SPE 114287

A New Porous Media Diffusivity Equation with the Inclusion of Rock and Fluid Memories

M. Enamul Hossain, SPE, S. Hossein Mousavizadegan*, and M. Rafiqul Islam**, SPE

Department of Civil & Resource Engineering, Dalhousie University

Sexton Campus, 1360 Barrington Street, Halifax, NS B3J 2X4, Canada

E-mail address: mehossain@dal.ca; hmousavi@dal.ca

*Currently with Amir Kabir University, Tehran, Iran

**Corresponding author. Tel.: +1.902.494.3980;

fax: +1. 902. 494.3108; rafiqul.islam@dal.ca

Summary

The complex rheological behavior of formation fluids and their structures depend on their history of formation and propagation. In petroleum engineering, this history tells the qualitative and quantitative measures of fossil fuels in a petroleum trap. The properties of fluid and formation are thoroughly investigated, as evidenced by the shear volume of publications on the topic. However, still there remains a formidable challenge on how to model the variation of these properties under complex in situ thermal and mechanical stress. These fluid properties as well as rock properties are assumed to be invariant with time in most of the available simulators. The variation of permeability is important when there are mineralizations in the pore network, which is subject to significant change during fluid extraction and depressurization. The main aim of this study is to model the variable permeability and viscosity over time. The concept of memory is applied to model these variations with time in the fluid flow through the porous medium.

A new rigorous model for the fluid flow inside the porous formation is introduced using the continuity equation and a new form of momentum balance equation based on the application of the memory concept. The proposed model is rigorous in the sense that the fluid and formation properties are considered as space and time dependent. This model can be used in any crude oil flow through porous media. This model can also be applicable to any non-Newtonian fluid flow during an enhanced oil recovery (EOR) process. It is also useful where variable rock compressibility exists and is affected by the pressure decline during the production life of a reservoir.

Introduction

Rock and fluid properties play an important role during fluid flow in porous media. Majority of rock and fluid properties are functions of pressure and temperature. Formation rock is the fluid transport media where fluid properties change during any pressure disturbance or thermal action in the formation. As a result, the rock properties such as permeability, porosity, pore volume, and, sometimes, formation wettability are greatly influenced by fluid properties. The available literature shows that existing fluid flow models are based on the above mentioned pressure and temperature related properties for both rock and fluid. However, there are some other naturally occurred actions such as mineral precipitation, chemical reaction between fluid and rock media, existence of solid particles in fluid may have some influence in the behavior of fluid flow in porous media. All the parameters mainly affect the flow path in the formation. Based on the situation and pathway traveled, the pore space may squeeze or enlarge which varies in space and time. The above actions that take place in the pore network may change permeability and porosity.

In the existing simulators, the rock and fluid properties are considered to be variable only with space. However, it is very important to consider permeability and viscosity as a function of time for applications involving geothermal actions or mineral precipitation and chemical reactions. The effect of changes on rock properties may range from negligible to substantial, depending on characteristics of the formation and property of fluid itself. The normal fluid motion equations such as Darcy's law do not allow one to consider the variation of fluid and rock properties in a proper way. Darcy's law has its own limitations of consideration of homogeneous media and constant fluid and rock properties. Therefore, it seems that the best way is to modify the Darcy's law in such a way that it may create some options of using the variable rock and fluid properties during the development of theoretical fluid flow models.

The concept of using variable rock and fluid properties can be modeled using the notion of memory. Eventually, we can define the properties of rock and fluid that help to describe the phenomena of changing permeability and viscosity with time and space as memory. Therefore, rock property such as permeability change with time may be modeled using a notion of fluid memory concept. This is true for fluid property and viscosity as well. During the thermal EOR process, or if there exist a hot water bank in the reservoir formation, geothermal actions take place. Even, during polymer flooding with shear-thinning fluid (Hossain et al., 2007a), changing permeability and viscosity with time concept can be applied.

Hossain and Islam (2006) presented an extensive review of fluid memory on the available literature and models. They showed that different researchers tried to identify and define the fluid memory with different fluid properties such as stress, density, free energies and others. Some researchers also mentioned the effects of pathway and dependence on history of the fluid to define the memory (Arenzon et al. 2003; Shin et al. 2003; Zhang 2003). Memory is a function of time and space and forward time events depend on previous time events (Zhang 2003).

Shin et al. (2003) studied the non-equilibrium mechanism in the transport of inertia-dominated particles. They explained the problem of particle deposition inside a turbulent boundary layer. They pointed out that a turbulent boundary layer is seriously affected by a non-equilibrium memory effect due to the inertia of particles and mean shearing of the carrier flows. While maintaining a partial memory of their earlier motion, part of the mean and fluctuating velocities at previous times are activated. This is called the non-equilibrium memory effect. The memory effect is sensitively dependent on the intermediate diffusion time scale and this has to be chosen depending on the characteristic time scale of the mechanism of interest. This model is for homogeneous surrounding media and is not sufficient to describe the full impact of fluid memory on flow behavior and in media.

Within a porous medium, mobilization and subsequent flow of a fluid with a yield stress can be explained well when the notion of memory is introduced (Chen et al. 2005). Here the modeled fluid behavior is a Bingham plastic using single-capillary expressions for the mobilization and flow within a pore-throat. To incorporate dynamic effects due to the viscous friction of mobilization, researchers introduced the concept of invasion percolation with memory (IPM). This concept explains the macroscopic threshold (minimum pressure gradient) which directly follows from the geometry of the path, along which mobilization first occurs. The minimum threshold path (MTP) is connected through nearest neighboring paths between two given boundaries (or points), along which the sum of thresholds is the minimum possible. Fundamental to this concept is the notion that specific, local thresholds must be exceeded across a given pore throat. Within this threshold, the fluid is to be mobilized, and these thresholds are distributed in the network. IPM addressed static properties of various problems with yield stress. These are the onset of the mobilization of a single-phase Bingham fluid in a porous medium, or foam formation and propagation in porous media in the absence of flow effects. However, it did not account for dynamic (viscous flow) effects of fluid by which mobilization occurs. In their calculations, Chen et al. (2005) discovered that the flow in an open path did not affect the distribution of pressure, so the identification of higher-energy paths was strictly a static (quasi-thermodynamic) process. In the case of Bingham fluids, this would correspond to a vanishing plastic viscosity. They explained how IPM works, however they did not construct a model which represents the notion of fluid memory.

Gatti and Vuk (2006) studied the motion of a linear viscoelastic fluid in a two-dimensional domain with periodic boundary conditions for the asymptotic behavior. They consider an isotropic homogeneous incompressible fluid of Jeffrey's type where the Reynolds number is equal to one. They also assumed that density is independent of time. In addition they assumed that pressure and velocity are independent of time. These assumptions follow the conventional models.

Theoretical development of model

Consider a porous media of ABCDEFGH as represented by **Fig. 1**. A differential element of the reservoir, A'B'C'D'E'F'G'H' is defined for a single phase fluid flow. The mathematical model which governs the process of a reservoir is formulated based on some basic equations. To understand the flow of fluids in porous media we must be able to postulate these equations that govern the behavior of these fluids. Conservation of mass, rate equation and equation of state are some of them.

Conservation of mass. The element of reservoir shown in **Fig. 1(b)** is taken into account to apply mass conservation. The fluid is entering through the face A'D'E'H' and going out through the face B'C'F'G' in the x-direction. The differential equation for the conservation of mass is governed by (Bird et al. 2002)

$$\frac{\partial(\rho\, u_x)}{\partial x}+\frac{\partial(\rho\, u_y)}{\partial y}+\frac{\partial(\rho\, u_z)}{\partial z}=-\frac{\partial(\rho\, \phi)}{\partial t} \qquad (1)$$

Rate equation. Temperature variation plays a great role in the behavior of fluid flow through some porous media such as formation zone with hot water bank, geothermal area, and during thermal operations or polymer flooding. Fluids may precipitate minerals in the pores of the porous medium during the heavy oil with asphaltene flow. Some fluids (e.g. crude oil with other minerals) carry solid particles that may impede some of the pores. The precipitation and obstruction may reduce the pore size and thus decrease the permeability with time. Some fluid may have chemically reacting behavior with the medium which may enlarge the pore size. These phenomena can lead to local mineralization and permeability changes in space and time. However, if permeability reduces with time, the effect of fluid pressure at the boundary on the flow of fluid through the medium is delayed. The rock property changes with time give important information about the pressure response at the boundary of the reservoir. The delaying response of pressure with time due to permeability change over time in the fluid flow may be modeled using the concept of the memory.

Caputo (1999) modified Darcy's law by introducing the memory represented by a derivative of fractional order of differentiation (α) and ratio of the pseudopermeability of the medium with memory to fluid viscosity (η). These two parameters simulate the effect of a variation of permeability and viscosity over time. If the fluid flows in x-direction, the mass flow rate equation may be written as;

$$q_x = -\eta \, \rho_o \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right], \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where,

$$\frac{\partial^\alpha}{\partial t^\alpha}\{p(x,t)\} = [1/\Gamma(1-\alpha)] \int_0^t (t-\xi)^{-\alpha}\left[\frac{\partial}{\partial \xi} p(x,\ t)\right] d\xi$$

with $0 \leq \alpha < 1$.

