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Effects of Criterion Values on Estimation of the Radius of Drainage and Stabilization Time

M. Enamul Hossain
Jamuna Oil Company Limited

M. Tamim
Bangladesh University of Engineering & Technology

N.M. Anisur Rahman
University of Alberta

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ABSTRACT

After a well starts flowing, a larger and larger portion of the reservoir contributes to production. At any given time, the radius of the portion of the reservoir that demonstrates a pressure gradient and contributes to production of fluid is the radius of drainage (ROD). The time required for the entire reservoir just to be able to contribute production is the stabilization time. Estimating the ROD and stabilization time is very important in well test design and production optimization. It has been a great challenge to estimate accurately the ROD and stabilization time because of inherent uncertainties with respect to the rock and fluid properties.

This study examines the effects of criterion values on the estimated values of the ROD and stabilization time. As expected, estimated values of the ROD and stabilization time vary considerably, depending on the suggested criteria. The primary objective of this study is to recognize and appreciate the importance of criterion values for defining the ROD and stabilization time. Generalized correlations have been proposed that allow one to determine the ROD and stabilization time as a function of the criterion values. The relationship between a pressure criterion and the corresponding rate criterion has been examined also.

INTRODUCTION

The concepts of ROD and stabilization time are commonly used in reservoir engineering and in well test analysis. Estimating the ROD is very important on many counts. A well-test analysis provides important reservoir information based on the area sampled within the ROD. It is important to know the extent of the reservoir that is being sampled when determining the parameters like permeability and storage capacity from a well-test analysis. In other words, the well test analysis provides the global values of the reservoir parameters that are valid over the radius of investigation.[1] Thus, the obtained reservoir information is good for the region within the ROD. In addition, knowing the ROD helps optimize the locations of new wells to be drilled in a field. It is very difficult to identify the well-test run time without an estimate of the ROD and stabilization time. The ROD concept has both quantitative and qualitative importance in well-test design and analysis. This distance is dependent on the way the pressure response propagates through the reservoir. It is also related to formation rock and fluid properties and elapsed time. Thus, the ROD concept presents a guide for well test design. This concept can be used also to estimate the time required to test to the desired depth in the formation. However, estimating any ROD has been dependent on the assumed level of the criterion for pressure or flow rate. As a result, there can be substantial variations in the estimated magnitude of the ROD.

Depending on the criterion parameters and their values, a number of definitions have been proposed for the ROD and stabilization-time equations. Daungkaew et al.[2] have provided a comprehensive account of these efforts.[3-16] Although most of these equations are not too fundamentally different from that of Muskat[3], Jones[7] and van Poolen[8] postulated ROD equations based on pressure criteria, and while Tek et al.[6] postulated the same based on a rate criterion. Recently, Hossain et al.[17] have demonstrated that there is a direct relationship between the pressure and the rate criteria for defining the ROD. For example, an ROD value, when defined as the distance experiencing less than 1% of the wellbore pressure drawdown, is equivalent to an ROD value defined as the radius of circumference across which less than 3.32% of the well-flow rate is occurring. This matter is discussed further later. Johnson[14] attempted to derive the ROD equation based on the amount of cumulative production. The current trend shows that the correlations by Daungkaew et al.[2] underestimate, and those of Tek et al.[6] and Jones[7] overestimate, the magnitude of the ROD, when compared to those of Muskat[3] and van Pollen.[8] In a nutshell, all these efforts have one thing in common – the equations for estimating the ROD have originated from the parabolic kind of diffusivity equations, which account for the undesirable paradox of an infinite velocity of pressure propagation.

Theoretically speaking, the time that is required for any reservoir to reach the stabilized condition for any reservoir is infinite. During the pseudosteady-state flow, the pressure drop is due to the expansion of the reservoir fluid in the reservoir that fills the void space created by fluid production. The time required to reach the pseudosteady state is finite, and it depends on the size of the reservoir. In this case, the early-time component of the solution of the diffusivity equation can be neglected. A stabilized condition is reached when the flow in the reservoir attains pseudosteady state. This condition can be expressed mathematically using a constant Cartesian derivative of pressure with respect to time. However, the derivative equation shows that it would attain a true constant value only at infinite time. This constant can be calculated easily using the appropriate equation, and its magnitude will vary depending on the reservoir parameters selected.

In this study, the effects of the pressure responses are considered as the criterion values for identifying the ROD and stabilization time. Correlations are proposed for estimating these important parameters.

GOVERNING EQUATIONS

In the literature, the diffusivity equation has been regarded as the basis for describing reasonably the flow in the reservoir. Thus, solutions to the diffusivity equation for a homogeneous domain, where production is occurring through the wellbore at a constant rate, are used in this study to investigate the issues of the ROD and stabilization time. However, the scope and capability of the diffusivity equation to describe the flow is not challenged. The major governing equations are presented in the Appendix.

Equations A-1 through A-4 are used for investigating the transient behavior in infinite-acting domains, and for examining and developing the ROD criteria. Equations A-5 through A-8 are used as the governing equations for analyzing the transient behavior, and for evaluating the stabilization-time criteria in bounded domains.

