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A NOVEL MATHEMATICAL TOOL FOR CHARACTERIZING PETROLEUM FLUID RHEOLOGY WITHIN POROUS MEDIA

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ABSTRACT
This paper introduces a novel mathematical tool to investigate temperature propagation pattern based on the dependence of various parameters such as production rate, injection rate, fluid velocity and pressure distribution in a petroleum reservoir during thermal operations. The rate equation and the conservation of the energy are applied to derive a mathematical model for the pressure and temperature distributions, using the modified Darcy's law, in which fluid memory is introduced as a continuous function of time. The unique feature of this model is the incorporation of fluid memory to capture the non-linearity of the fluid and media properties.

1 INTRODUCTION

At present, many conventional petroleum reservoirs are reaching maturity, reflected in their reduced productivity. At this stage of petroleum production, the water-to-oil ratio (WOR) is increased drastically, making it difficult to produce hydrocarbons under reasonable economic constraints. In a high-WOR reservoir, it is important to maintain flow rates below the critical rates for which water coning is formed [1]. The temperature profile can be used to identify water or gas entries in the vicinity of a petroleum well. The same profile can also be used to determine fracture characteristics of a reservoir. The temperature sensors are also useful for guiding the action of sliding sleeves or other down-hole flow control devices.

Yoshioka et al. [2] presented a model for predicting the temperature profile in a horizontal well. Their model applies to steady-state flow conditions. They assumed the reservoir to be ideally isolated with each segment, within a box-shaped homogeneous body. With this configuration, they investigated the effects of production rate, permeability and fluid types on temperature profiles. However, they did not investigate the effects of formation fluid and steam injection velocity. Dawkrajai et al. [3] studied the water entry location identification by temperature profile in a horizontal well. They varied the production rate and types of oil to observe their effects on the temperature profile. They assumed that flowing fluid and rock temperature are the same. They did not check the effects of fluid velocity and injection steam velocity on temperature distribution. Jiang and Lu [4] investigated fluid flow and convective heat transfer of water in sintered bronze porous plate channels. The numerical simulations assumed a simple cubic structure with homogeneous particles. They also considered a small contact area and a finite-thickness wall subject to a constant heat flux at the surface. They also studied temperature distributions in the porous media. They also conducted limited investigation of the effects of fluid velocity on temperature distribution. They recommended further investigation of the boundary characteristics and internal phenomena controlling heat transfer in porous media. In the present study, these criteria have been investigated to determine the role of these parameters in temperature distribution throughout the reservoir.

Hossain et al. [5] investigated temperature propagation pattern and its dependence on various parameters during thermal operations. They completed an extensive review of temperature propagation and its dependence on several parameters [5-6]. They solved both 1-energy and 2-energy balance equations. The comparison between these two models showed that the fluid and rock matrix temperature difference is negligible. Results show that formation fluid velocity, steam injection velocity, and time have an impact on temperature profiles behavior. They identified that temperature distribution is much more sensitive to time, and formation fluid velocity. It is also sensitive to steam or hot water injection rate or velocity. They assumed a linear function for fluid velocity in the formation. Due to temperature variation in the formation, the porosity, permeability and also the rheological behavior of reservoir may also change [7-8]. This change of pore space may greatly be influenced by the fluid memory especially in some geothermal area [8-9]. Therefore, the present study includes the effects of fluid memory in the fluid flow behavior at porous media.

2 MATHEMATICAL MODEL DEVELOPMENTS

Two energy balance equations are applied to model rock and fluid in petroleum reservoir. These two equations are coupled through fluid/rock interaction. The governing partial differential equations have a familiar form because the system has been averaged over representative elementary volumes (REV). A right handed Cartesian coordinate system is considered, for which the *x*-axis is along the formation length. The mass flow rate and conservation energy equations are considered to develop the model for temperature distribution in porous media. To introduce the notion of fluid memory in temperature distribution with space and time, modified Darcy's law is used as the flow rate equation which may be written as given in Caputo [10]:

$$u = -\eta\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right], \text{ where } \frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right) = \frac{\int_0^t (t-\xi)^{-\alpha}\left[\frac{\partial^2 p}{\partial \xi\, \partial x}\right]\partial\xi}{\Gamma(1-\alpha)} \text{ with } 0 \le \alpha < 1 \tag{1}$$

Two cases such as different temperature, same rock and fluid temperature are considered during the formulation of the temperature profile models with memory. The final form of complete energy balance equations can be written as shown below for these two cases [5, 7-8]:

Case I

Conservation of energy equations for both solid rock and fluid can be written in dimensionless form when different rock and fluid temperature are considered.

$$\frac{\partial T_s^*}{\partial t^*} - M_1\frac{\partial^2 T_s^*}{\partial x^{*2}} + M_2\left(T_s^* - T_f^*\right) = 0 \tag{2}$$

$$\frac{\partial T_f^*}{\partial t^*} + M_3\frac{\partial T_f^*}{\partial x^*} - M_4\frac{\partial^2 T_f^*}{\partial x^{*2}} + M_5\left(T_f^* - T_s^*\right) = 0 \tag{3}$$

where,

$$M_1 = \frac{\phi_i\,\mu\, c_t\, k_s}{k_{pi}(1-\phi_i)\rho_s c_{ps}}, \quad M_2 = \frac{\phi_i\,\mu\, c_t\, L\, h_c}{k_{pi}(1-\phi_i)\rho_s c_{ps}}, \quad M_3 = u\,\frac{\mu\, c_t\, L}{k_{pi}}, \quad M_4 = \frac{k_f\,\mu\, c_t}{k_{pi}\,\rho_f c_{pf}}, \quad M_5 = \frac{h_c\,\mu\, c_t\, L}{\rho_f\, c_{pf}\, k_{pi}}$$

$$\rho_f c_f = \rho_w c_{pw} S_w + \rho_o c_{po} S_o + \rho_g c_{pg} S_g, \qquad k_f = k_w + k_o + k_g \qquad \text{and} \qquad S_w + S_o + S_g = 1.$$

