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A NOVEL FLUID FLOW MODEL WITH MEMORY FOR POROUS MEDIA APPLICATIONS

M. Enamul Hossain*, S. Hossein Mousavizadegan** and M. Rafiqul Islam*[1]
*Department of Civil & Resource Engineering, Dalhousie University
**Marine Engineering Department, Amirkabir University, Tehran, Iran

ABSTRACT
A new model for describing fluid flow inside porous media is introduced. The memory concept is applied to modify the equation of motion and take into account the variation of fluid and formation properties with time. The combination of this equation of motion and the continuity equation gives rise to an integro-differential equation that is fully nonlinear. A numerical formulation of the problem is obtained using an implicit-explicit finite difference method. The solution provides important information regarding the importance of the effect of variation of fluid and formation properties with time.

1 INTRODUCTION

The big challenge in petroleum industry is to increase the oil recovery. The Technological advancement may create an opportunity in this industrial era. This advancement is linked to accurate precision of rock-fluid behavior. In this computational and information age, it is possible to model the complex and naturally occurring phenomena. However, to-date, majority of the existing fluid flow through porous media models are based on some assumptions that are not reflective of the reality. Darcy's law is the best example in this case which is used as the basic model for more than last hundred years. The assumptions behind Darcy's law such as homogeneous and isotropic porous medium, uniform thickness, rock and fluid properties independent of pressure and laminar flow are reported in many literatures [1-3]. If at least one of the assumptions is eliminated, the newly developed model will be closer to natural phenomena and may contribute to improved predictive ability.

Moreover, existing simulators deem the rock and fluid properties to be constant in time. It is very important to consider the variation of properties such as permeability, viscosity with time. Conventional fluid motion equations do not allow us to consider the time variation of fluid and rock properties in a proper way. Therefore, Darcy's law should be modified in such a way that it may create some options of using the variable rock and fluid properties during the development of theoretical fluid flow models. Following Caputo [4], we use the memory concept to take into account the time in the computation of flow properties. The effect is introduced by modification of the equation of motion and using the fractional derivative with respect to time. The combination of the memory based equation of motion with the continuity equation gives rise to an integro-differential equation that we call the "memory based diffusivity" equation. This equation is a generalization of the normal diffusivity equation [5]. Several applications of the memory based formulation for the equation of motion are given in Hossain et al. [6 – 9].

2 MODEL BASED ON MEMORY

To take into account the variation of fluid and formation properties with time, the Darcy's law is modified using fractional derivative as given in Caputo [4].The fluid velocity is related to the pressure variation along the reservoir while the time variation is introduced with a fractional derivative. In other word, the fluid velocity is proportional to the fractional derivative of the pressure variation along the reservoir with respect to time, $u_x = -\eta\left[\frac{\partial^\alpha}{\partial t^\alpha}(\partial p/\partial x)\right]$. Here, the notation η is the ratio of the pseudo-permeability of the medium to fluid viscosity and $0 \le \alpha \le 1$ is fractional order of differentiation. The parameter η is used instead of the permeability and viscosity in conventional Darcy's law. This equation of motion is called the memory based mathematical model. The combination of this equation of motion with the continuity equation leads to the following equation.

$$\frac{1}{\eta}\frac{\partial\eta}{\partial x}\left\{\frac{\int_0^t (t-\xi)^{-\alpha}\left[\frac{\partial^2 p}{\partial\xi\partial x}\right]\partial\xi}{\Gamma(1-\alpha)}\right\} + c_f\frac{\partial p}{\partial x}\left\{\frac{\int_0^t (t-\xi)^{-\alpha}\left[\frac{\partial^2 p}{\partial\xi\partial x}\right]\partial\xi}{\Gamma(1-\alpha)}\right\} + \frac{\partial}{\partial x}\left\{\frac{\int_0^t (t-\xi)^{-\alpha}\left[\frac{\partial^2 p}{\partial\xi\partial x}\right]\partial\xi}{\Gamma(1-\alpha)}\right\} = \frac{\phi\, c_t}{\eta}\frac{\partial p}{\partial t} \qquad (1)$$