DEPENDENCE OF ROD ON CRITERION VALUES

Previous Efforts

In the literature, most of the investigators[2-17] have postulated the equations for the ROD in a dimensionally consistent form as

$$r_d = D\sqrt{\frac{kt}{\phi\mu c_t}} \quad \text{......} (1)$$

However, Equation 1 can be written in a dimensionless form as

$$r_{dD} = D\sqrt{t_D} \quad \text{......} (2)$$

Various values for D have been proposed[2-16], based on different criterion parameters and their values. These values lie in the range between 0.379 and 4.29, suggesting that an estimation of the ROD can be subject to variations of 1000%. Moreover, the uncertainty in estimating the ROD does not include the effect of reservoir heterogeneity and uncertainties in other reservoir conditions and parameters.

More Efforts

In this section we consider four more approaches, extending what has been done in the literature to define the ROD.

a. Extension of Jones' Approach

Hossain[20] has extended Jones' approach[7], and has proposed a generalized correlation for the ROD with the criterion value as a parameter. Despite having a serious limitation of being derived for a semi-infinite, linear-flow system with an analogy between heat conduction through solids and fluid flow through porous media, Equation A-1 provides reasonable solution for a radial-flow system. Figure 1 shows the variation of p_{rD} with the dimensionless variable η. It has been found that the error-function component of the solution reaches a value of 0.9999999831 when η is 4.055. This fact can be used to define a characteristic distance called the ROD, r_d, corresponding to a given time t. Thus, the ROD can be defined as that distance over which the pressure change is equal to 0.0000016% of the pressure change at the wellbore, corresponding to $\eta = 4.055$. In contrast, a theoretical consideration will lead to the fact that r_d is located where $p_{rD} = 0$, as η tends to infinity; however, this approach does not lead to any practical solution. A finite value for η (e.g., 4.055) leads to a practical estimate of r_d, and yet, the value of η depends on the criterion value used (e.g., 0.0000016%). Thus, from a practical standpoint, the upper limit of the parameter $D = 2(4.055) = 8.11$. As all of the pore volume, as determined by $D = 8.11$, may not be effective,[8] one needs to introduce a criterion parameter to define an effective ROD.

From the plot of D vs. p_{rD} in Figure 3, the following empirical correlation can be developed by a best-fit approach:

$$D = -0.74995\log(p_{rD}) + 3.8703 \quad \text{......} (3)$$

In Equation 3, p_{rD} can be considered as the criterion value – the percentage of the pressure change that can be taken as a benchmark for the ROD. This correlation allows one to determine the corresponding ROD equation for a pre-determined criterion value. Here, the range of pressure disturbance is considered from 1% to 0.0000016%.

Figure 4 shows the expected errors corresponding to the estimated values of D from Equation 3. From this empirical correlation, it can be shown that for a value of $D = 3.8703 \approx 4.0$, an error of – 4.7% is introduced. Within the range considered, the maximum possible error is 4.7%.

b. Pressure Approach

Lee[12] considered a maximum pressure disturbance at the ROD with respect to time. However, he ended up with an identical equation to those of Muskat[3] and van Pollen.[8]

We are going to examine the exponential-integral solution, Equation A-2, to the diffusivity equation, which is based on a line-sink well in an infinite medium. When the logarithmic approximation to this equation is appropriate, the solution can be written as[19]

$$p_{rD} = 0.5\left[\ln\left(\frac{t_D}{r_D^2}\right) + 0.80907\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

At the ROD, the dimensionless pressure value in Equation becomes zero; thus, we can write

$$\ln\left(\frac{r_D^2}{t_D}\right)_{r_D = r_{dD}} = 0.80907\,,$$

which leads to a new equation for the ROD:

$$r_d = 1.4986\sqrt{\frac{kt}{\phi\mu c_t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

A comparison of Equation 5 to Equation 1 suggests that $D = 1.4986$ (≈ 1.5), as in Equation 5. van Poolen[8] pointed out his personal communication (as Reference 12 in his paper), indicating that Hutchinson and Kern had obtained $D = 1.5$ by means of a differential network. Besides this, no more evidence in regards to this work has been found in the literature.

c. Time-Derivative Approach

Daungkaew et al.[2] have demonstrated that test-period estimations using the equations proposed in References 3 through 16, fall short of identifying a situation where the wellbore is located near a sealing fault. Hence, with this very specific purpose in mind, it has been proposed that the D value in Equation 1 should lie in between 0.379 and 1.623. However, we will examine a producing well-sealing fault system to investigate the pressure-transient behavior and propose a new version of the ROD equation.

Here the fault boundary is located at a known value of r_{dD} from the well. Figure 2 shows the semi-log (dimensionless time) derivative of the dimensionless pressure responses, as developed from Equation A-4 for different values of r_{dD}. The flow period with $p_{wD\ L}' = 0.5$ shows the infinite-acting radial flow regime, and the period with $p_{wD\ L}' = 1.0$ shows the stabilized flow, highlighting the doubling-of-slope phenomenon in the transition period. This means, the infinite-acting radial flow period ends as soon as the effect of the sealing fault is felt. In effect, this can happen only when the ROD is just equal to the distance to the sealing fault, r_d, as in this case. The responses at the wellbore are considered to have reached the specified dimensionless distance (r_{dD}) when the $p_{wD\ L}'$ value just exceeds its infinite-acting radial flow value of 0.5 by 1%. Based on this criterion, the travel times of pressure responses are estimated and presented in Table 1. These findings can lead to an estimation of the ROD. This follows that the average value of $t_D/r_{dD}^2 = 0.21714$ for the system. This results in $D = 2.146$ which, of course, is based on the criterion with the semi-log derivative.

