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MODIFIED ENGINEERING APPROACH WITH THE VARIATION OF PERMEABILITY OVER TIME USING THE MEMORY CONCEPT

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ABSTRACT

Reservoir simulation, as practiced in the oil industry, is well established and is the standard tool for solving reservoir engineering problems. The conventional major steps involved in the development of a reservoir simulator can be characterized as formulation, discretization, well representation, linearization, solution, and validation. Recently the engineering approach has been introduced that would allow one bypass linearization during the development of model equations. To avoid the hidden misconceptions in formulation, engineering approach with a new form of motion equation based on the application of memory concept is used to develop a mathematical model for the fluid flow in porous media. The proposed model involves discretization, followed by formulation to obtain the flow equations in the integral form. The model equations are written for a given gridblock in space at a given time level. The equations reflect the flow equations in an algebraic form. The most important feature of this model is the consideration of fluid and rock properties as a time dependent variable without linearization. This model can be used in reservoir simulation and well testing.

1. INTRODUCTION

The modeling of fluid movement through porous media is the most important but very intricate and difficult research task in petroleum engineering due to the complex fluid rheological behavior and geological structure. Rheology is usually defined as the branch of physics that studies the deformation and flow of matter [1]. In simpler terms, a rheological measurement indicates how “fluid-like” or “solid-like” a material is. However, in rheology interest is usually focused on those materials that possess both elastic and viscous properties. The geological structure of reservoir formation is dependent on the deposition or sedimentation of earth with time. This structure of reservoir formation is the key to model the behavior of fluid movement within the pore network where grid size, grid block are very important issues. Therefore, the rheology of fluid and the structure of the formation are the most influential motivating criteria in developing the true fluid flow models for porous media application. More generally, understanding the flow of rheologically complex fluids through porous media is important in many other engineering applications. The knowledge of the behavior of polymer solutions in rocks can be applied in ground water hydrology, soil mechanics, industrial pollutant infiltration and chemical flooding processes. Therefore, fluid rheology is an important issue for any reservoir management.

The big challenge is that almost all the properties of any fluid/material are time dependent. However, another problem with "time" is the scale of time. The time dimension itself is a complex presentation in reservoir simulation. This concept will be used during the development of reservoir simulation models. Therefore, in this advanced technological and information age, it is not unrealistic to expect accurate computational efficiency after including such an important dimension as time. As a matter of fact, this history is the indication of time dependency which can be defined as the pathway travelled by both fluid and formation with time. The concept of using variable rock and fluid properties with time can be modeled using the notion of “memory”. In this article, the definition of “memory” is stated as “the properties of rock and fluid that help to account for changes in rock properties (such as permeability, porosity) and fluid properties (such as pressure dependent fluid properties, viscosity) with time and space” [1]. Even though the memory concept is old knowledge, with frequent citations in holy book [2], modern researchers are only beginning to model this concept. Some researchers revealed the effects of pathway and dependency on history of the fluid to define the memory concept [3 - 5]. Chen et al. [6] argued that mobilization and subsequent flow in a porous medium of a fluid with a yield stress can be explained well when the notion of memory is introduced.

It is well known that the petroleum industry drives the energy sector, which in turn drives the modern civilization. While significant research has been conducted in this area, focused on improving oil recovery with different techniques, theoretical prediction methods for recovery schemes – a matter of utmost importance with cost implications in the millions of dollars on wrong uses for a single oil field – still suffer from certain shortcomings in their mathematical modeling [7]. The shortcomings that this dissertation wants to address include: 1) insufficient description of rock/fluid interaction, particularly under thermal constraints; 2) linearization of rheological data; and 3) linearization of governing equations. Based on an insight in the information age, it has become possible to include such phenomena that are considered to be intangible and beyond the scope of computational mathematics [8]. It is very important to look further into fluid rheology and fluid memory. To overcome those shortcomings, the most recent approach, called, "Engineering Approach" is becoming extremely popular within the researchers, academician and for new simulator development [7]. This article introduces a new rigorous fluid flow model bypassing inherent linearization of formulation using the "Engineering Approach". This model will be useful for reservoir simulation, well testing and reservoir performance predictions.

