ROLES OF PRESSURE AND FLOW RATE IN DEFINING THE RADIUS OF DRAINAGE

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Abstract For a given elapsed time since the production has begun through the well bore, pressure changes occur everywhere within certain region around the well bore. The apparent radius of this region is termed as the radius of drainage. An estimation of this depends upon the criteria based on pressure and flow differentials. Previous studies have proposed correlations for the radius of drainage, based on the criteria with either pressure or production rate. This study examines the cross relationships of these considerations.

Keywords: Radius of drainage, Criterion values (pressure response and flow rate)

NOMENCLATURE

c_t	=	Total compressibility, Lt^2/m
erf η	=	Error function of η, $\int_0^{\eta} e^{-\eta^2} d\eta / \int_0^{\infty} e^{-\eta^2} d\eta$ [Important properties: erf 0 = 0, erf ∞ =1.0, erf $(-\eta)$ = erf η]
h	=	Net formation thickness, L
k	=	Reservoir rock permeability, L^2
p_i	=	Initial reservoir pressure, m/Lt^2
$p_{r,t}$	=	Pressure at radial distance r and time t, m/Lt^2
p_{wf}	=	Flowing bottom hole pressure, m/Lt^2
q	=	Flow rate, L^3/t
q_{rd}	=	Flow rate through the radius of drainage into drainage area, L^3/t
q_{wf}	=	Production rate through the well bore, L^3/t
r_d	=	Drainage radius, L
t	=	Time, t
x	=	Linear distance, L
$\Delta p_{r,t}$	=	Pressure drop at radial distance r and time t, m/Lt^2
$\Delta p_{x,t}$	=	$(p_i - p_{x,t})$, pressure drop at linear distance x and time t, m/Lt^2
Δp_{wf}	=	$(p_i - p_{wf})$, pressure drop at well bore, m/Lt^2
ϕ	=	Porosity, fraction
η	=	$\sqrt{\dfrac{\phi\mu c_t x^2}{4kt}}$
μ	=	Viscosity, m/Lt

INTRODUCTION

Estimating the radius of drainage is very important on many counts. A well-test analysis provides important reservoir information on an average basis. This information is good for the region within the radius of drainage. Thus, it is important to know how much of the reservoir is participating to provide the parameters like permeability and storage capacity of the reservoir from a well-test analysis. Another important aspect of knowing the radius of drainage is to optimize the locations of new wells to be drilled in a field. However, it has been observed that an estimation of any radius of drainage is dependent on the assumed level of criterion on pressure or flow rate. As a result, there can be substantial variations of an estimated amount of the radius of drainage.

Most of the well-test analyses assume a single well producing at a constant rate in an infinitely acting reservoir. It can be defined as the case in which the pressure responses at the well bore in a porous medium have not been tempered with by the boundary effects.

A number of investigators in the field have proposed different equations for computing the radius of drainage, based on the criteria on pressure or flow rate. *Tek et al.* (1957) defined the radius of drainage as the location across which fluid influx occurs at a rate of 1% of the flow rate at the well bore. Solution of diffusivity equation for an infinite-acting reservoir in terms of the exponential integral function has been the basis of this kind of analysis.

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Hurst et al. (1961) introduced a radius of drainage formula. This formula is based on a constant pressure at the exterior boundary with constant rate production. *Hurst et al.* cited their radius of drainage equation by graphical presentation.

Jones (1962), in discussing reservoir limit tests on gas wells, asserted that field measurements of the time it takes for a pressure disturbance at one well to be detected at a neighboring well. This equation is based on principle of superposition (image well system). He considered the distance to a fault, the interference from a gas-water contact, the evaluation of the formation constants, and the effects of anisotropic permeability.

van Poolen (1964) used the error function solution of *Bird et al.* (1960) as developed from the heat transfer concept. This solution is analogous to that for heat flow through a semi-infinite slab, so that the temperature distribution in the slab resembles the pressure response in a reservoir.

Hurst (1968) introduced a radius of drainage formula on the basis of the Lord Kelvin effect (the pressure plots linearly with the logarithm of time). *Hurst* ignored the early time region part (i.e., Bessel function's part) for the solution of diffusivity equation when he derived the radius of drainage equation.