d. Space-Derivative Approach

This space-derivative approach is going to involve the rate of fluid flow. The equation of the derivative of the dimensionless pressure with respect to dimensionless radius can be derived from Equation A-2 as

$$-\left[r_D\,\frac{dp_{rD}}{dr_D}\right]_{r_D = r_{dD}} = e^{-0.25D^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Ideally, the right-hand side of Equation 6 should become zero, signifying the fact that no fluid crosses the boundary at the ROD. Theoretically, this means $D \to \infty$. However, the left-hand side of Equation 6 is a very important entity; when compared to the flow equation of Tek et al.[6], it follows

$$\frac{q_{r_d}}{q} = e^{-0.25D^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

where $q_{r\,d}/q$ is the ratio of the rate of fluid influx at the ROD to the rate of production at the wellbore.

Equation 7 demonstrates a very powerful development that relates the location of an ROD with the amount of fluid crossing the assumed boundary at the ROD. In effect, the parameter $e^{-0.25D^2}$ quantifies this ratio. For example, $D = 2$, as suggested by van Poolen[8], means that the boundary at the ROD is subject to a fluid influx rate of 36.8% of the production rate at the wellbore. Hossain[20] has reported that van Poolen's suggestion is equivalent to a 17.7% pressure differential. Thus, it also implies that a 17.7% pressure differential at the ROD is equivalent to allowing 36.8% influx through an imaginary boundary located at the ROD. It is also evident that the pressure criterion is less sensitive than the rate (or space-derivative) criterion. Table 2 compares the fluid influx through the RODs, defined as different D values. This illustrates the fact that the higher the value of D, the larger the ROD at a given time, and the lower the rate of influx at the defined ROD.

Discussion of ROD

As shown above, allowing too much fluid influx through the ROD, e.g., 36.8% for $D = 2$, would obviously jeopardize the notion of the ROD. Unfortunately, most of the D values in the literature are in and around 2, which means that the computed ROD values are underestimated by at least 100%. It is imperative that defining the ROD in terms of a pressure criterion should be complemented by the rate-criterion check with Equation 7.

DEPENDENCE OF STABILIZATION TIME ON CRITERION VALUES

The most common definition of stabilization time describes it as the elapsed time when the reservoir attains either the steady- or pseudosteady-state flow. The general form of the equation for defining the time to stabilization is given by

$$t_s = T_{DS} \frac{\phi \mu c_t r_e^2}{k} \quad \text{..........} (8)$$

where T_{DS} is the value of dimensionless time T_D (based on the radius at the extreme boundary) at stabilization. This dimensionless value depends on the criterion parameters and their values.

Cases of closed outer-boundary and constant pressure outer-boundary reservoirs are considered to develop the proposed generalized correlations for stabilization time.

Closed Outer-Boundary Reservoir

A closed outer-boundary reservoir is defined as one in which the well is assumed to be located in the center of a cylindrical reservoir with no flow across the exterior boundary. For no flow across the exterior boundary, the pressure gradient at the boundary becomes zero. When this occurs, the reservoir depletion becomes pseudosteady state. During this state, the pressure in the reservoir declines at a constant rate throughout the reservoir.

Theoretically, the exponential term in Equation A-6 vanishes at large values for dimensionless time. At that point, the pressure response reaches the outer boundary of the reservoir. Figure 5 shows the dependence of the Cartesian derivative of dimensionless pressure responses on dimensionless time, T_D. Eventually, the pseudosteady-state condition is reached, and a constant value of the Cartesian derivative is attained. This value can be calculated to be $p_{wD}' = 1.257 \times 10^{-5}$ (perfect stabilized condition) for $r_{eD} = 1000$. The computed numerical values of T_{DS} are presented in Table 3, using different criterion values as the percentage difference from a perfect stabilized condition. Figure 6 shows how the value of T_{DS} varies with the definition of an apparent stabilized condition.

From the trend in Figure 6, a correlation between T_{DS} and the percentage difference from a perfect stabilized condition, can be developed as

$$T_{DS} = -0.156806 \log(\% \, Difference) + 0.4375 \quad \text{..........} (9)$$

With Equation 9, one can estimate a T_{DS} value upon choosing a criterion value (as % difference from a perfect stabilized condition). In contrast, Brownscombe and Kern[4] defined the stabilization time as the time required to reach within 2% of the stabilized condition. Figure 7 shows the amount of error to be incurred when the T_{DS} value is estimated from the above correlation. The percentage of error is expected to be in between

– 0.02117 and 0.001114, for the range of dimensionless time values between 0.25 and 0.70.

Constant Pressure Outer-Boundary Reservoir

Constant pressure outer-boundary reservoirs are defined with a well at the center of a cylindrical area, and with a constant pressure along the outer boundary. The effect of a constant-pressure boundary ultimately causes the well-pressure response to achieve steady state. Chatas[5] estimated the stabilization time for a linear system.