2. THEORETICAL MODEL DEVELOPMENT

Fluid properties that are needed to model single-phase fluid flow include those that appear in the flow equations, namely, density (ρ), formation volume factor (B), viscosity, (μ). Fluid density is needed for the estimation of fluid gravity (γ) using

$$\gamma = \gamma_c \, \rho \, g \tag{1}$$

A conservative equation describes the rate of fluid movement into or out the reservoir volume element. The modified Darcy's law in 1D can be written as

$$u_x = -\beta_c \, \eta \left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left(\frac{\partial \Phi}{\partial x} \right) \right\} \tag{2}$$

where,

$$\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial \Phi}{\partial x}\right) = [1/\Gamma(1-\alpha)] \int_0^t (t-\xi)^{-\alpha} \frac{\partial}{\partial \xi}\left(\frac{\partial \Phi}{\partial x}\right) \partial \xi, \quad with\; 0 \le \alpha < 1$$

$$= \frac{\int_0^t (t-\xi)^{-\alpha} \left[\frac{\partial^2 \Phi}{\partial \xi \partial x}\right] \partial \xi}{\Gamma(1-\alpha)}$$

The potential is related to pressure through the following relationship

$$\Phi - \Phi_{\text{ref}} = \left(p - p_{ref}\right) - \gamma\left(Z - Z_{ref}\right)$$

Therefore,

$$\frac{\partial \Phi}{\partial x} = \frac{\partial p}{\partial x} - \gamma \frac{\partial Z}{\partial x} \tag{3}$$

2.1 Derivation of the 1D Flow Equation in Cartesian Coordinates

Figure 1 shows block *i* and its neighboring blocks *i* - 1 and *i* + 1 in the *x*-direction. At any instant in time, fluid enters block *i*, coming from block *i* - 1 across its $x_{i-1/2}$ face at a mass rate of $w_x\rfloor_{x_{i-1/2}}$ and leave to block *i* + 1 across its $x_{i+1/2}$ face at a mass rate of $w_x\rfloor_{x_{i+1/2}}$. The fluid also enters block *i* through a well at a mass rate of q_{m_i}. The mass of fluid contained in a unit volume of rock in block *i* is m_{v_i}. Therefore, the material balance equation for block *i* written over a time step $\Delta t = t^{n+1} - t^n$ can be written as

$$m_i\rfloor_{x_{i-1/2}} - m_o\rfloor_{x_{i+1/2}} + m_{s_i} = m_{a_i} \tag{4}$$

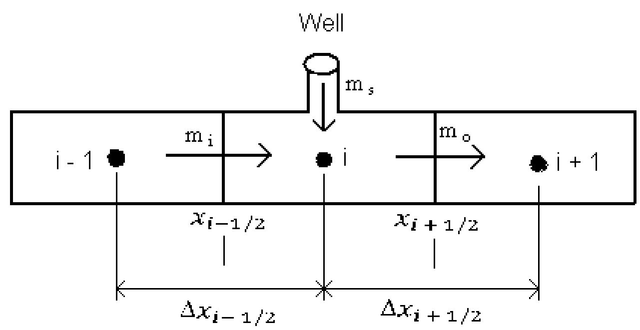

Figure 1. *Block i as a reservoir volume element in 1D flow (redrawn from [7]).*

Terms like $w_x\rfloor_{x_{i-1/2}}$, $w_x\rfloor_{x_{i+1/2}}$, and q_{m_i} are functions of time only because space is not a variable for an already discretized reservoir. Abou-Kassem et al. [7] have explained this very well. Therefore,

$$m_i\rfloor_{x_{i-1/2}} = \int_{t^n}^{t^{n+1}} w_x\rfloor_{x_{i-1/2}} dt \tag{5}$$