It is clear that the memory introduced in Eq. (2) to describe the flow of the fluid implies the use of two parameters, namely α and η. These two parameters are used instead of the permeability and viscosity in conventional Darcy's law. Equation (2) can be written for fluid velocity which is related to pressure gradient as;

$$u_x = -\eta \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Substituting Eq. (3) in to continuity equation, Eq. (1), for 1D yields;

$$\frac{\partial}{\partial x}\left\{-\rho\,\eta\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right]\right\} = -\frac{\partial(\rho\,\phi)}{\partial t}$$

In the right-hand side of equation, porosity is a function of pressure and time. So the equation can be expanded to eliminate the porosity and density as:

$$\frac{\partial}{\partial x}\left\{-\rho\,\eta\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right]\right\} = \phi\frac{\partial \rho}{\partial t} + \rho\frac{\partial \phi}{\partial t} \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Equation of state. The equation of state is needed to express the density in terms of pressure. Most oil field liquid systems are considered to be slightly compressible. In this case, equation of state is:

$$\rho = \rho_o \, e^{c_f\,(p-p_o)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Now to derive the general governing equation with memory, it can be shown that porosity is related to the pressure and formation compressibility because the pressure difference between overburden and internal pore pressure is referred to as the effective overburden pressure. During pressure depletion operations, the internal pore pressure decreases which leads to increase of the effective overburden pressure. This increase causes two effects such as the reduction of bulk volume of the reservoir rock and expansion of sand grains within the pore spaces (Ahmed 2001). These two volume changes tend to reduce the pore space and, therefore, the porosity of the rock. For most petroleum reservoirs, the rock and bulk compressibility are considered small in comparison with the pore compressibility. The formation compressibility is the term commonly used to describe the total compressibility of the formation (c_s) which can be set equal to pore compressibility. The formation compressibility is defined as the fractional change in pore volume of the rock with a unit change in pressure at an isothermal condition and given by the following relationship:

$$c_s = -\frac{1}{V_p}\left(\frac{\partial V_p}{\partial p}\right)_T$$

where p is the pore pressure, v_p is the pore volume. The above equation can be expressed in terms of the porosity ϕ by noting that ϕ increases with the increase in the pore pressure;

$c_s = \frac{1}{\phi}\frac{\partial \phi}{\partial p}$, which becomes as, $\phi = e^{c_s\,(p - p_o)}$.

As $\phi = f(p)$ and $p = f(t)$, applying the chain rule of differentiation to $\partial\phi/\partial p$;

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p}\frac{\partial p}{\partial t}$$

so, $c_s = \frac{1}{\phi}\left(\frac{\partial \phi}{\partial t}/\frac{\partial p}{\partial t}\right)$

$$\phi\, c_s \frac{\partial p}{\partial t} = \frac{\partial \phi}{\partial t} \text{..} (6)$$

Substituting Eq. (6) into Eq. (4) yields;

$$\frac{\partial}{\partial x}\left(\rho\,\eta\,\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right]\right) = \phi\,\frac{\partial \rho}{\partial t} + \phi\,\rho\,c_s\frac{\partial p}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\rho\,\eta\,\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right]\right) = \phi\,\left\{\frac{\partial \rho}{\partial t} + \rho\,c_s\frac{\partial p}{\partial t}\right\} \text{............} (7)$$

Equation (7) is the basic partial differential equation with memory for any axial flow of a single phase fluid in a porous medium. By using the chain rule for differentiation in the LHS of Eq. (7) yields;

$$\frac{\partial \eta}{\partial x}\rho\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right] + \eta\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right]\frac{\partial \rho}{\partial x} + \rho\,\eta\,\frac{\partial}{\partial x}\left(\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right]\right)$$
$$= \phi\,\left\{\frac{\partial \rho}{\partial t} + \rho\,c_s\frac{\partial p}{\partial t}\right\} \text{...................................} (8)$$

Substituting Eq. (5) into Eq. (8) yields;

$$\frac{\partial \eta}{\partial x}\left[\rho_o\,e^{c_f\,(p - p_o)}\right]\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right]$$
$$+\,\eta\,\frac{\partial\left[\rho_o\,e^{c_f\,(p - p_o)}\right]}{\partial x}\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right]$$
$$+\,\eta\,\left[\rho_o\,e^{c_f\,(p - p_o)}\right]\frac{\partial}{\partial x}\left[\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial p}{\partial x}\right)\right]$$
$$= \phi\,\left\{\frac{\partial\left[\rho_o\,e^{c_f\,(p - p_o)}\right]}{\partial t} + c_s\left[\rho_o\,e^{c_f\,(p - p_o)}\right]\frac{\partial p}{\partial t}\right\}$$

If we assume that the compressibility of fluid is constant over the pressure range of p and p_0, the above equation can be differentiated with respect to x.

$$\frac{\partial \eta}{\partial x}\left[\rho_o\, e^{c_f\,(p - p_o)}\right]\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right]$$

$$+\,\eta\, \rho_o c_f\, e^{c_f\,(p - p_o)}\frac{\partial p}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right]$$

$$+\,\eta\, \rho_o\, e^{c_f\,(p - p_o)}\frac{\partial}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right]$$

$$=\ \phi\ \left[\rho_o\, c_f\, e^{c_f\,(p - p_o)}\frac{\partial p}{\partial t} +\ c_s\, \rho_o\, e^{c_f\,(p - p_o)}\ \frac{\partial p}{\partial t}\right]$$

$$\frac{\partial \eta}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right] +\ \eta\, c_f \frac{\partial p}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right] +\ \eta\ \frac{\partial}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right]$$

$$=\ \phi\left(c_f + c_s\right)\frac{\partial p}{\partial t}$$

$$\frac{1}{\eta}\frac{\partial \eta}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right] + c_f\,\frac{\partial p}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right]$$

$$+\frac{\partial}{\partial x}\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right] = \frac{\phi\, c_t}{\eta}\,\frac{\partial p}{\partial t} \qquad \ldots(9)$$

Using the definition of Eq. (2) to replace the fractional derivative in Eq. (9), we can substitute that into Eq. (9);

$$\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right) = [1/\Gamma(1-\ \alpha)]\ \int_0^t (t-\ \xi)^{-\,\alpha}\left[\frac{\partial}{\partial \xi}\left(\frac{\partial p}{\partial x}\right)\right] d\xi$$

with $0\ \leq\ \alpha <\ 1$.

As α is related to time and space, the above equation can be written as;

$$\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right) = \frac{\int_0^t (t-\ \xi)^{-\,\alpha}\left(\frac{\partial^2 p}{\partial \xi\, \partial x}\right) d\xi}{\Gamma(1-\ \alpha)}$$

Substituting the above equation in Eq. (9) yields;

$$\frac{1}{\eta}\frac{\partial \eta}{\partial x}\left[\frac{\int_0^t (t-\xi)^{-\,\alpha}\left(\frac{\partial^2 p}{\partial \xi\, \partial x}\right) d\xi}{\Gamma(1-\alpha)}\right] + c_f\,\frac{\partial p}{\partial x}\left[\frac{\int_0^t (t-\xi)^{-\,\alpha}\left(\frac{\partial^2 p}{\partial \xi\, \partial x}\right) d\xi}{\Gamma(1-\alpha)}\right]$$

$$+\frac{\partial}{\partial x}\left[\frac{\int_0^t (t-\xi)^{-\,\alpha}\left(\frac{\partial^2 p}{\partial \xi\, \partial x}\right) d\xi}{\Gamma(1-\alpha)}\right] = \frac{\phi\, c_t}{\eta}\,\frac{\partial p}{\partial t} \qquad \ldots(10)$$

Equation (10) represents the general form of diffusivity equation with memory. Equation (10) is a non-linear diffusivity equation considering fluid memory for an axial flow of any single phase fluid in porous media. Equation (10) is strictly non-linear because both the first part (due to dependence of η on pressure) and the second part are non-linear. In addition, the third part consists of a derivative of a pressure derivative with memory part. The fourth part consists of pressure derivative with time, η compressibility and porosity multiplication which also makes this term nonlinear. If the effect of memory is neglected (i.e. $\alpha = 0$), this equation reduces to the conventional form of diffusivity equation (Hossain et al. 2008).

Definition of the composite variable, η

The most important parameter of Eq. (10) is the variable η, which depends on formation and fluid properties. Thus it is very essential to define the type of crude oil and the type of formation because different crude oils have different viscosity and properties. This is true also for reservoir formation. Different reservoirs have different types of rocks, heterogeneity and complexity. Therefore, the use of the proposed diffusivity equation with memory (Eq. (10)) is more sensitive to its parameters definition. To solve this equation as an example case, Almehaideb (2003) viscosity models and İscan et al. (2006) permeability correlation are used. To solve the diffusivity equation with memory, other correlations such as Beal's (1946), Chew and Conally's (1959), Beggs and Robinson (1975), Vazquez and Beggs (1980), Khan et al. (1987) and Petrosky and Farshad (1995) can be easily used in computing viscosity. The viscosity models developed by Almehaideb (2003) are based on regional data, such as Standing's for California crudes, Petrosky and Farshad's for Gulf of Mexico crudes, and Glaso's for North Sea crudes. He cited that the use of these regional correlations is more appropriate for crudes from the same basins for which the correlation is derived. He also used other correlations, such as the Vasquez and Beggs correlation, which are based on data from a very large number of samples coming from multiple regions.