Equation A-7 defines the dimensionless pressure responses at the wellbore, and Equation A-8 defines the Cartesian derivative of these responses. Figure 8 shows the dependence of the Cartesian derivative on dimensionless time. Theoretically, the steady state condition attained when $p_{wD}' = 0$ (perfect stabilized condition). Unlike the closed-boundary case, defining a stabilized condition with respect to p_{wD}' is not easy. However, this matter can be taken care of with the p_D diagram shown in Figure 9. Here, it is demonstrated that the steady-state condition is attained at $p_D = 35.337$ for $r_{eD} = 1000$. Both Figures 8 and 9 indicate that steady state is reached at some dimensionless time between 0.25 and 1.0. A similar approach to that used in closed-boundary case is taken to define apparent stabilized conditions.

Apparent stabilized dimensionless times (T_{DS}) are determined, and are presented as a function of percentage difference from a perfect stabilized condition in Figure 10. Using the regression analysis, the following correlation has been developed:

$$T_{DS} = -0.39812 \log(\%\, Difference) + 0.5408 \;\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

With Equation 10, one can estimate T_{DS} for a given criterion value. Figure 11 shows the error that results when a T_{DS} value is calculated using Equation 10. The percentage of error is expected to be in between –0.01561 and 0.067355, within the dimensionless-time range of 0.25 to 1.0.

Remarks on Correlations for Stabilization Time

Two correlations, Equations 9 and 10, have been proposed for dimensionless stabilized times, based on two different outer-boundary conditions. These correlations are similar in nature, but with different coefficients. These correlations are compared in Figure 12. Apparently, both correlations provide an identical value of dimensionless stabilized time ($T_{DS} = 0.37$) at a criterion value of 2.70%. One can choose either of the correlations as the stabilization-time equation in the range of 0.30-0.45, because the estimates of T_{DS} would not differ by more than 10%.

CONCLUSIONS

1. The definition of the ROD depends on the criterion value used. The impact of the criterion values used should be appreciated when evaluating the ROD. From a practical standpoint, the upper limit of D is 8.11.
2. A new generalized correlation, which has been proposed in this study, allows one to have the flexibility of choosing one's own criterion value in estimating the ROD as needed.
3. The definition of the stabilization time depends on the criterion values used.
4. The new generalized correlations based on closed and constant-pressure boundary conditions are flexible enough to allow one to choose one's own model for stabilization time.

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NOMENCLATURE

B = Formation volume factor

c_t = Compressibility, Lt^2/m

D = Coefficient for ROD (Equation 1)

$Ei(-x)$ = Exponential-integral function, $-\int_{x}^{\infty} \frac{e^{-u}}{u} du$

$erf(x)$ = Error function, $\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-u^2} du$

h = Net formation thickness of the reservoir, L

k = Formation permeability, L^2

p_i = Initial reservoir pressure, m/Lt^2

p_{rD} = Dimensionless pressure at distance r, as a function of time –
$[p_i - p_{r,t}]/[p_i - p_{wf}]$ for Equations 3, A-1;
$(2\pi kh/q\mu B)\,[p_i - p_{r,t}]$ for Equations 4, 6, A-2;

$p_{r,t}$ = Pressure at radial distance r and time t, m/Lt^2

p_{wD} = Dimensionless pressure at wellbore, $(2\pi kh/q\mu B)[p_i - p_{wf}]$

$p_{wD}{}'$ = Cartesian derivative of pressure responses at wellbore, dp_{wD}/dt_D

$p_{wD\,L}{}'$ = Semi-log derivative of pressure responses at wellbore, $dp_{wD}/dlnt_D$

p_{wf} = Flowing bottom-hole pressure, m/Lt^2

q = Flow rate, L^3/t

q_{r_d} = Influx rate at the drainage boundary, L^3/t

r = Radial coordinate, L

r_d = Drainage radius, L

r_{dD} = r_d/r_w

r_D = Dimensionless radius, r/r_w

r_e = Location of the extreme boundary, L

r_{eD} = r_e/r_w

r_w = Wellbore radius, L

t = Time variable, t

t_s = Elapsed time to stabilization, t

t_D = Dimensionless time, $kt/\phi\,\mu\,c_t\,r_w^2$

T_D = Dimensionless time based on r_e, $kt/\phi\,\mu\,c_t\,r_e^2$ (also $T_D = t_D / r_{eD}^2$)

T_{DS} = Dimensionless time at an apparent stabilized condition

u = Dummy variable

λ_l = lth eigenvalue for the r direction problem (Reference 18)

ϕ = Porosity

η = Dimensionless variable, $\sqrt{\frac{\phi\mu c_t r^2}{4kt}}$

μ = Viscosity, m/Lt

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APPENDIX: GOVERNING EQUATIONS

Infinite Systems

Extension of Jones' Approach

The following solution[7] to the diffusivity equation, in terms of error function, is used

$$p_{rD} = 1 - erf(\eta) \quad \text{...(A-1)}$$

Line-Sink Solution

The dimensionless pressure responses at a distance r_D from a line-sink well, subject to a constant rate of production, can be expressed as[19]

$$p_{rD} = -\frac{1}{2} Ei\left(-\frac{r_D^2}{4t_D} \right) \quad \text{..(A-2)}$$

Producing Well-Sealing Fault System

The principle of superposition can be used for the system when a sealing fault is located at a dimensionless distance of r_{dD} from the wellbore. In this case an image well of the same type can be considered with the producing wellbore in an infinite-acting system. With these wells located $2r_{dD}$ apart, the dimensionless pressure responses at the active-well location can be expressed as[2]

$$p_{wD} = -\frac{1}{2} Ei\left(-\frac{1}{4t_D} \right) - \frac{1}{2} Ei\left(-\frac{r_{dD}^2}{t_D} \right) \quad \text{......................(A-3)}$$