This equation is an integro-differential equation that is fully nonlinear. It may be considered as a general form of the normal diffusivity equation that is obtained by the combination of the Darcy law with the continuity equation. If it is taken into account that $\alpha = 0$, $\frac{\partial \eta}{\partial x} = 0$ and $\eta = k/\mu$, the Eq. (1) is simplified as $c_f \left[\frac{\partial p}{\partial x}\right]^2 + \frac{\partial^2 p}{\partial x^2} = \frac{\phi\,\mu\,c_t}{k}\frac{\partial p}{\partial t}$. Since the variation of pressure along the reservoir is small, the square of the pressure variation along the reservoir can be ignored and finally we get the normal diffusivity equation $\frac{\partial^2 p}{\partial x^2} = \frac{\phi\,\mu\,c_t}{k}\frac{\partial p}{\partial t}$. The first part of the diffusivity equation with memory Eq. (1) is nonlinear since η is a pressure dependent parameter. The relation for η is depend on the properties of the reservoir fluid. For crude oil, it may be express in the form $\eta = \frac{[3.0\,(p/6894.76)^{-0.31}+10.5]\times 10^{-12}}{\mu_{ob}e^{8.422\times 10^{-5}(p-p_b)}}\,(t)^{\alpha}$ that is obtained following Almehaideb [10].

3 The computational method

The Eq. (1) can be solved numerically. We applied the finite difference method to find the result. An implicit-explicit scheme is applied to make the solution amenable. The Eq. (1) can be recast in the form

$$\frac{1}{\eta}\frac{\partial \eta}{\partial x}\sum_{k=1}^{n}(t-\xi_k)^{-\alpha}\frac{\partial^2 p}{\partial \xi \partial x}W_k + c_f\frac{\partial p}{\partial x}\sum_{k=1}^{n}(t-\xi_k)^{-\alpha}\frac{\partial^2 p}{\partial \xi \partial x}W_k + \sum_{k=1}^{n}(t-\xi_k)^{-\alpha}\frac{\partial^3 p}{\partial \xi \partial x^2}W_k = \frac{\phi\,c_t}{\eta}\Gamma(1-\alpha)\frac{\partial p}{\partial t} \qquad (2)$$

where W_k is a weighting function for the numerical integration. Using the finite difference method it can be written

$$\begin{aligned}
&\frac{1}{\eta}\frac{\partial \eta}{\partial x}\frac{1}{2\,\Delta t^{(\alpha+1)}\Delta x}\left\{\sum_{k=0}^{n}\frac{W_k}{(n-k)^{\alpha}}\left[(p_{j+1})^{k+1}-(p_{j+1})^{k}-(p_{j-1})^{k+1}+(p_{j-1})^{k}\right]\right\}\\
&+\frac{c_f}{4\,\Delta t^{(\alpha+1)}(\Delta x)^2}\left[(p_{j+1})^{n}-(p_{j-1})^{n}\right]\sum_{k=0}^{n}\frac{W_k}{(n-k)^{\alpha}}\left[(p_{j+1})^{k+1}-(p_{j+1})^{k}-(p_{j-1})^{k+1}+(p_{j-1})^{k}\right]\\
&+\frac{1}{\Delta t^{(\alpha+1)}(\Delta x)^2}\sum_{k=0}^{n}\frac{W_k}{(n-k)^{\alpha}}\left[(p_{j+1})^{k+1}-(p_{j+1})^{k}-2(p_j)^{k+1}+2\,(p_j)^{k}+(p_{j-1})^{k+1}-(p_{j-1})^{k}\right]=\frac{\phi\,c_t}{\eta}\Gamma(1-\alpha)\frac{(p_j)^{n+1}-(p_j)^{n}}{\Delta t} \qquad (3)
\end{aligned}$$

that is a simultaneous algebraic equation. The main problem in solution of this simultaneous equations is the singular form of the equation when $k = n$. We use different numerical techniques to remove these singularities and find the solution.

