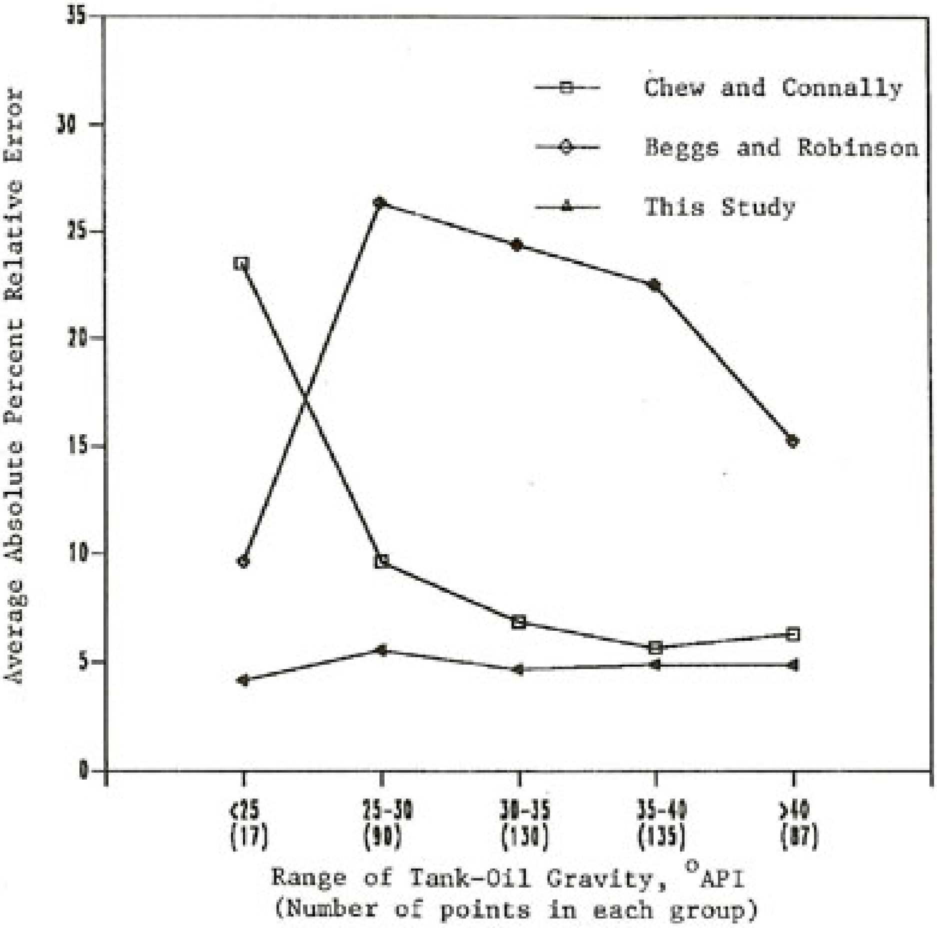

Figure 3. Average Error for Bubblepoint Oil Viscosity Grouped by Tank-Oil Gravity.

The new correlation produced the lowest error in the average value and in the standard deviation. Beggs and Robinson's is the worst of the three in both categories. These tendencies were observed with the data set as a whole as well as to groupings within the set as shown in Table 4. The average error and standard deviation data of Table 4 are plotted in Figures 3 and 4. The Beggs and Robinson's correlation exhibited the greatest error even though it is based on a larger dataset with somewhat wider temperature and oil gravity ranges which should have theoretically produced a more universal correlation.

Table 4. Statistical Accuracy of Correlations for Data Grouped by Stock Tank Oil Gravity.