The properties of the crudes used in this computation had the following ranges of measured properties (Almehaideb 2003):

$30.9 < API < 48.6$

$190 < T < 306\,F$

$501 < p_b < 4822\,psia$

$1.142 < B_o < 3.562\ bbl/stb$

$128 < R_s < 3871$

$0.746 < \gamma_g < 1.116$

Oil viscosity above the bubble point:

$$\mu_{ab} = \mu_{ob} e^{8.422\times10^{-5}\,(p - p_b)} \quad \text{.........................(11)}$$

where,

$$\mu_{ob} = 6.59927 \times 10^5 R_s^{-0.597627} T^{-0.941624} \times \gamma_g^{-0.555208} API^{-1.487449}$$

$$p_b = -620.592 + 6.23087 \frac{R_s\,\gamma_o}{\gamma_g\,B_o^{1.38559}} + 2.89868\,T$$

$$B_o = B_{ob} e^{-C_o(p - p_b)}$$

$$B_{ob} = 1.122018 + 1.410 \times 10^{-6} \frac{R_s T}{\gamma_o^2}$$

$$c_o = (-70603.2 + 98.404\,R_s + 378.266\,T - 6102.03\,\gamma_g + 755.345\,API)/(p + 3755.53)$$

The above equations are based on oil field units such as oil formation volume factor (B_o), rb/stb; oil compressibility (C_o), psi^{-1}; oil formation volume factor at bubble point(B_{ob}), rb/stb; oil viscosity at the bubble point(μ_{ob}), cp; oil viscosity above the bubble point(μ_{ab}), cp; solution gas oil ratio (R_s), scf/stb; temperature of crude oil (T), ^{o}F; specific gravity of gas (γ_g), lb_m/ft^3; specific gravity of oil(γ_o), lb_m/ft^3; and API oil gravity (*API*), ^{0}API.

İscan et al. (2006) permeability correlation is based on the data which are correlated by a power-law decaying equation. The equation was originally proposed by Civan (2000) for pore-throat plugging. The equation was extended and verified by İscan and Civan (2005) for plugging criteria in formation damage. Finally, İscan et al. (2006) modified their equation by changing the parameters and coefficients. Therefore, the independent correlation obtained directly applicable for stress–permeability relationships is given as:

$$k = A\,(p)^{-B} + C \text{.....................................(12)}$$

where,

A, B, C = Empirical coefficient.

İscan et al. (2006) presented both the stress–strain relationships with the permeability of the reservoir rock samples obtained from experimental data on rock samples. The rocks were assumed homogeneous and isotropic when they are not under mechanical stress. They also analyzed stress–strain applications, porosity, and permeability measurements and permeability variation with effective overburden stress with four limestone samples from southeast Turkey, i.e. limestone-1, limestone-2, limestone-3, limestone-4, respectively. Based on their findings, the empirical correlation (Eq. (12)) for limestone-3 is given as:

$$k = 0.003\,(p)^{-0.31} + 0.0105 \text{........................(13)}$$

where,

p = pressure, psi

k = permeability, Darcy

The definition of η can be expanded in the form of permeability, viscosity and time as:

$$\eta = \frac{m^3 s^{1+\alpha}}{kg} = \left[\frac{m^2}{kg/m\,s}\right](s^{\alpha}) = \frac{k}{\mu_{ab}}(t)^{\alpha}$$

Substituting Eqs. (11) and (13) into the above relation where μ_{ab} unit is replaced from oil field unit to SI unit.

$$\eta = \frac{k}{\mu_{ab}}(t)^{\alpha} = \frac{0.003\,(p)^{-0.31} + 0.0105}{\mu_{ob} e^{8.422\times10^{-5}\,(p - p_b)}}\,(t)^{\alpha} \text{..........(14)}$$

Numerical Solution

Numerical simulation using finite differences has been selected to model the diffusivity equation with memory (Eq. (10)). The steps to model the equation are outlined in the solution procedure section. A reservoir of length (L = 5000.0 m), width ($w = 300\,m$) and height, ($H = 20\,m$) has been considered. The initial porosity(ϕ_o) and permeability (k_o) of the reservoir are 30% and $15 \times 10^{-15}\,m^2$, respectively. The reservoir is completely sealed and produces at a constant rate, for which the initial pressure is $p_o = 48{,}263{,}299\,pa$. The fluid is assumed to be API 31 gravity crude oil with the initial properties such as oil compressibility; $c_o = 1.7404421 \times 10^{-9}\ 1/pa$, $\mu_o = 0.12\,pa.s$ at 200°F (other fluid properties are presented in solution procedure section). Formation rock compressibility, $c_s = 5.80147 \times 10^{-10}\ 1/pa$, initial production rate, $q_i = 2.024144 \times 10^{-4}\ m^3/s$ have been considered. It is also deemed as fractional order of differentiation, $\alpha = 0.2 \sim 0.8$, $\Delta x = 20\,m$, $\Delta t = 7200\,s$, and $t = 10$ months. In solving this diffusivity equation with memory, trapezoidal method is used. All computation is carried out by Matlab 6.5.

The direct use of Eqs. (11), (13) and (14) into Eq. (10) has some restriction because of unit conversion. Therefore, it is necessary first to transform the units. In the computation with Eq. (11), $R_s = 129.0\ scf/stb$, $T = 200\ {}^oF$, $\gamma_g = 0.748\ lb_m/ft^3$ and $\gamma_o = 0.862\ lb_m/ft^3$ are considered. When μ_{ab} is used during the computation of Eq. (10), it is transformed into SI unit as pa-s and Eq. (11) can be written as;

$$\mu_{ab} = \mu_{ob} e^{8.422\times10^{-5}(p - p_b)} \times 10^{-3} \quad \ldots\ldots\ldots\ldots(15)$$

In this study, SI unit is considered, therefore, it is also necessary to transform Eq. (13) into SI as pressure in pa (1 psi = 6894.76 pa) and permeability in m^2 (1 darcy = 10^{-12} m^2), then the equation becomes as,

$$k = [0.003\ (p/6894.76)^{-0.31} + 0.0105] \times 10^{-12}$$

$$k = [3.0\ (p/6894.76)^{-0.31} + 10.5] \times 10^{-15} \quad \ldots(16)$$

Substituting Eqs. (15) and (16) into Eq. (14) where μ_{ab} and k units are replaced from oil field unit to SI unit.

$$\eta = \frac{k}{\mu_{ab}}(t)^{\alpha} = \frac{[3.0\ (p/6894.76)^{-0.31}+10.5]\times 10^{-15}}{\mu_{ob} e^{8.422\times10^{-5}(p - p_b)} \times 10^{-3}}\ (t)^{\alpha}$$

$$\eta = \frac{[3.0\ (p/6894.76)^{-0.31}+10.5]\times 10^{-12}}{\mu_{ob} e^{8.422\times10^{-5}(p - p_b)}}\ (t)^{\alpha} \quad \ldots\ldots\ldots(17)$$

To solve the Eq. (10) based on the above data, the equation can be written as

$$\frac{1}{\eta}\frac{\partial \eta}{\partial x} Z + c_f \frac{\partial p}{\partial x} Z + \frac{\partial Z}{\partial x} = \frac{\phi\, c_t}{\eta}\frac{\partial p}{\partial t} \quad \ldots\ldots\ldots\ldots(18)$$

where, $Z = \dfrac{\int_0^t (t-\xi)^{-\alpha} \frac{\partial^2 p}{\partial \xi \partial x} \partial \xi}{\Gamma(1-\alpha)}$

In solving Eq. (18), initially it is assumed that Darcy diffusivity equation is valid and the variation of the permeability with space and time is constant. The initial computation then used to solve Eq. (18) where reservoir formation is homogeneous (e.g. precipitation is constant in every place in x-direction) and isotropic (e.g. precipitation is not constant in every place in other directions). Permeability is varied only with respect to time globally not locally. As a result, $\eta = \eta(p,\ t)$ and $Z = Z(p,\ t)$. It is also assumed that all coefficient approximated using pressure value at time step *n,* for the grid point *i*. Therefore, $\eta = \eta(p_i^n)$ and $Z = Z(p_i^n)$.

Equation (18) can be solved using two concepts. These are based on space and pressure derivative concepts.

Space derivative concept. Equation (18) can be expanded using the space derivative concept as

$$\phi\, c_t \frac{\partial p}{\partial t} = \frac{\partial \eta}{\partial x} Z + \eta \frac{\partial Z}{\partial x} + c_f\, Z\, \eta \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} = \frac{1}{\phi\, c_t}(C_1\, Z + C_2\, \eta) + \frac{c_f\, Z\, \eta}{\phi\, c_t}\frac{\partial p}{\partial x} \quad \text{..................(19)}$$

where, $C_1 = \frac{\partial \eta}{\partial x}$ and $C_2 = \frac{\partial Z}{\partial x}$.