$$m_o\rfloor_{x_{i+1/2}} = \int_{t^n}^{t^{n+1}} w_x\rfloor_{x_{i+1/2}} dt \tag{6}$$

$$m_{s_i} = \int_{t^n}^{t^{n+1}} q_{m_i} dt \tag{7}$$

Using Eqs. (5) through (6), Eq. (4) can be rewritten as

$$\int_{t^n}^{t^{n+1}} w_x\rfloor_{x_{i-1/2}} dt - \int_{t^n}^{t^{n+1}} w_x\rfloor_{x_{i+1/2}} dt + \int_{t^n}^{t^{n+1}} q_{m_i} dt = m_{a_i} \tag{8}$$

The mass accumulation is defined as

$$m_{a_i} = \Delta_t (V_b \, m_v)_i = V_{b_i}\left(m_{v_i}^{n+1} - m_{v_i}^{n}\right) \tag{9}$$

Now mass rate and mass flux are related through

$$w_x = \dot{m}_x\, A_x \tag{10}$$

Mass flux ($\dot{m}_x$) can be expressed in terms of fluid density and volumetric velocity,

$$\dot{m}_x = \alpha_c\, \rho\, u_x \tag{11}$$

Mass of fluid per unit volume or rock (m_v) can be expressed in terms of fluid density and porosity,

$$m_v = \phi\, \rho \tag{12}$$

Mass of injected or produced fluid (q_m) can be expressed in terms of well volumetric rate (q) and fluid density,

$$q_m = \alpha_c\, \rho\, q \tag{13}$$

Substituting Eqs. (9) and (10) into Eq. (8) yields

$$\int_{t^n}^{t^{n+1}} (\dot{m}_x\, A_x)\rfloor_{x_{i-1/2}} dt - \int_{t^n}^{t^{n+1}} (\dot{m}_x\, A_x)\rfloor_{x_{i+1/2}} dt + \int_{t^n}^{t^{n+1}} q_{m_i} dt = V_{b_i}\left(m_{v_i}^{n+1} - m_{v_i}^{n}\right) \tag{14}$$

Substituting Eqs. (11) through (13) into Eq. (14) yields

$$\int_{t^n}^{t^{n+1}} (\alpha_c\, \rho\, u_x\, A_x)\rfloor_{x_{i-1/2}} dt - \int_{t^n}^{t^{n+1}} (\alpha_c\, \rho\, u_x\, A_x)\rfloor_{x_{i+1/2}} dt + \int_{t^n}^{t^{n+1}} (\alpha_c\, \rho\, q)_i\, dt = V_{b_i}\left[(\phi\, \rho)_i^{n+1} - (\phi\, \rho)_i^{n}\right] \tag{15}$$

An equation of state describes the density of fluid as a function of pressure and temperature. For single-phase fluid,

$$B = \frac{\rho_{sc}}{\rho} \tag{16}$$

Substitution of Eq. (16) into Eq. (15), dividing by $\alpha_c\, \rho_{sc}$ yields

$$\int_{t^n}^{t^{n+1}} \left(\frac{u_x\, A_x}{B}\right)\Big\rfloor_{x_{i-1/2}} dt - \int_{t^n}^{t^{n+1}} \left(\frac{u_x\, A_x}{B}\right)\Big\rfloor_{x_{i+1/2}} dt + \int_{t^n}^{t^{n+1}} \left(\frac{q}{B}\right)_i dt = \frac{V_{b_i}}{\alpha_c}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right]$$

Substituting $q_{sc} = q/B$ in the above equation yields

$$\int_{t^n}^{t^{n+1}} \left(\frac{u_x\, A_x}{B}\right)\Big\rfloor_{x_{i-1/2}} dt - \int_{t^n}^{t^{n+1}} \left(\frac{u_x\, A_x}{B}\right)\Big\rfloor_{x_{i+1/2}} dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right] \tag{17}$$