°API Range	< 25	25-30	30-35	35-40	> 40
Number of Points	17	90	130	135	87
Average Absolute Percent Relative Error, $\bar{E}_a$					
Chew and Connally	23.48	9.62	6.88	5.67	6.29
Beggs and Robinson	9.65	26.36	24.39	22.52	15.27
This Study	4.15	5.54	4.66	4.87	4.86
Standard Deviation, s					
Chew and Connally	9.62	10.30	6.38	6.93	7.76
Beggs and Robinson	13.07	9.86	6.02	8.93	9.23
This Study	4.81	6.31	5.36	5.65	5.86

4. CONCLUSIONS

A simple equation was derived to correlate the bubble-point viscosity of a crude oil to its relative density at the bubble point. The equation provides

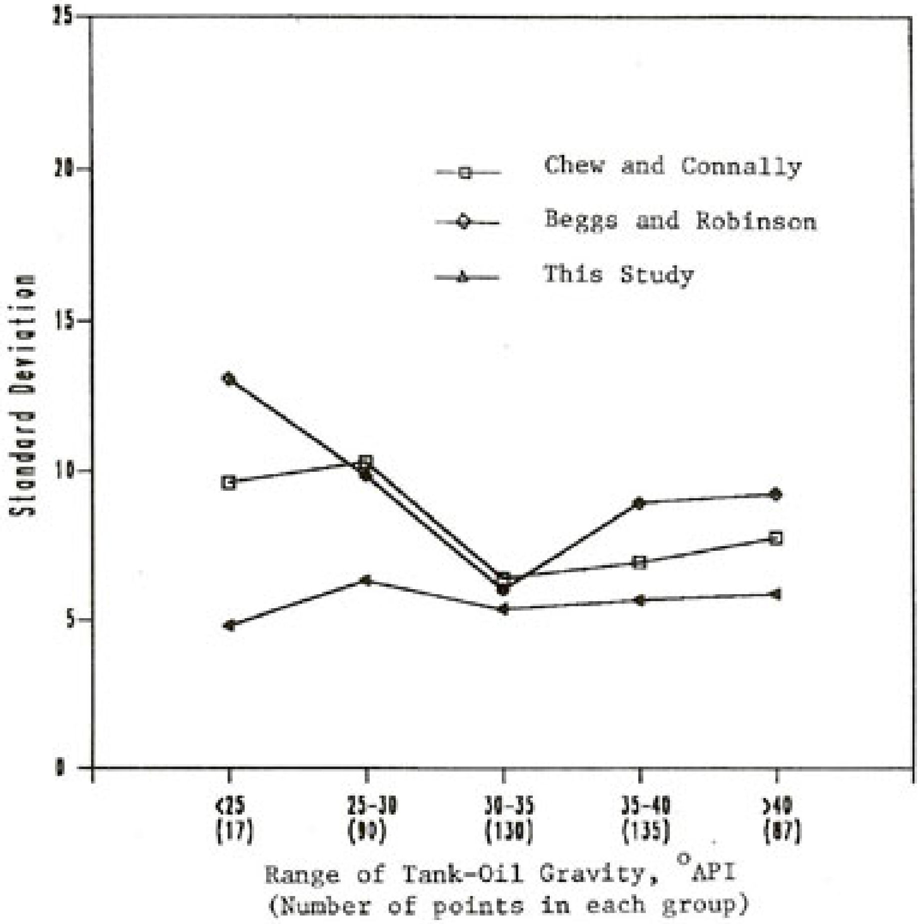

Figure 4. Standard Deviation for Bubblepoint Oil Viscosity Grouped by Tank-Oil Gravity.

more accurate predictions for Middle Eastern and Canadian data than the correlations of Chew and Connally and Beggs and Robinson. This conclusion applies to all segments of this particular data set and is expected to apply to other Middle Eastern data as well.

NOTATION

a_0 = constant
a_1 = constant
°API = tank-oil gravity, °API
B_{ob} = bubble-point formation volume factor, bbl/STB
E_a = absolute percent relative error
$\bar{E}_a$ = average absolute percent relative error
E_r = percent relative error
$\bar{E}_r$ = average percent relative error
F = correlation quantity
i = observation index
m = number of observations
P = pressure, psia
P_b = bubble-point pressure, psia
R = coefficient of correlation
R_s = solution gas/oil ratio, SCF/STB
s = standard deviation
T = temperature, °F
γ_g = gas relative density at 14.7 psia and 60 °F (air = 1)
γ_o = oil relative density at 14.7 psia and 60 °F (water = 1)
γ_{ob} = oil relative density at the bubble-point

μ_o = oil viscosity, cP
μ_{od} = dead-oil viscosity, cP
μ_{ob} = bubble-point oil viscosity, cP
μ_{est} = estimated viscosity, cP
μ_{exp} = measured viscosity, cP

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