Equation (19) is discretized using finite difference method as

$$\frac{p_i^{n+1} - p_i^n}{\Delta t} = \frac{1}{\phi_i^n\, c_t}(C_{1\,i}^n\, Z_i^n + C_{2\,i}^n\, \eta_i^n) + \frac{c_f\, Z_i^n\, \eta_i^n}{\phi_i^n\, c_t}\frac{p_{i+1}^n - p_i^n}{\Delta x}$$

$$p_i^{n+1} = p_i^n + \frac{\Delta t}{c_t}\left(\frac{C_{1\,i}^n\, Z_i^n + C_{2\,i}^n\, \eta_i^n}{\phi_i^n}\right) + \frac{\Delta t\, c_f}{\Delta x\, c_t}\frac{Z_i^n\, \eta_i^n}{\phi_i^n}(p_{i+1}^n - p_i^n)$$

$$p_i^{n+1} = p_i^n + a + b(p_{i+1}^n - p_i^n) \quad \text{..................(20)}$$

In Eq. (20),

$a = a_1 a_2$, $b = a_3 a_4$, $a_1 = \frac{\Delta t}{c_t}$, $a_2 = \frac{C_{1\,i}^n\, Z_i^n + C_{2\,i}^n\, \eta_i^n}{\phi_i^n}$, $a_3 = \frac{\Delta t\, c_f}{\Delta x\, c_t}$ and $a_4 = \frac{Z_i^n\, \eta_i^n}{\phi_i^n}$

Pressure derivative concept. Equation (18) can be expanded using the pressure derivative concept as

$$\phi\, c_t \frac{\partial p}{\partial t} = \frac{\partial \eta}{\partial x} Z + \eta \frac{\partial Z}{\partial x} + c_f\, Z\, \eta \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} = \frac{1}{\phi\, c_t}\left(\frac{\partial \eta}{\partial x} Z + \eta \frac{\partial Z}{\partial x}\right) + \frac{c_f\, Z\, \eta}{\phi\, c_t}\frac{\partial p}{\partial x}$$

Applying the chain rule in the above equation, it becomes as;

$$\frac{\partial p}{\partial t} = \frac{1}{\phi\, c_t}\left(\frac{\partial \eta}{\partial p}\frac{\partial p}{\partial x} Z + \eta \frac{\partial Z}{\partial p}\frac{\partial p}{\partial x}\right) + \frac{c_f\, Z\, \eta}{\phi\, c_t}\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} = \frac{1}{\phi\, c_t}\left\{\frac{\partial \eta}{\partial p} Z + \eta \frac{\partial Z}{\partial p} + c_f\, Z\, \eta\right\}\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} = \frac{1}{\phi\, c_t}\left\{D_1\, Z + \eta D_2 + c_f\, Z\, \eta\right\}\frac{\partial p}{\partial x} \quad \text{................(21)}$$

where, $D_1 = \frac{\partial \eta}{\partial p}$ and $D_2 = \frac{\partial Z}{\partial p}$.

Equation (21) is discretized using finite difference method as

$$\frac{p_i^{n+1} - p_i^n}{\Delta t} = \frac{1}{\phi_i^n\, c_t}\left(D_{1\,i}^n\, Z_i^n + D_{2\,i}^n\, \eta_i^n + c_f\, Z_i^n\, \eta_i^n\right)\frac{p_{i+1}^n - p_i^n}{\Delta x}$$

$$p_i^{n+1} = p_i^n + \frac{\Delta t}{2\,\Delta x\, c_t}\left(\frac{D_{1\,i}^n\, Z_i^n + D_{2\,i}^n\, \eta_i^n + c_f\, Z_i^n\, \eta_i^n}{\phi_i^n}\right)(p_{i+1}^n - p_{i-1}^n)$$

$$p_i^{n+1} = p_i^n + a\, b(p_{i+1}^n - p_{i-1}^n) \quad \text{......................(22)}$$

In Eq. (22),

$$a = \frac{\Delta t}{2\,\Delta x\, c_t},\quad b = \left(\frac{D_{1\,i}^n\, Z_i^n + D_{2\,i}^n\, \eta_i^n + c_f\, Z_i^n\, \eta_i^n}{\phi_i^n}\right)$$

Solution Procedure

To solve the diffusivity equation with memory (Eq. (20) or Eq. (22)), the following steps were introduced;

i) Conventional Darcy's law is used for initial computation of pressure variation with time and space;

ii) To initialize the solution, conventional diffusivity equation is considered where no memory is used;

iii) Solve the Darcy diffusivity equation to find out the pressure (p) as a function of time and space. Also computed is the $\partial^2 p/\partial x \partial t$ term;

iv) Finite difference method is used to solve the diffusivity equation and SP-line function is used to compute the $\partial^2 p/\partial x \partial t$;

v) Based on the above steps, the initial Z-value is computed for different α values;

vi) A logical relationship of η and pressure (p) based on the definition of η (i.e., $\eta = m^3 s^{1+\alpha}/kg = m^3 s. s^{\alpha}/kg$) is chosen. The choice depends on the fluid and formation type. η is sensitive to fluid and rock type. The definition of η varies with fluid (i.e., crude oil) and reservoir rock type. Therefore, it is necessary to identify which correlation should be used to represent the pressure dependent η value. In this computation, Almehaideb (2003) viscosity correlations and İscan et al. (2006) permeability correlations were used;

vii) At the initial condition (t = 0), $\alpha = 0$, therefore, η_o is calculated as: $\eta_o = m^2 s^{\alpha}/(kg/m\ s)$ $= k_o \times s^{\alpha}/\mu_o = k_o/\mu_o$;

viii) Based on the above steps, solve the Eq. (20) or Eq. (22) for pressure distribution in space and time;

ix) Repeat the all steps until the convergence of pressure distribution (i.e., $p_n - p_{n-1} < \in$)

Conventional diffusivity equation. To initialize the solution of Eq. (20) or Eq. (22), the pressure distribution is assumed to be modeled through the diffusivity equation in porous media. This equation has been derived by combining the continuity equation with the Darcy's law as the momentum equation.

$$\frac{\partial^2 p}{\partial x^2} = \frac{\phi\, \mu_0 c_t}{k}\frac{\partial p}{\partial t} \qquad (23)$$

It is defined that $a_2 = \phi\, \mu_o\, c_t/k$ in Eq. (23) and therefore, it may be written that

$$\frac{\partial p}{\partial t} = \frac{1}{a_2}\frac{\partial^2 p}{\partial x^2}$$

$$p_i^{n+1} = p_i^n + \frac{1}{a_2}\frac{\Delta t}{(\Delta x)^2}(p_{i+1}^n - 2p_i^n + p_{i-1}^n)$$

$$p_i^{n+1} = (1 - 2a_1)p_i^n + a_1(p_{i+1}^n + p_{i-1}^n), \qquad (24)$$

where, $h = \Delta t/(\Delta x)^2$ and $a_1 = h/a_2$. This equation can be solved using the specified initial and boundary conditions. In solving Eq. (24), the following initial and boundary conditions are considered.

Initial condition. $p(x,0) = p_i$

Boundary condition. The external boundary is considered as no flow boundary i.e. closed reservoir. The interior boundary is considered as a constant production rate boundary.

The outer boundary. According to Darcy's law,

$$u_{x=L} = -\frac{k}{\mu}\frac{\partial p}{\partial x} = 0, \Rightarrow \left.\frac{k}{\mu}\frac{\partial p}{\partial x}\right|_{x=L} = 0, \Rightarrow \left.\frac{\partial p}{\partial x}\right|_{x=L} = 0$$

$$\frac{p_{i+1}^n - p_{i-1}^n}{2\,\Delta x} = 0, \Rightarrow\ p_{i+1}^n = p_{i-1}^n$$

The inner boundary. According to Darcy's law,

$$q_{x=0} = A\,u_x = -\frac{k\,A_{yz}}{\mu}\frac{\partial p}{\partial x}$$

$$q = \left(-\frac{k\,A_{yz}}{\mu}\right)\left(\frac{p_{i+1}^n - p_{i-1}^n}{2\,\Delta x}\right) = \left(-\frac{k\,A_{yz}}{2\mu\,\Delta x}\right)(p_{i+1}^n - p_{i-1}^n)$$

$$q = e_1(p_{i+1}^n - p_{i-1}^n)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

where $e_1 = -\frac{k\,A_{yz}}{2\mu\,\Delta x}$.

Equation (25) can be written as $p_{i+1}^n = p_{i-1}^n + q/e_1$. This is used in the solution of Eq. (20) or Eq. (22).

Results and Discussion

Fig. 2 presents the variation of pressure with distance from the wellbore towards the outer boundary of the reservoir based on Darcy's diffusivity equation. Pressure response increases towards the boundary with time. The maximum pressure drop is in the wellbore and it gradually increases up to the initial reservoir pressure with distance. Pressure drop reaches at initial pressure at a distance approximately 850 m after 10 month whereas it reaches at around 200 m after 1 month of production.

The effects of Z-values with distance are depicted in **Fig. 3**. The variation is shown for 1 and 10 months where Darcy diffusivity and $\alpha = 0.2$ are considered during computation. Z-values increase very fast around the wellbore and reach its pick at a distance of 120 m and 170 m for 1 and 10 months respectively. The decreasing trend begins after the pick and becomes zero after 350 m and 1200 m. The effects of z-values increase at a wider range of reservoir area from the wellbore with the increase of production life of reservoir. Therefore, it can be concluded that at the beginning of production life, there is a great impact around the wellbore and as time passes, this impact affects throughout the reservoir which becomes 0 at the outer boundary of the reservoir.