However, the semi-log derivative of the dimensionless pressure responses with respect to dimensionless time from Equation A-3 can be expressed as

$$p_{wDL}{}' = \frac{1}{2} e^{-\frac{1}{4t_D}} + \frac{1}{2} e^{-\frac{r_{dD}}{t_D}} \quad \text{.......................................(A-4)}$$

Bounded Systems

Closed Outer Boundary

The solution to the diffusivity equation for the dimensionless pressure at the wellbore for a reservoir whose outer boundary is closed to any fluid communication can be constructed from the solutions of Rahman and Bentsen[18] as

$$p_{wD} = \frac{4\pi t_D}{r_{eD}^2} + \frac{4\pi}{r_{eD}^2} \sum_{l=1}^{\infty} \frac{J_O(\lambda_l)\left(1-e^{-\lambda_l^2 t_D}\right)}{J_O^2(\lambda_l r_{eD})\lambda_l^2} \qquad \text{.........(A-5)}$$

The Cartesian derivative with respect to dimensionless time of Equation A-5 can be shown to be:

$$p'_{wD} = \frac{4\pi}{r_{eD}^2} + \frac{4\pi}{r_{eD}^2} \sum_{l=1}^{\infty} \frac{J_0(\lambda_l)}{J_0^2(\lambda_l r_{eD})} e^{-\lambda_l^2 t_D} \qquad \text{.........................(A-6)}$$

Constant-Pressure Outer Boundary

The solution to the diffusivity equation for the dimensionless pressure at the wellbore for a reservoir with a constant-pressure outer boundary can also be constructed from Reference 18:

$$p_{wD} = \frac{4\pi}{r_{eD}^2} \sum_{l=1}^{\infty} \frac{J_0(\lambda_l)\left(1-e^{-\lambda_l^2 t_D}\right)}{J'^2_0(\lambda_l r_{eD})\lambda_l^2} \qquad \text{...............................(A-7)}$$

The Cartesian derivative with respect to dimensionless time of Equation A-7 can be shown to be:

$$p'_{wD} = \frac{4\pi}{r_{eD}^2} \sum_{l=1}^{\infty} \frac{J_0(\lambda_l) e^{-\lambda_l^2 t_D}}{J'^2_0(\lambda_l r_{eD})} \qquad \text{.......................................(A-8)}$$

TABLE 1: Dimensionless Travel Time for Transient Responses in a Well-Sealing Fault System.

Dimensionless Distance to Sealing Fault, r_{dD}	Dimensionless Time Elapsed, t_D
50	542.871
100	2171.439
200	8685.924

TABLE 2: Significance of the D Values as Percentages of Fluid Influx Rate at the DOR.

Reference(s)	D	q_{rd}/q, %
2	0.379–1.623	96.47–51.76
This study	1.5	56.97
4	1.783	45.16
3, 8, 12	2	36.79
This study	2.146	31.62
14	2.81	13.89
6	4	1.83
7	4.29	1.00

TABLE 3: Values of T_{DS} for a Closed-Outer Boundary Reservoir.

T_{DS}	p_{wD}'	Percentage Difference from a Perfect Stabilized Condition
0.25	1.45396E-05	15.70259581
0.26	1.427E-05	13.55713147
0.27	1.40373E-05	11.70512498
0.28	1.38364E-05	10.10631012
0.29	1.36629E-05	8.725999091
0.3	1.35132E-05	7.534286666
0.31	1.33839E-05	6.505365874
0.32	1.32722E-05	5.616994813
0.33	1.31758E-05	4.849947565
0.34	1.30926E-05	4.187664058
0.35	1.30207E-05	3.615820347
0.36	1.29587E-05	3.12206601
0.37	1.29051E-05	2.695745621
0.38	1.28589E-05	2.327636153
0.39	1.28189E-05	2.009795774
0.4	1.27844E-05	1.735356948
0.41	1.27547E-05	1.498391153
0.42	1.2729E-05	1.293781559
0.43	1.27068E-05	1.117119572
0.44	1.26876E-05	0.964577517
0.45	1.2671E-05	0.832860886
0.46	1.26567E-05	0.719136721
0.47	1.26444E-05	0.620938121
0.48	1.26337E-05	0.536148325
0.5	1.26166E-05	0.399720708
0.52	1.26038E-05	0.298012742
0.54	1.25943E-05	0.222183369
0.56	1.25872E-05	0.165643576
0.58	1.25819E-05	0.123499347
0.6	1.25779E-05	0.092074203
0.62	1.2575E-05	0.068646595
0.64	1.25728E-05	0.05117934
0.66	1.25712E-05	0.038152508
0.68	1.25699E-05	0.028444057
0.7	1.2569E-05	0.021210465

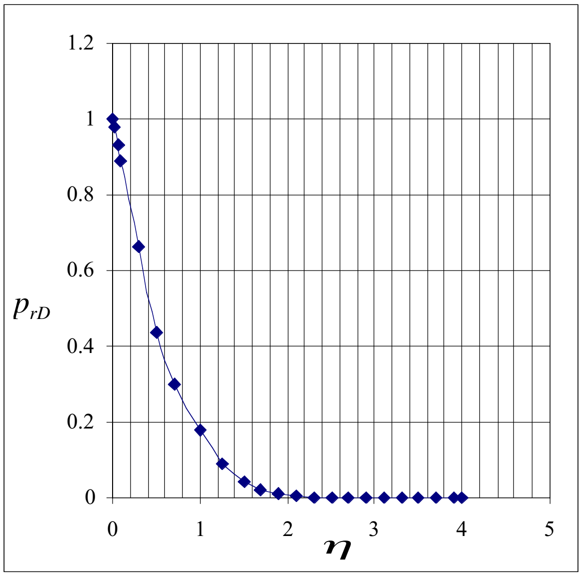

Figure 1: Dependence of p_{rD} on η.