The pressure and flow rate criteria, used to define the radius of drainage, in the literature are related. This study examines the cross relationships of these considerations.

SOLUTION OF DIFFUSIVITY EQUATION

A well is considered to be producing under the following assumptions:

- the well produces at a constant rate,
- a line sink well is considered,
- the reservoir is at uniform pressure, p_i before production begins,
- the well drains an infinite area which has a cylindrical shape,
- the reservoir is homogeneous and isotropic.

Assuming a linear segment of the radial system, the solution of the diffusivity equation can be developed with the error function (abbreviated as erf), satisfying the above assumptions in a semi-infinite reservoir. This solution to the diffusivity equation can be written as:

$$\frac{\Delta p_{x,t}}{\Delta p_{wf}} = 1 - \text{erf}\ \eta \quad \ldots\ldots(1)$$

Using the solution presented in Equation (1), Figure 1 shows the relative pressure variations along the distance from the location of the well bore.

DEVELOPMENT OF A NEW RADIUS OF DRAINAGE EQUATION

Since the definition of the radius of drainage depends on the definition of an "undetectable" pressure response, there have been a variety of definitions in the literature. *van Poolen* assumed a criterion that the radius of drainage is a point at which 1% of pressure change occurs with respect to well bore pressure change.

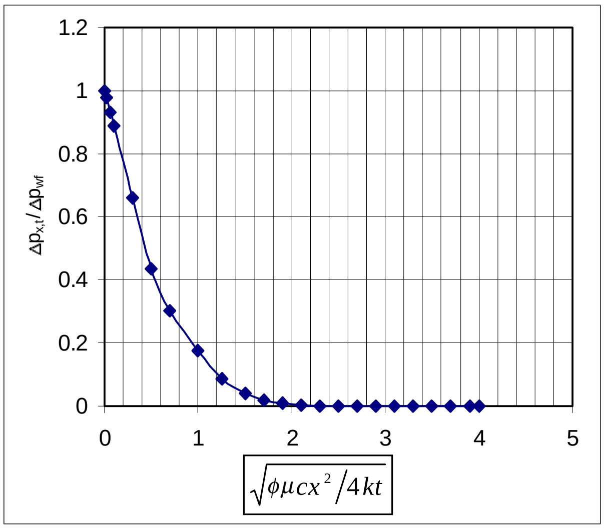


Fig.1 The pressure distribution with variable, η, for an infinite-acting reservoir.

The error function is a monotone increasing function ranging from 0 to 1. In other words, erf η = 1 when $\eta = \infty$. But for practical considerations as in Equation (1), a value of 0.999999984 for the error-function component is found when η [i.e., $(p_i - p_{x,t})/(p_i - p_{wf})$ = 1.59×10^{-08}] = 4.055 is considered. This fact can be used to define a boundary-drainage distance, x, as the distance for which the pressure change, $(p_i - p_{x,t})$, has dropped to a value of 0.000000016 $(p_i - p_{wf})$. That is, the drainage distance can be defined as the distance at which the pressure change is equal to 0% of pressure change in the well bore. Here, it should be mentioned that the pressure change would be zero at an infinite distance. In this study, considering the pressure change of 0.0000016% has been taken as zero from a practical point of view. Therefore, Equation (1) can be written as:

$$\text{erf}\ \eta = 1 \quad \ldots\ldots(2)$$

In this case, considering η = 4, Equation (1) becomes

$$x = 8\sqrt{\frac{kt}{\phi\mu\, c_t}} \quad \ldots\ldots(3)$$

This can easily be applied to a radial system considering the radial flow at constant terminal rate case. So x can be termed as the drainage radius and be expressed as

$$r_d = 8\sqrt{\frac{kt}{\phi\mu\ c_t}} \quad \text{.......................(4)}$$

This is the proposed equation for radius of drainage for an infinite-acting reservoir when an idealistic criterion is considered. The significance of this approach is discussed with respect to the contribution of the flow rate at the radius of drainage in the next section. This means, we are going to relate the pressure criteria with the rate criteria.