Variation of η with distance. Fig. 4 shows the variation of η with distance from the wellbore towards the outer boundary of the reservoir for $\alpha = 0.1$. This figure compares the Darcy diffusivity equation and proposed diffusivity equation with memory. **Fig. 4(a)** is plotted for 1 month after the start of production and **Fig. 4(b)** is for 2 months. There is no difference of change of η with respect to distance at the initial stage of the reservoir production (Fig. 4(a)). However, the difference of changing η becomes significant with production time (Fig. 4(b). η variation is more sensitive with time and around the wellbore of the reservoir which is captured by the notion of "memory". The numerical value of η is substantially reducing when production continues with time and the effects spread through the reservoir with time (Fig. 4(b)) which is only capturing by proposed model. So, it can be concluded that there is a strong effects of "memory" in describing fluid flow through porous media.

Variation of η with reservoir pressure. The variation of η with reservoir pressure is depicted by **Fig. 5** for the proposed model. The trend of the curve is highly non-linear and it is an elliptical shape. At the beginning of pressure, η variation is very high and it starts to decrease with the increase of pressure. The variation of η becomes stable at very high pressure where there is no change of η value.

The pressure response with respect to the variation of η is compared for both Darcy model and proposed model in **Fig. 6** when $\alpha = 0.1$. **Fig. 6(a)** is plotted when reservoir production time is considered as 1 month and **Fig. 6(b)** is shown the comparison for 2 months of production life. It is very interesting that after 1 month of production, there is no pressure variation over time or over η when proposed model is used to calculate the pressure variation throughout the reservoir (Fig. 6(a)). However, the variation of pressure starts to decline with η after one month. This only due to the memory effects on fluid and rock which is not possible to capture by conventional diffusivity equation. The declining pressure variation response is dominant with time. As production time of the reservoir increases, the effect of memory becomes dominant and gives a substantial difference in pressure response (Fig. 6(b)).

Porosity change with distance. The variation of porosity over distance from the wellbore towards the outer boundary of the reservoir is shown in Figure 7 using the conventional diffusivity equation and proposed model where $\alpha = 0.1$ is considered. At the beginning of the production, there is no substantial difference of porosity change with the initial porosity of 0.30 **(Fig. 7(a))**. Use of Darcy diffusivity equation, does not give any significant variation of porosity. However, use of proposed model gives a difference of porosity value of 0.292 which represent a contribution of memory effect on rock property. This change is almost same throughout the reservoir. The variation of porosity becomes significant with the hydrocarbon production life. **Fig. 7(b)** shows the variation of porosity over time at different reservoir position.

Conclusions

A diffusivity equation with rock/fluid memory has been developed by invoking time-dependence to permeability and viscosity. The resulting highly non-linear theoretical model has an option of triggering the memory variable, depending on the applicability. These equations were solved in their non-linear form, showing the difference in prediction between the conventional diffusivity equation and the new memory-induced diffusivity. The results indicate that this model can be used for a wide range of applications. The variation of η for different reservoir parameters is identified and the effects of "memory" is shown using both Darcy model and proposed model. The findings of this research establish the contribution of memory in reservoir fluid flow through porous media.

Nomenclature

A_{yz} = Cross sectional area of rock perpendicular to the flow of flowing fluid, m^2

c_f = $c_o + c_w$ = total fluid compressibility of the system, $1/pa$

c_s = formation rock compressibility of the system, $1/pa$

c_t = $c_f + c_s$ = total compressibility of the system, $1/pa$

c_w = formation water compressibility of the system, $1/pa$

k = initial reservoir permeability, m^2

L = distance between production well and outer boundary along x direction, m

p = pressure of the system, N/m^2

p_i = initial pressure of the system, N/m^2

p_o = a reference pressure of the system, N/m^2

q_i = Au = initial volume production rate, m^3/s

q_x = fluid mass flow rate per unit area in x-direction, kg/m^2s

t = time, s

u = filtration velocity in x direction, m/s

u_x = fluid velocity in porous media in the direction of x axis, m/s

α = fractional order of differentiation, dimensionless

ρ = density at pressure p, kg/m^3

ρ_o = density at a reference pressure p_o, kg/m^3

ϕ = porosity of fluid media at pressure p, m^3/m^3

ϕ_o = porosity of fluid media at reference pressure p_o, m^3/m^3

μ = fluid dynamic viscosity, pas

η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3s^{1+\alpha}/kg$

ξ = a dummy variable for time i.e. real part in the plane of the integral, s

Acknowledgements

The authors would like to thank the Atlantic Canada Opportunities Agency (ACOA) for funding this project under the Atlantic Innovation Fund (AIF). The first author would also like to thank Natural Sciences and Engineering Research Council of Canada (NSERC) for funding.

References

Ahmed, T. ed. 2001. *Reservoir Engineering Handbook*, Second edition, Boston, USA: Gulf Professional Publishing.

Almehaideb, R.A. 2003. Improved Correlations for Fluid Properties of UAE Crude Oils. *Petroleum Science and Technology* **21**(11 & 12): 1811–1831.

Arenzon, J.J., Levin, Y., and Sellitto, M. 2003. Slow Dynamics Under Gravity: a Nonlinear Diffusion Model. *Physica A* **325**: 371–395.

Beal, C. 1946. The Viscosity of Air, Natural Gas, Crude Oil and its Associated Gases at Oil Field Temperature and Pressure. *Trans. AIME* **145**: 94–112.

Beggs, H.D., Robinson, J.R. 1975. Estimating the Viscosity of Crude Oil Systems. *JPT* **27**:1140–1141.

Bird, R.B. Stewart, W.E. Lightfoot, E.N. ed. 2002. *Transport Phenomena*. 2nd edition, New York, USA: J. Wiley.

Caputo, M. 1999. Diffusion of Fluids in Porous Media with Memory. *Geothermics* **23**: 113-130.

Chen, M., Rossen, W., and Yortsos, Y.C. 2005. The Flow and Displacement in Porous Media of Fluids with Yield Stress. *Chemical Engineering Science* **60**: 4183 – 4202.

Chew, J., Conally, Jr. C. A. 1959. A Viscosity of Crude Oil Systems. *Trans. AIME* **216**: 23–25.

Civan, F. ed. 2000. *Reservoir Formation Damage*. 742, Houston, Texas: Gulf Publishing Company.

Gatti, S. and Vuk, E. 2006. Singular Limit of Equations for Linear Viscoelastic Fluids with Periodic Boundary Conditions. *International Journal of Non-Linear Mechanics*, article in press.

Hossain, M E., Mousavizadegan, S.H. and Islam, M.R. 2008. A Novel Fluid Flow Model with Memory for Porous Media Applications. paper 108, will be presented at Mechanics of Time Dependent Materials Conference of 2008 (MTDM 2008), Monterey, California, USA, March 30–April 4.

Hossain, M.E., Liu, L. and Islam, M.R. 2007a. Inclusion of the Memory Function in Describing the Flow of Shear-thinning Fluids in Porous Media. *Physics Letters A*, submitted.

Hossain, M.E. and Islam, M.R. 2006. Fluid Properties with Memory–A Critical Review and Some Additions. CIE – 00778, *Proc.* 36th International Conference on Computers and Industrial Engineering, Taipei, Taiwan, June 20-23.

Khan, S.A., Al-Marhoun, M.A., Duffuaa, S.O., Abu-Khamsin, S.A. 1987. Viscosity Correlations for Saudi Arabian Crude Oils. SPE 15720 presented at the 5th SPE Middle East Oil Show held in Manama, Bahrain, 7–10 March.

Petrosky, G.E.Jr., Farshad, F.F. 1995. Viscosity Correlations for Gulf of Mexico Crude Oils. SPE 29468 presented at SPE Production Operations Symposium, Oklahoma city, Dallas, Texas, USA, 2–4 April.

Shin, M., Kim, D.S., and Lee, J.W. 2003. Deposition of Inertia-dominated Particles Inside a Turbulent Boundary Layer. *International Journal of Multiphase Flow* **29**: 893–926.

Vasquez, M., Beggs, H.D. 1980. Correlations for Fluids Physical Property Prediction. JPT **32**:968–970.

Zhang, H.M. 2003. Driver Memory, Traffic Viscosity and a Viscous Vehicular Traffic Flow Model. *Transportation Research Part B* **37**: 27–41.

İscan, A.G., Civan, F. 2005. Correlation of Criteria for Perforation and Pore Plugging by Particles. *Journal of Porous Media*, accepted on September.

İscan, A.G., Kök, M.V. and Bagci, A.S. 2006. Estimation of Permeability and Rock Mechanical Properties of Limestone Reservoir Rocks Under Stress Conditions by Strain Gauge. *Journal of Petroleum Science and Engineering* **53**: 13 – 24.

SI Metric Conversion Factors

$\mathrm{cp} \times 1.0^{*}$	$\mathrm{E} - 03 = \mathrm{pa.s}$	
$\mathrm{psi} \times 6.894\,757$	$\mathrm{E} + 03 = \mathrm{pa}$	
$\mathrm{bbl} \times 1.589\,873$	$\mathrm{E} - 01 = \mathrm{m}^3$	
$\mathrm{lb_m} \times 4.535\,924$	$\mathrm{E} - 01 = \mathrm{kg}$	
$\mathrm{ft} \times 3.048^{*}$	$\mathrm{E} - 01 = \mathrm{m}$	
$\mathrm{ft}^3 \times 2.831\,685$	$\mathrm{E} - 02 = \mathrm{m}^3$	
$^{0}\mathrm{F}$	$(°\mathrm{F} + 459.67)/1.8$	$= \mathrm{k}$
$\mathrm{md} \times 1.0$	$\mathrm{E} - 15 = \mathrm{m}^2$	

M. Enamul Hossain is a PhD Candidate in Civil and Resource Engineering and MBA student at Dalhousie U., Canada. Hossain received his BSc in Mechanical Engineering and MSc in Petroleum and Mineral Resources Engineering from Bangladesh U. of Eng. and Tech. (BUET). He has a remarkable academic career. He got five awards throughout his academic and professional career. He published over thirty technical papers on reservoir engineering, EOR, drilling activities and risk management in different referred journals and conferences. His research interest is in the Field of Petroleum Engineering and Risk Management. Hossain is an active member of SPE, Engineers Institution Bangladesh. Moreover, he has 10 years professional experience in the petroleum industry.