Fluid volumetric velocity (flow rate per unit cross-sectional area) from block $i-1$ to block i ($u_x\rfloor_{x_{i-1/2}}$) at any time instant t is given by the algebraic analog of Eq. (2)

$$u_x\rfloor_{x_{i-1/2}} = \frac{\beta_c}{\Gamma(1-\alpha)}\, \eta_x\rfloor_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \frac{\left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i-1} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i}\right\}}{\Delta x_{i-1/2}}\, \partial\xi\right] \tag{18}$$

Likewise, fluid flow per unit cross-sectional area from block i to block $i+1$ is

$$u_x\rfloor_{x_{i+1/2}} = \frac{\beta_c}{\Gamma(1-\alpha)}\, \eta_x\rfloor_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \frac{\left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i+1}\right\}}{\Delta x_{i+1/2}}\, \partial\xi\right] \tag{19}$$

Substitution of Eqs. (18) and (19) into Eq. (17) yields

$$\int_{t^n}^{t^{n+1}} \frac{\beta_c}{\Gamma(1-\alpha)}\, \eta_x\rfloor_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \frac{\left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i-1} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i}\right\}}{\Delta x_{i-1/2}}\, \partial\xi\right]\left(\frac{A_x}{B}\right)\Bigg\rfloor_{x_{i-1/2}} dt$$

$$- \int_{t^n}^{t^{n+1}} \frac{\beta_c}{\Gamma(1-\alpha)}\, \eta_x\rfloor_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \frac{\left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i+1}\right\}}{\Delta x_{i+1/2}}\, \partial\xi\right]\left(\frac{A_x}{B}\right)\Bigg\rfloor_{x_{i+1/2}} dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right]$$

Or,

$$\int_{t^n}^{t^{n+1}} \left[\frac{A_x\, \beta_c\, \eta_x}{\mathrm{B}\, \Delta x\, \Gamma(1-\alpha)}\right]_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i-1} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} \left[\frac{A_x\, \beta_c\, \eta_x}{\mathrm{B}\, \Delta x\, \Gamma(1-\alpha)}\right]_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i+1} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right]$$

$$\int_{t^n}^{t^{n+1}} T_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i-1} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} T_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i+1} - \frac{\partial^2 \Phi}{\partial\xi \partial x}\Big\rfloor_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right] \tag{20}$$

Where,

$$T_{x_{i\mp 1/2}} = \left[\frac{A_x\, \beta_c\, \eta_x}{\mathrm{B}\, \Delta x\, \Gamma(1-\alpha)}\right]_{x_{i\mp 1/2}}$$

The accumulation term in Eq. (20) can be expressed in terms of the change in the pressure of block *i* as shown in Eq. (21)

$$\int_{t^n}^{t^{n+1}} T_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial^2\Phi}{\partial\xi\partial x}\Big]_{i-1} - \frac{\partial^2\Phi}{\partial\xi\partial x}\Big]_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} T_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial^2\Phi}{\partial\xi\partial x}\Big]_{i+1} - \frac{\partial^2\Phi}{\partial\xi\partial x}\Big]_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left(\frac{\phi}{B}\right)_i \left[p_i^{n+1} - p_i^n\right] \qquad (21)$$

Or,

$$\int_{t^n}^{t^{n+1}} T_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\left[\frac{\partial}{\partial\xi}\left(\frac{\partial\Phi}{\partial x}\right)\right]_{i-1} - \left[\frac{\partial}{\partial\xi}\left(\frac{\partial\Phi}{\partial x}\right)\right]_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} T_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\left[\frac{\partial}{\partial\xi}\left(\frac{\partial\Phi}{\partial x}\right)\right]_{i+1} - \left[\frac{\partial}{\partial\xi}\left(\frac{\partial\Phi}{\partial x}\right)\right]_{i}\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left(\frac{\phi}{B}\right)_i \left[p_i^{n+1} - p_i^n\right]$$