RELATIONSHIP BETWEEN PRODUCTION RATE AND PRESSURE RESPONSES

The line-sink solution of diffusivity equation (*Tek et al.*) for an infinite-acting reservoir can be written as:

$$p_{r,t} = p_i + \frac{q_{wf}\,\mu}{4\pi kh}\int_x^{\infty}\frac{e^{-u}}{u}du \quad \text{..................(5)}$$

Applying the Darcy's law and partial differential properties in evaluating the flow rate at radius, *r*, and time, *t* the following relationship can be written as:

$$\frac{q_{r_d}}{q_{wf}} = e^{-\frac{\phi\mu\ c_t r_d^2}{4\ kt}} \quad \text{.......................(6)}$$

The above relationship in Equation (6) expresses the relative flow rate with respect to the production at the well bore at a distance, r_d, which is estimated as a radius of drainage. Thus, it has been established that the radius of drainage is related to the flow rate as well as pressure response of the well. Equations (1) and (6) have been used to develop Table 1. This table shows the cross relationship between the criteria based on pressures and flow rates.

DISCUSSION

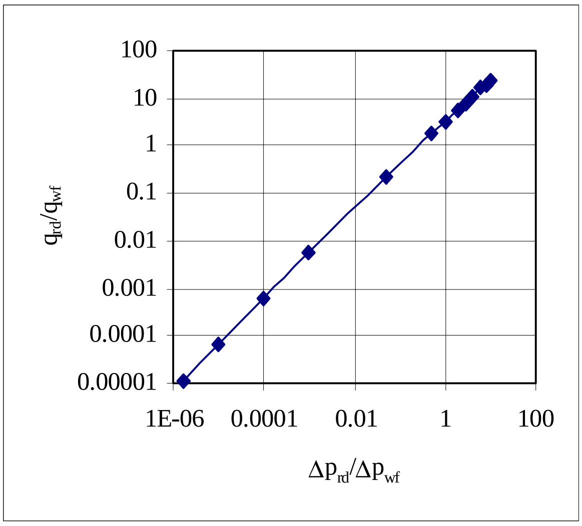
In the present study, it has been established that the radius of drainage is related to the criterion values of pressure response of the well. The variation on percentage of pressure response will change the percentage of flow rate at the drainage boundary. Figure 2 presents the roles of percentage pressure responses and flow rate changes with respect to well bore. With the increase of percentage of pressure response, the percentage of flow rate will increase. Theoretically the drainage radius is defined, as the radius beyond which there would be no fluid flow through the drainage boundary of the reservoir.

As shown in Table 1 and Figure 2, 1% of pressure response introduces 3.32% of fluid influx through the drainage boundary. But ideally this fluid flow should be

Table 1 Values showing a relationship between pressure and flow rate differentials in percentages.

$\Delta p_{r_d}/\Delta p_{wf}$ %	q_{r_d}/q_{wf} %
10.0	23.13
8.0	18.45
6.0	16.16
4.0	10.54
3.0	7.73
2.0	5.56
1.0	3.32
0.5	1.79
0.05	0.22
0.001	0.00556
0.0001	0.00061
0.00001	0.000067
0.0000016	0.000011

zero. So a consideration of taking 1% of pressure response as a criterion value is an approximation of the ideal situation. If 0.0000016% of pressure response is considered at the drainage boundary, the fluid influx is 0.000011% of well bore flow. When one requires a higher precision in estimating the radius of drainage, the newly proposed equation (Equation (4)) will be useful. Simultaneously, it is possible to be aware of the corresponding relative rate of fluid influx into the drainage area across the assumed drainage radius. Table 1 shows that the estimated drainage radius as with Equation (4) is subject to 0.000011% influx into the drainage area. There is always some room for argument that the 3.32% relative rate of fluid influx corresponding to the 1% pressure change criterion into the drainage area is too much for defining a radius of drainage. Therefore, the consideration of the idealistic situation as with Equation (4) will allow one to choose a realistic



value of criterion based on pressure or flow rate. In other words,

Fig.2 Effects of pressure response on flow rate at the drainage boundary.

The proposed Equation (4) for radius of drainage will be useful as a benchmark.

CONCLUSIONS

- The criteria based on pressure and flow rate to define the radius of drainage are related.
- As the criterion gets close to the ideal condition in terms of pressure (e.g., $\Delta p \rightarrow 0$), the estimated drainage of radius increases twice as much that predicted using the criteria of 1% pressure differential. However, the contribution of flow rate decreases dramatically from 3.32% to 0.000011%.

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