4 RESULTS AND DISCUSSION

Fig. 1 shows the variation of η with distance from the wellbore towards the outer boundary of the reservoir

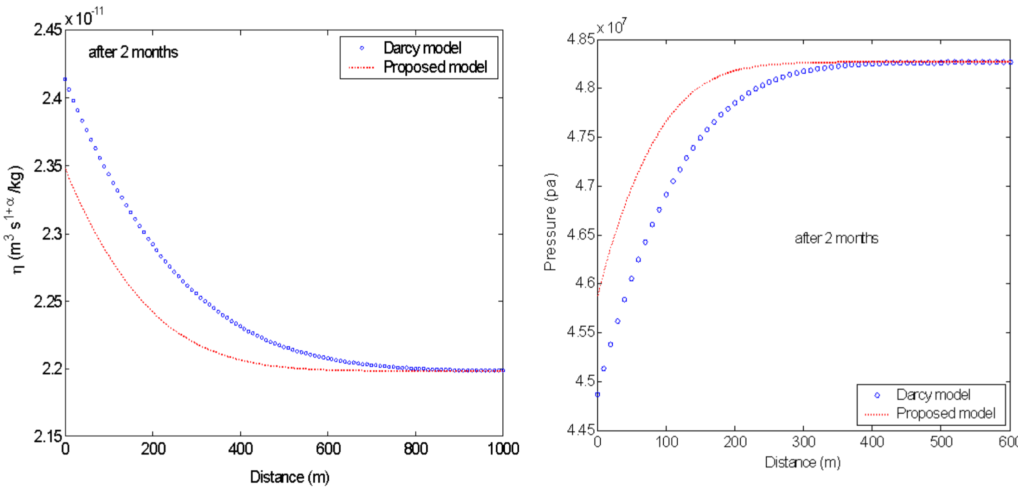

Figure 1: (a) η variation for Darcy model and proposed model, (b) pressure variation with distance for Darcy and proposed model

where pressure distribution is computed using Darcy Diffusivity equation and proposed diffusivity equation with memory (Eq. (1)). η decreases with distance very fast around the wellbore (Fig. 1(a)). As the pressure response towards the boundary is not 'felt', η does not vary after 1000 m from the wellbore for both cases. The Darcy model gives higher η values whereas the proposed model gives lower values.

The pressure data as a function of distance from the wellbore toward the outer boundary of the reservoir are depicted in Fig. 1(b). In this figure, Darcy diffusivity and proposed diffusivity equation with memory are compared after two months of production. Both models predict sharp pressure gradient near the wellbore. These pressure gradients gradually become blunter towards the outer boundary of the reservoir. The two month prediction results show that the pressure response has reached up to 540 m and 370 m from the wellbore for Darcy and the proposed model, respectively. The trend of the pressure gradient is similar for both cases. However, the actual pressure values predicted using Darcy's model is consistently lower than those of the proposed model. The propagation of the pressure shock is steeper for the memory case. The proposed model gives higher pressure for a particular point from the wellbore because of the rock/fluid properties dependency with respect to time. This would result in more optimistic production data with the new model.

5 CONCLUSIONS

A comparison of the proposed model with conventional diffusivity equation is shown in this study. If the notion of memory is ignored, the new model turns to the conventional Darcy diffusivity equation which is derived as a special case of the proposed diffusivity equation with memory. The importance of time dependent fluid and media properties are established here using the pressure response criteria. A period of 2 months and fractional order of differentiation $\alpha = 0.1$ is considered during the computation of η and pressure response.

NOMENCLATURES

c_f = $c_o + c_w$ = total fluid compressibility of the system
c_w = formation water compressibility of the system, $1/pa$
c_t = $c_f + c_s$ = total compressibility of the system
c_s = formation rock compressibility of the system, $1/pa$
k = initial reservoir permeability, m^2
L = distance between production well and outer boundary along x direction, m
p = pressure of the system, pa
p_i = initial pressure of the system, pa,
p_o = a reference pressure of the system, N/m^2
q_i = Au = initial volume production rate, m^3/s
q_x = fluid mass flow rate per unit area in x-direction, kg/m^2s
u = filtration velocity in x direction, m/s
u_x = fluid velocity in porous media in the direction of x axis, m/s
α = fractional order of differentiation, dimensionless
ρ = density at pressure p,
ρ_o = density at a reference pressure p_o , kg/m^3
ϕ = porosity of fluid media at pressure p,
ϕ_o = porosity of fluid media at reference pressure p_o, m^3/m^3
μ = fluid dynamic viscosity,
η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3s^{1+\alpha}/kg$

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