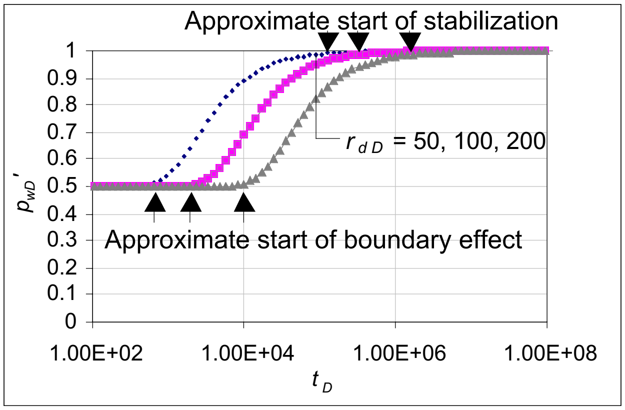

Figure 2: Semi-log derivative profiles for a well-fault system.

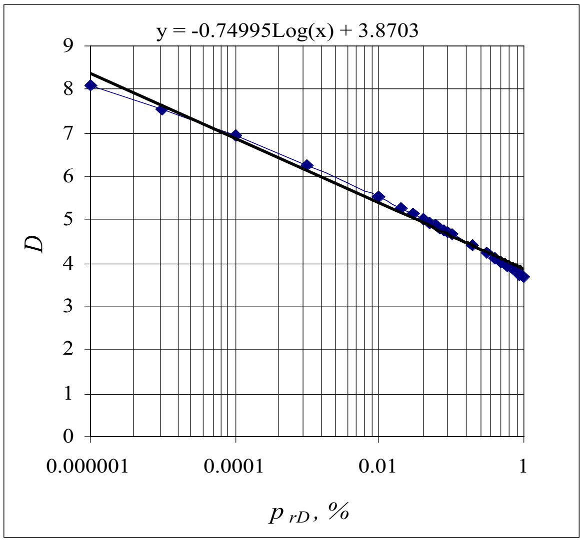

Figure 3: Best-fit line through the *D* vs. p_{rD} plot.

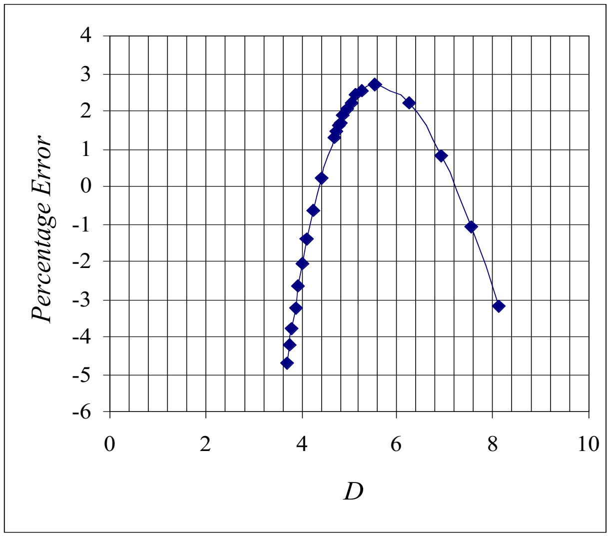

Figure 4: Expected percentage of error in estimating *D* from the proposed correlation.

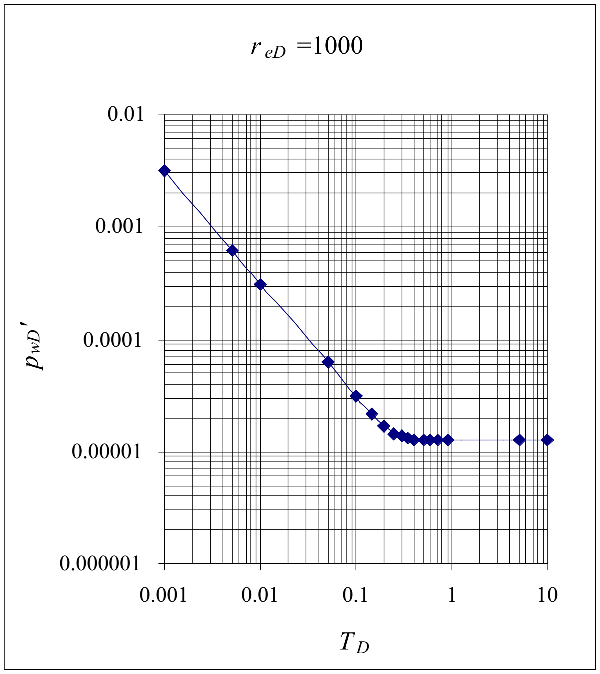

Figure 5: Cartesian derivative vs. dimensionless time.