Substitution of Eq. (3) yields

$$\int_{t^n}^{t^{n+1}} T_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial}{\partial\xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i-1} - \gamma_{i-1/2}\left(\frac{\partial Z}{\partial x}\right)_{i-1}\right] - \frac{\partial}{\partial\xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i} - \gamma_{i-1/2}\left(\frac{\partial Z}{\partial x}\right)_{i}\right]\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} T_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial}{\partial\xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i+1} - \gamma_{i+1/2}\left(\frac{\partial Z}{\partial x}\right)_{i}\right] - \frac{\partial}{\partial\xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i+1} - \gamma_{i+1/2}\left(\frac{\partial Z}{\partial x}\right)_{i}\right]\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left(\frac{\phi}{B}\right)_i \left[p_i^{n+1} - p_i^n\right]$$

Or,

$$\int_{t^n}^{t^{n+1}} T_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial}{\partial\xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i-1} - \left(\frac{\partial p}{\partial x}\right)_{i}\right] - \frac{\partial}{\partial\xi}\left(\gamma_{i-1/2}\right)\left[\left(\frac{\partial Z}{\partial x}\right)_{i-1} - \left(\frac{\partial Z}{\partial x}\right)_{i}\right]\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} T_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial}{\partial\xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i+1} - \left(\frac{\partial p}{\partial x}\right)_{i}\right] - \frac{\partial}{\partial\xi}\left(\gamma_{i+1/2}\right)\left[\left(\frac{\partial Z}{\partial x}\right)_{i+1} - \left(\frac{\partial Z}{\partial x}\right)_{i}\right]\right\} \partial\xi\right] dt + \int_{t^n}^{t^{n+1}} q_{sc_i}\, dt = \frac{V_{b_i}}{\alpha_c}\left(\frac{\phi}{B}\right)_i \left[p_i^{n+1} - p_i^n\right] \qquad (22)$$

Equation (22) represents the general form of diffusivity equation with memory using the Engineering Approach. Equation (22) is a non-linear diffusivity equation considering fluid memory for an axial flow of any single phase fluid in porous media. Equation (22) is strictly non-linear because all the parts are non-linear because it is derived bypassing the Taylor approximation and in its algebraic form. In addition, it consists of a derivative of a pressure derivative with memory part, pressure derivative with time, η, compressibility and porosity multiplication which also makes the terms nonlinear.

3. CONCLUSIONS

A diffusivity equation with rock/fluid memory has been developed by invoking time-dependence to permeability and viscosity using Engineering approach. The resulting highly non-linear theoretical model has an option of triggering the memory variable, depending on the applicability. The findings of this research establish the contribution of memory in reservoir fluid flow through porous media. This representation accounts for the chaotic behavior of several non-Newtonian fluids and time dependent rock properties are due to the memory of fluid rheological and formation rock. Therefore, it may be concluded that although other causes such as heterogeneity, anisotropy and inelasticity of the matrix, may be invoked to interpret certain phenomena, the memory mechanism could help in interpreting part of the phenomenology. If the memory function is invoked, the resulting equation is complete. This formulation allows one to investigate fluid flow in porous media without resorting to a priori approximations and subsequent linearization.

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5. NOMENCLATURE

A_x = Cross sectional area of rock perpendicular to the flow of flowing fluid, m^2
B = formation volume factor, $m^3/std\, m^3$
g = gravitational acceleration, m/s^2
m_v = mass of fluid contained in a unit volume or rock, *kg*
$m_{v_i}^n$ = mass of fluid contained in a unit volume or rock at block *i* at time step n, *kg*
$m_{v_i}^{n+1}$ = mass of fluid contained in a unit volume or rock at block *i* at time step n+1, *kg*
$m_i\rfloor_{x_{i-1/2}}$ = mass of fluid entering the reservoir volume element at boundary, $x_{i-1/2}$, *kg*
$m_o\rfloor_{x_{i+1/2}}$ = mass of fluid leaving the reservoir volume at boundary, $x_{i+1/2}$, *kg*
m_{s_i} = the mass of fluid entering or leaving the reservoir volume, *i* element externally through wells
$\dot{m}_x$ = mass flux at point *x*, *kg*