The equations (2) and (3) are in dimensionless form where the dimensionless parameters are defined:

$$T^* = T/T_i, \quad T_s^* = T_s/T_i, \quad T_f^* = T_f/T_i, \quad x^* = x/L, \quad p^* = p/p_i,$$

$$q^* = q/q_i, \quad t^* = k_{pi}\, t/\phi_i\, \mu\, c_t\, L^2 \quad \text{and} \quad \xi^* = k_{pi}\, \xi/\phi_i\, \mu\, c_t\, L^2.$$

The partial differential Eqs. (2) and (3) should be solved simultaneously to find the temperature distribution for fluid temperature, T_f^* and rock matrix temperature, T_s^* in the formation. The initial and boundary conditions are defined

$$T_f^*(x^*, 0) = T_s^*(x^*, 0) = 1, \qquad T_f^*(0, t^*) = T_s^*(0, t^*) = T_{steam}/T_i \text{ and } \qquad T_f^*(1, t^*) = T_s^*(1, t^*) = 1$$

that are in dimensionless form.

Case II

If the temperature of the fluid and rock matrix are the same, the energy balance equations can be combined together and expressed as a single equation in the form of dimensionless equation [5, 7-8];

$$\frac{\partial T^*}{\partial t^*} + M_6\frac{\partial T^*}{\partial x^*} - M_7\frac{\partial^2 T^*}{\partial x^{*2}} = 0 \tag{4}$$

where,

$$M_6 = \frac{\phi_i\,\rho_f c_{pf}\,\mu\, c_t\, L}{M\, k_{pi}}\, u, \qquad M_7 = \left(k_s + k_f\right)\frac{\phi_i\,\mu\, c_t}{M\, k_{pi}},$$

$$M = (1-\phi_i)\rho_s c_{ps} + \phi_i\,\rho_w c_{pw} S_w + \phi_i\,\rho_o c_{po} S_o + \phi_i\,\rho_g c_{pg} S_g.$$

This equation gives the dimensionless temperature profile along the formation length when the rock and fluid temperature are considered to be same. The initial and boundary conditions are $T^*(x^*, 0) = 1$, $T^*(0, t^*) = T_{steam}/T_i$ and $T^*(1, t^*) = 1$.

4 CONCLUSIONS

Mathematical models with the inclusion of "memory" concept are presented for a petroleum reservoir under thermal operations. The proposed models are suitable for handling variable rock and fluid properties with time and space. The effects of temperature on different reservoir parameters can also be investigated using the proposed mathematical tool.

NOMENCLATURES

A_{yz} = cross sectional area of rock perpendicular to the flow of flowing fluid, m^2
c_{pf}, c_{pg}, c_{po}, c_{pw}, c_{ps}= specific heat capacity of injected fluid, steam, oil, water and solid rock matrix, kJ/kgk
sg = gravitational acceleration in *x* direction, m/s^2
h_c = convection heat transfer coefficient, kJ/hm^2k
k_{pi} = permeability, m^2
k_f, k_g, k_o, k_w, k_s = thermal conductivity of fluid, steam, oil, water and solid rock matrix, $kJ/h\ m\ k$
L = distance between production and injection well along x direction, m
L^* = dimensionless length of the reservoir
M = average volumetric heat capacity of the fluid-saturated rock, kJ/m^3k
p, p_i, p_0 = pressure, initial pressure and a reference pressure of the system, pa
q_{inj}, q_{prod}, q_i = *Au* = injection, production volume flow rate of steam, oil, and initial volume production rate, m^3/s
q_x = fluid mass flow rate per unit area in *x*-direction, kg/m^2s
S_{wi}, S_w, S_g, S_o = initial water, water, gas and oil saturation, volume fraction
t, t^* = time and dimensionless time respectively, *s*
T, T_f, T_i, T_r, T_s = temperature, temperature of injected fluid, initial reservoir temperature, reference temperature of injected fluid, average temperature of solid rock matrix, k
T^* = dimensionless temperature
u, u_i = fluid velocity in porous media and initial fluid velocity in *x* direction, m/s
u^* = dimensionless velocity
x^* = dimensionless distance
ϕ = porosity of the rock, volume fraction
ρ = reference density, kg/m^3
ρ_f, ρ_o, ρ_w, ρ_g, ρ_s = density of fluid, oil, water, steam and solid rock matrix, kg/m^3
= density of steam, μ = fluid dynamic viscosity, $pa\ s$
c_f, c_s, c_t = total fluid, formation rock and total compressibility of the system respectively, $1/pa$
α = fractional order of differentiation, dimensionless
η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3s^{1+\alpha}/kg$
ξ = a dummy variable for time i.e. real part in the plane of the integral, *s*

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