S. H. Mousavizadegan is currently an Assistant Professor of Mechanical Engineering at Amir Kabir University in Tehran. He received his PhD degree from Dalhousie University in 2005, following which he spent several years as a post-doctoral fellow under the supervision of M.R. Islam. Dr. Mousavizadegan has written many papers on reservoir simulation and recently co-authored a book on knowledge-based reservoir simulation to be published in 2008.

M. Rafiqul Islam is a Professor of Petroleum Engineering at Dalhousie U. where he joined as in 2000 as the Killam Chair in Petroleum Engineering. Islam holds a Dip. Ing. D'Etat degree from the Algerian Petroleum Inst. as well as Masters and PhD degrees from U. of Alberta. He is the Editor-in-Chief of *The Journal of Nature Science and Sustainable Technology*, and is on the editorial board of a number of journals. He has served SPE through various committees and positions, including the SPE Editorial Board (SPEREE), ATCC technical committees (Drilling and Production Operations), the SPE Technical Forum committee, as an SPE Faculty sponsor (at two universities), and as Director of Executive Committee (Atlantic SPE) and in other capacities as well. At present, he is introducing an SPE Student chapter at Dalhousie U.

Appendix A: Figures

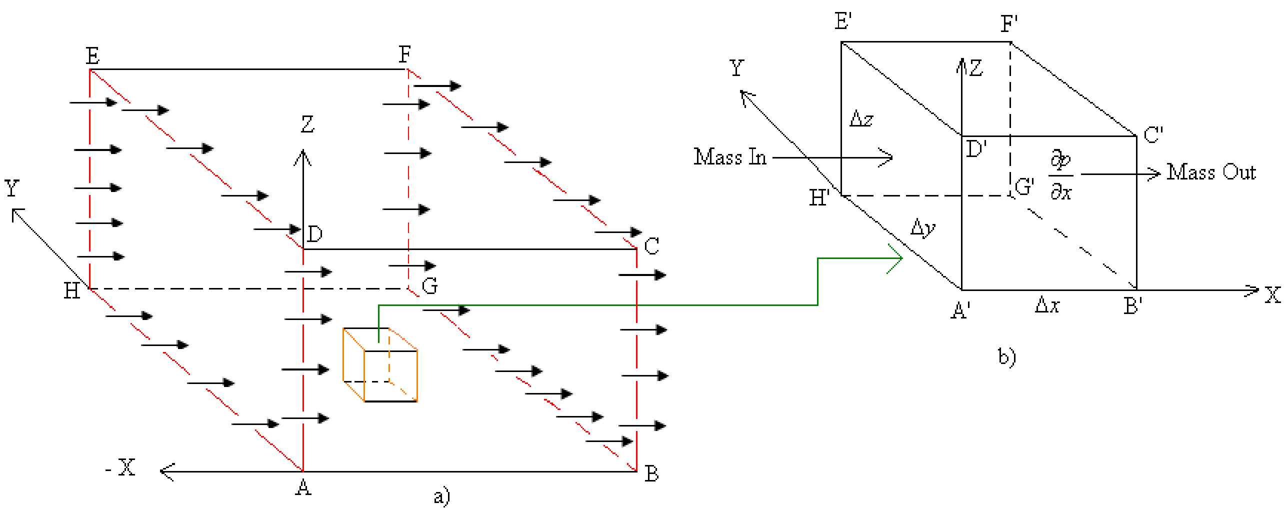

Fig. 1– A rectangular block of a porous media

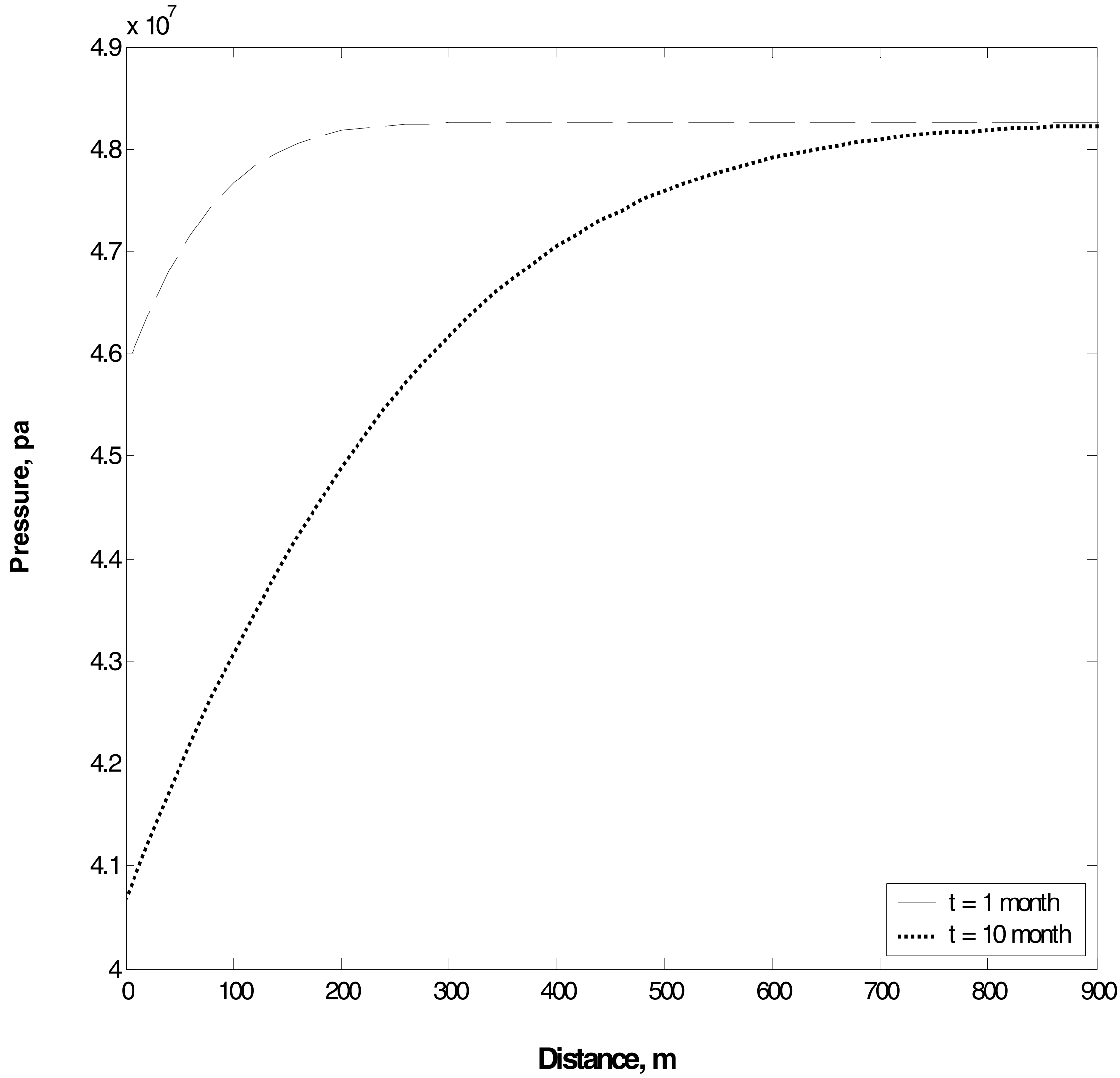

Fig. 2–Pressure variation with distance based on Darcy diffusivity equation

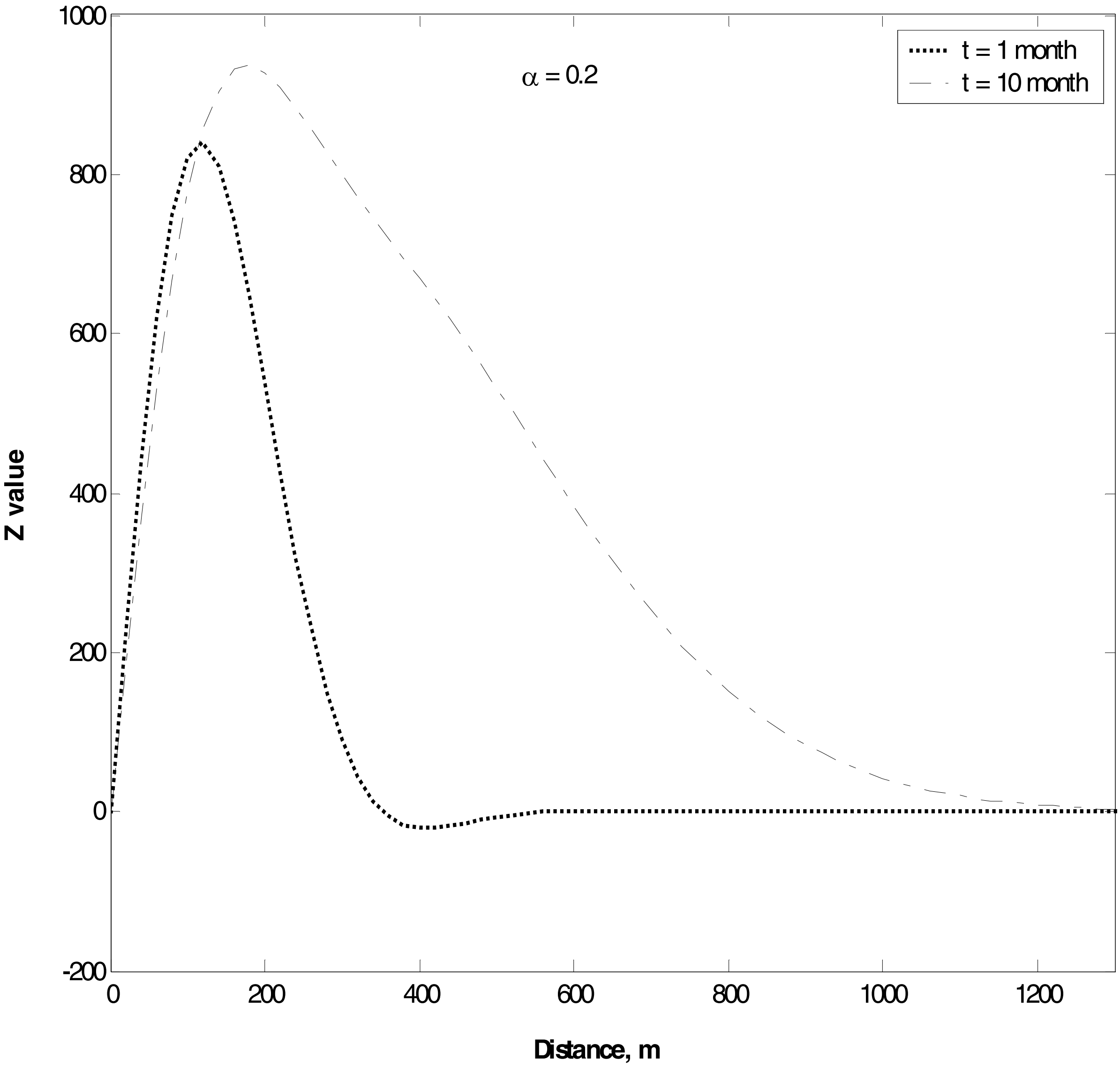

Fig. 3 – Z variation with distance based on Darcy diffusivity equation for $\alpha = 0.1$

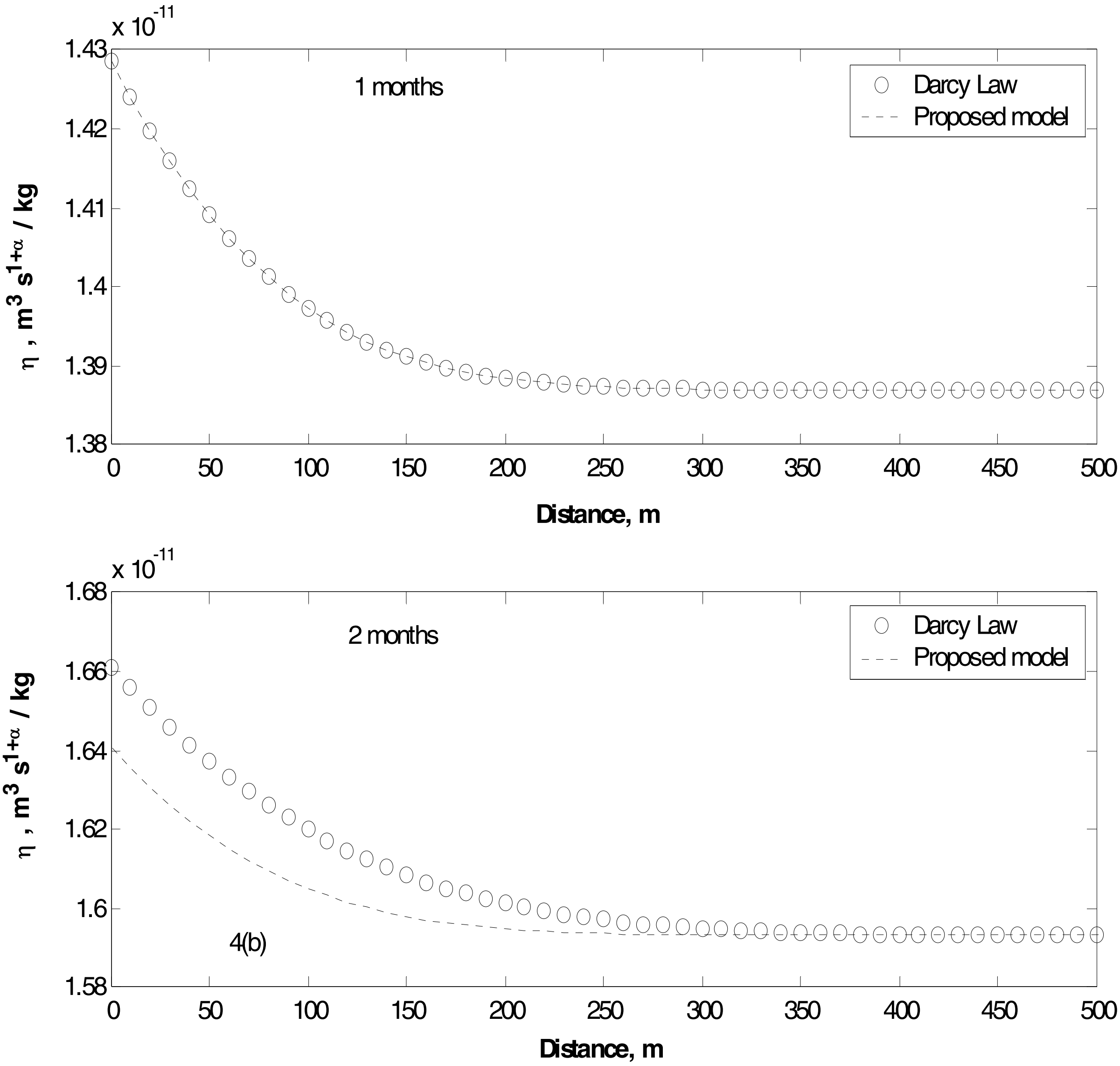

Fig. 4– η variations with distance for $\alpha = 0.1$

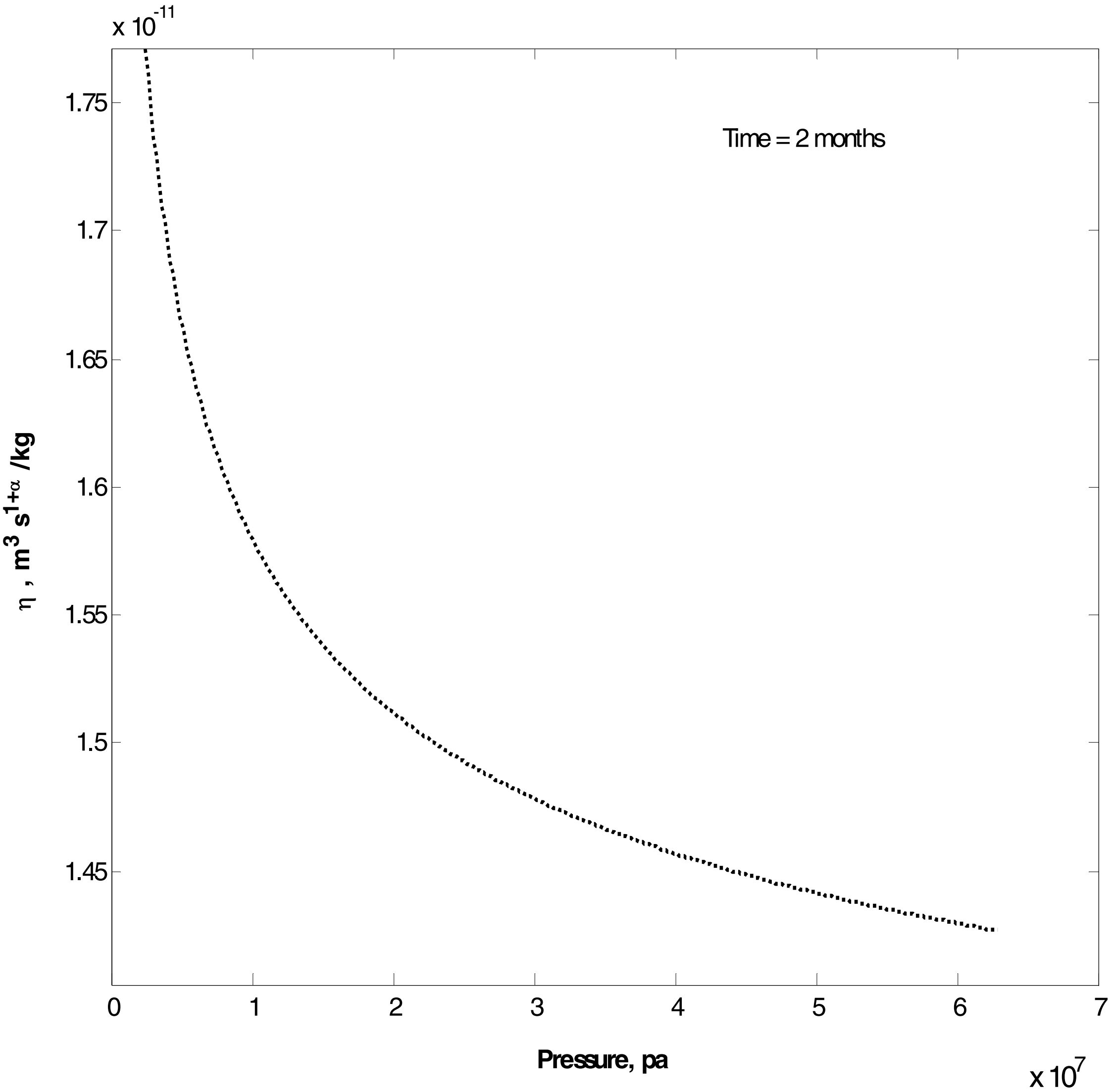

Fig. 5– Variation of η with pressure using proposed model with memory for $\alpha = 0.1$.