m_{a_i} = the mass of excess fluid stored in or depleted from the reservoir volume element over a time interval
p = pressure at any time, *t*, *pa*
p_{ref} = pressure at a reference point at any time, *t*, *pa*
q = fluid flow rate, $std\ m^3/d$
q_{sc} = fluid flow rate at standard condition, $std\ m^3/d$
q_{m_i} = mass rate enters through the well at block *i*, kg/s
t^n = time at step n, *day*
t^{n+1} = time at step n+1, *day*
w_x = mass rate of fluid, kg/s
$w_x\rfloor_{x_{i-1/2}}$ = mass rate of fluid at $x_{i-1/2}$, kg/s
$w_x\rfloor_{x_{i+1/2}}$ = mass rate of fluid at $x_{i+1/2}$, kg/s
u_x = velocity normal to the flow direction *x*, m/s
V_b = block bulk volume, m^3
V_{b_i} = block bulk volume at block *i*, m^3
Z = elevation from datum, with positive values downward, *m*
Z_{ref} = the datum reference point, *m*
ρ = density, kg/m^3
γ = fluid gravity, kg/m^3
γ_c = gravity conversion factor
η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3 s^{1+\alpha}/kg$
ξ = a dummy variable for time i.e. real part in the plane of the integral, *s*
α = fractional order of differentiation, dimensionless

β_c = the transmissibility conversion factor
Φ = potential, pa
Φ_{ref} = potential at a reference point, pa
ρ_{sc} = fluid density at standard condition, $kg/\ m^3$
Δt = time step, *day*
α_c = volume conversion factor,
$\eta_x\rfloor_{x_{i-1/2}}$ = between block *i* – 1and *i* that are separated by a distance $\Delta x_{i-1/2}$
$\left.\frac{\partial^2\Phi}{\partial\xi\partial x}\right\rfloor_{i-1}$ = potentials derivative with respect to time of block *i* – 1
$\left.\frac{\partial^2\Phi}{\partial\xi\partial x}\right\rfloor_{i}$ = potentials derivative with respect to time of block *i*

6. REFERENCES

[1] Hossain, M.E., "An Experimental and Numerical Investigation of Memory-Based Complex Rheology and Rock/Fluid Interactions", *PhD dissertation*, Dalhousie University, Halifax, Nova Scotia, Canada, 2008, p. 773.
[2] The Holy Al-Qur"an, 17:44; 57:1; 62:1; 64:1.
[3] Arenzon, J.J., Levin, Y., and Sellitto, M., "Slow Dynamics Under Gravity: A Nonlinear diffusion model", *Physica A, Vol. 325, 2003, p. 371 – 395*.
[4] Shin, M., Kim, D.S., and Lee, J.W., "Deposition of Inertia-Dominated Particles Inside a Turbulent Boundary Layer" *International Journal of Multiphase Flow, Vol. 29, 2003, p. 893–926*.
[5] Zhang, H.M., "Driver Memory, Traffic Viscosity and a Viscous Vehicular Traffic Flow Model", *Transportation Research Part B, Vol. 37, 2003, p. 27–41*.
[6] Chen, M., Rossen, W., and Yortsos, Y.C., "The Flow and Displacement in Porous Media of Fluids with Yield Stress", *Chemical Engineering Science, Vol. 60, 2005, p. 4183 – 4202*.
[7] Abou-Kassem, J.H., Farouq Ali, S.M., and Islam, M.R., "Petroleum Reservoir Simulation: A Basic Approach", *Gulf Publishing Company*, Houston, TX, USA, 2006, p. 445.
[8] Islam, M.R., "Computing for the Information Age, Keynote speech", *Proceeding of 36th Int. Conf. Comp. Ind. Eng.*, June 20-23, 2006, Taipei, Taiwan.