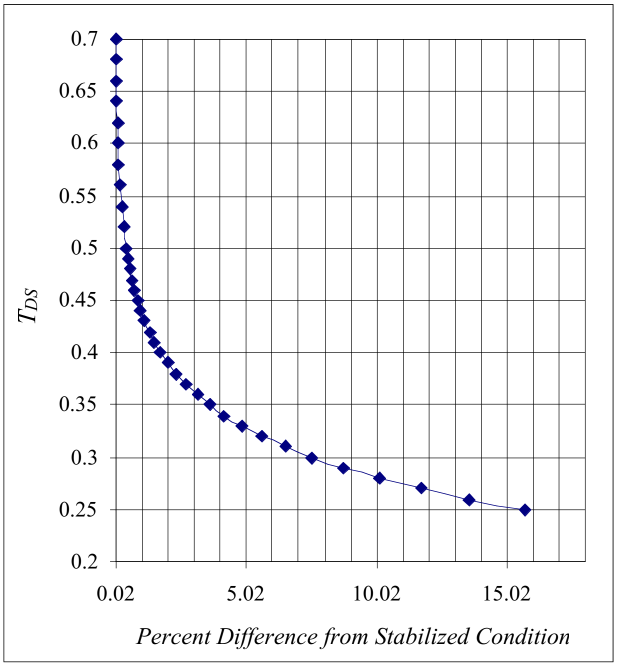

Figure 6: Variation of T_{DS} with percentage difference from a perfect stabilized condition in a closed outer-boundary reservoir.

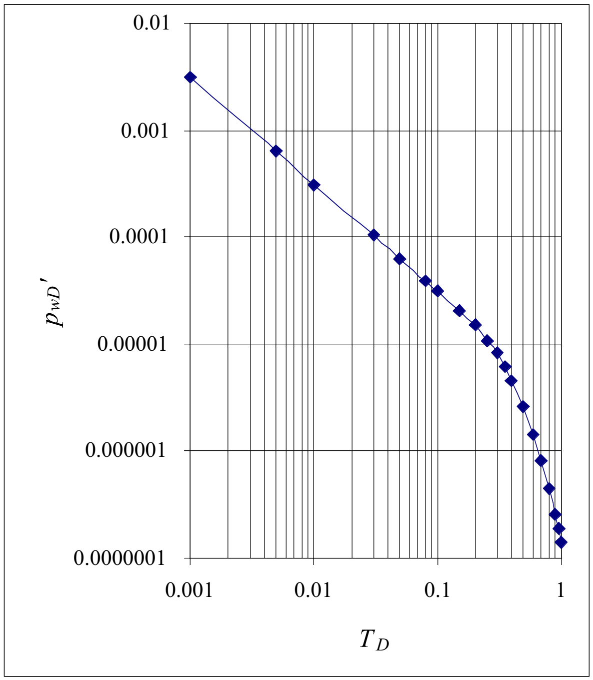

Figure 8: Variation of Cartesian derivative with dimensionless time in a constant pressure outer-boundary reservoir.

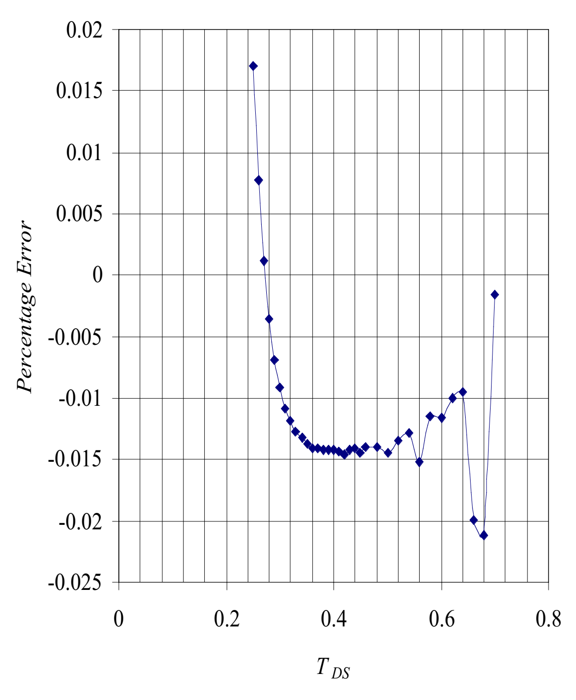

Figure 7: Expected errors in T_{DS}, when calculated from the proposed correlation.

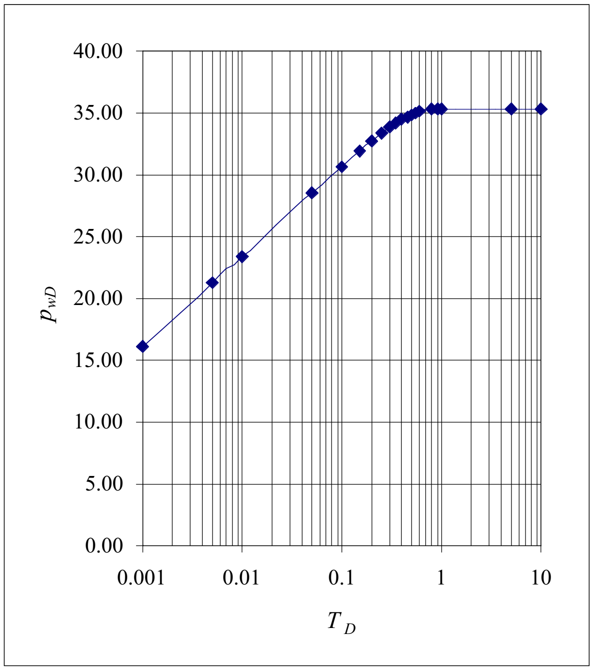

Figure 9: Variation of dimensionless pressure with dimensionless time in a constant pressure outer-boundary reservoir.