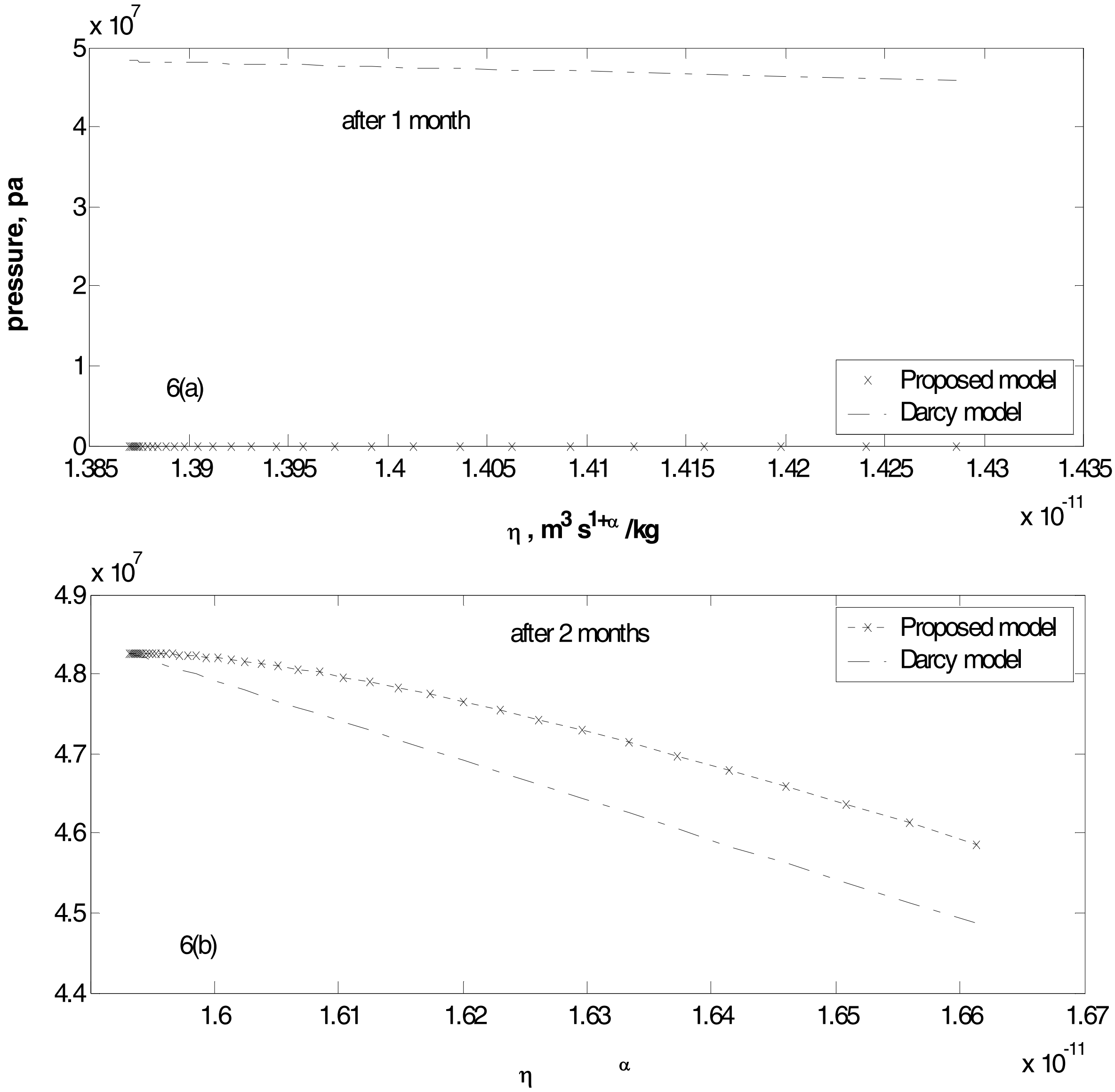

Fig. 6–Variation of pressure with η based on Darcy diffusivity equation and proposed diffusivity equation with memory for $\alpha = 0.1$.

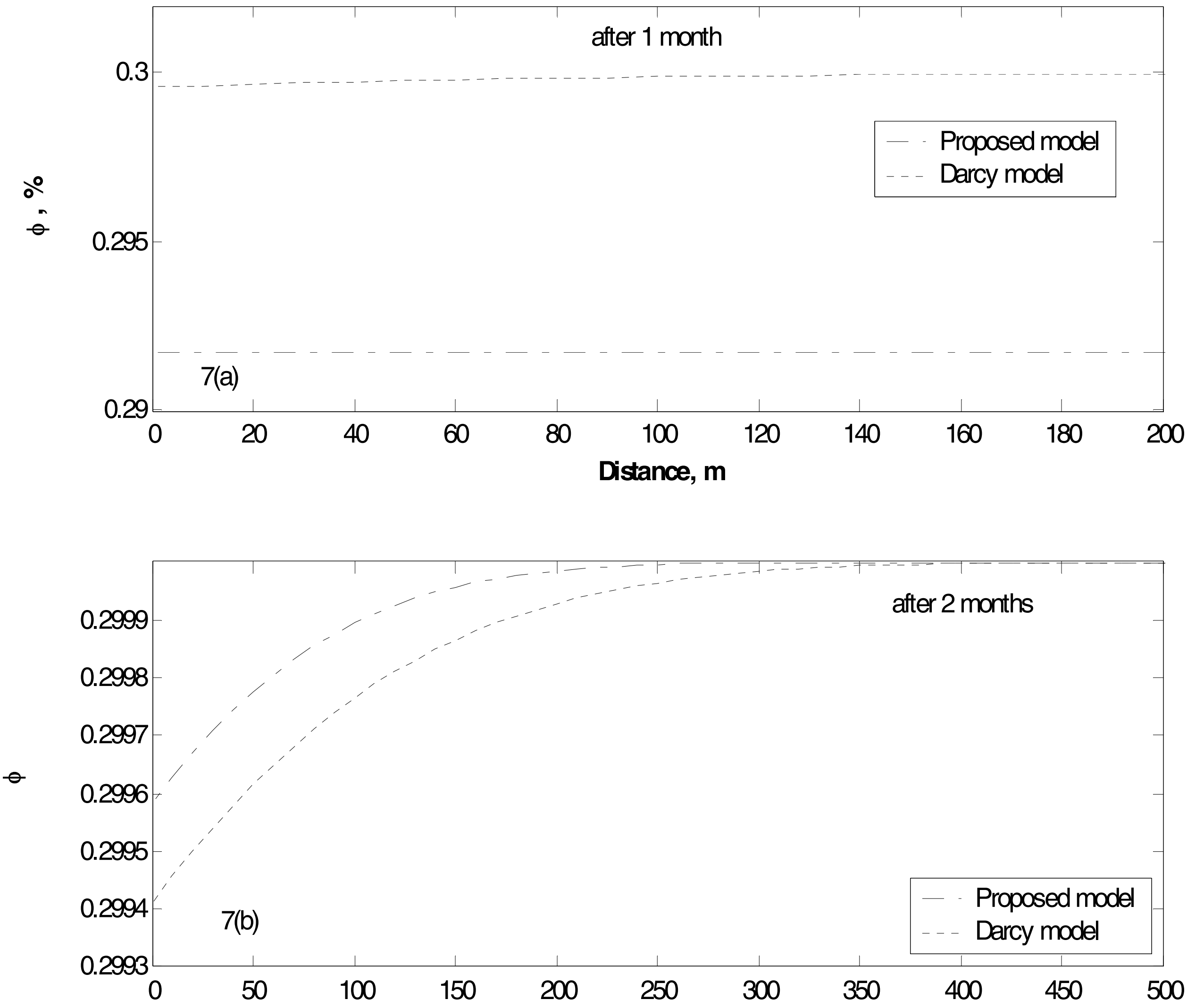

Fig. 7–Variation of porosity and distance using Darcy diffusivity equation and proposed diffusivity equation with memory for $\alpha = 0.1$.