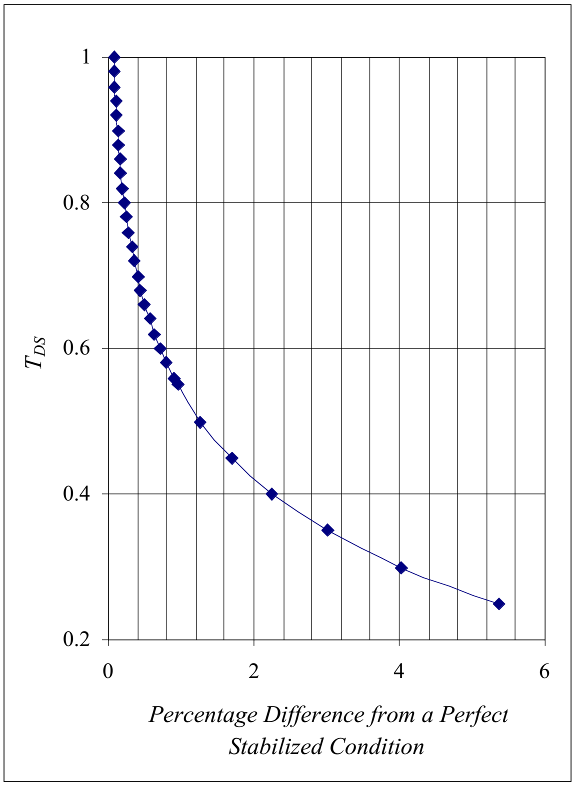

Figure 10: Dependence of T_{DS} on the percentage difference from a stabilized condition in a constant pressure outer-boundary reservoir.

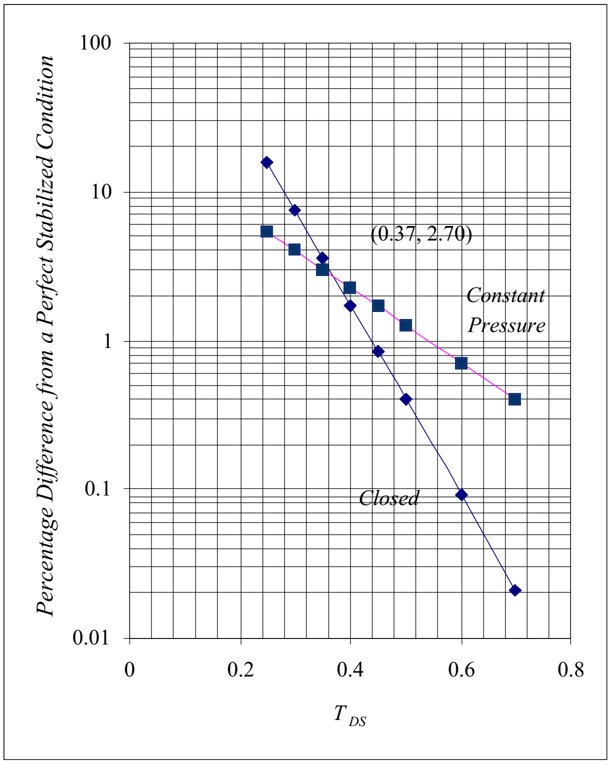

Figure 12: Comparison of two correlations for stabilized conditions.

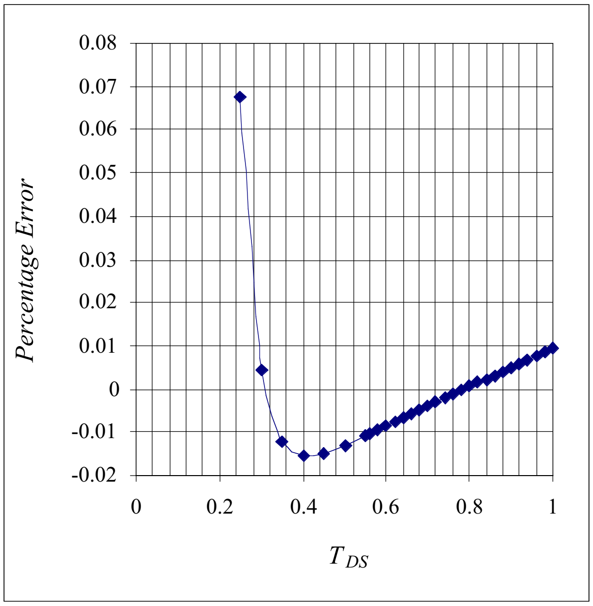

Figure 11: Expected error in T_{DS} when estimated from the proposed correlation for a constant pressure outer-boundary reservoir.