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A RIGOROUS DIFFUSIVITY EQUATION WITH THE INCLUSION OF VARIABLE ROCK/FLUID PROPERTIES FOR POROUS MEDIA

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ABSTRACT

The complex rheological behavior of formation fluids and their structures depend on their history of formation and propagation. In petroleum engineering, this history tells the qualitative and quantitative measures of fossil fuels in a petroleum trap. The properties of fluid and formation are thoroughly investigated, as evidenced by the shear volume of publications on the topic. However, still there remains a formidable challenge on how to model the variation of these properties under complex in situ thermal and mechanical stress. These fluid properties as well as rock properties are assumed to be invariant with time in most of the available simulators. The variation of permeability is important when there is mineralization in the pore network, which is subject to significant change during fluid extraction and depressurization. The main aim of this study is to model the variable permeability and viscosity over time. The concept of memory is applied to model these variations with time in the fluid flow through the porous medium. A new rigorous model for the fluid flow inside the porous formation is introduced using the continuity equation and a new form of momentum balance equation based on the application of the memory concept. The proposed model is rigorous in the sense that the fluid and formation properties are considered as space and time dependent. This model can be used in any crude oil flow through porous media.

Keywords: Complex rheology; reservoir formation; time function; rock-fluid properties; pore network; Darcy model.

1. INTRODUCTION

Rock and fluid properties play an important role during fluid flow in porous media. Majority of rock and fluid properties are functions of pressure and temperature. Formation rock is the fluid transport media where fluid properties change during any pressure disturbance or thermal action in the formation. As a result, the rock properties such as permeability, porosity, pore volume, and, sometimes, formation wettability are greatly influenced by fluid properties. The available literature shows that existing fluid flow models are based on the above mentioned pressure and temperature related properties for both rock and fluid. However, there are some other naturally occurred actions such as mineral precipitation, chemical reaction between fluid and rock media, existence of solid particles in fluid may have some influence in the behavior of fluid flow in porous media. All the parameters mainly affect the flow path in the formation. Based on the situation and pathway traveled, the pore space may squeeze or enlarge which varies in space and time. The above actions that take place in the pore network may change permeability and porosity. The normal fluid motion equations such as Darcy's law do not allow one to consider the variation of fluid and rock properties in a proper way. Darcy's law has its own limitations of consideration of homogeneous media and constant fluid and rock properties. Therefore, Hossain et al. [1 – 3] used the modified Darcy's law in such a way that it may create some options of using the variable rock and fluid properties during the development of theoretical fluid flow models.

Hossain and Islam [4] presented an extensive review of fluid memory on the available literature and models. They showed that different researchers tried to identify and define the fluid memory with different fluid properties such as stress, density, free energies and others. Some researchers also mentioned the effects of pathway and dependence on history of the fluid to define the memory [5 – 7]. Memory is a function of time and space and forward time events depend on previous time events [7]. The memory effect is sensitively dependent on the intermediate diffusion time scale and this has to be chosen depending on the characteristic time scale of the mechanism of interest. This model is for homogeneous surrounding media and is not sufficient to describe the full impact of fluid memory on flow behavior and in media. Within a porous medium, mobilization and subsequent flow of a fluid with a yield stress can be explained well when the notion of memory is introduced [8]. This concept explains the macroscopic threshold (minimum pressure gradient) which directly follows from the geometry of the path, along which mobilization first occurs. The minimum threshold path (MTP) is connected through nearest neighboring paths between two given boundaries (or points), along which the sum of thresholds is the minimum possible. Gatti and Vuk [9] studied the motion of a linear viscoelastic fluid in a two-dimensional domain with periodic boundary conditions for the asymptotic behavior. They consider an isotropic homogeneous incompressible fluid of Jeffrey's type where the Reynolds number is equal to one. They also assumed that density is independent of time. In addition they assumed that pressure and velocity are independent of time. These assumptions follow the conventional models.

This article introduces a rigorous, new fluid flow model where the variable rock/fluid properties are considered. The full length derivation and mathematical formulation are available at Hossain and co-author's references [1 – 3, 10]. This model can also be applicable to any non-Newtonian fluid flow during an enhanced oil recovery (EOR) process. It is also useful where variable rock compressibility exists and is affected by the pressure decline during the production life of a reservoir.

2. MATHEMATICAL MODEL

A general form of diffusivity equation with memory has been derived which is a non-linear diffusivity equation with fluid memory for an axial flow of any single phase fluid in porous media [1 – 3, 10]. Equation (1) is strictly non-linear because both the first part (due to dependence of η on pressure) and the second part are non-linear. In addition, the third part consists of a derivative of a pressure derivative with memory part. The fourth part consists of pressure derivative with time, η compressibility and porosity multiplication which also makes this term nonlinear. If the effect of memory is neglected (i.e. $\alpha = 0$), this equation reduces to the conventional form of diffusivity equation.

$$\frac{1}{\eta}\frac{\partial \eta}{\partial x}\left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial \xi\, \partial x}\right) d\xi}{\Gamma(1-\alpha)}\right] + c_f \frac{\partial p}{\partial x}\left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial \xi\, \partial x}\right) d\xi}{\Gamma(1-\alpha)}\right]$$

$$+\frac{\partial}{\partial x}\left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial \xi\, \partial x}\right) d\xi}{\Gamma(1-\alpha)}\right] = \frac{\phi\, c_t}{\eta}\frac{\partial p}{\partial t} \qquad (1)$$

This equation is solved using the initial condition, $p(x,0) = p_i$ and two boundary conditions as the external boundary with no flow boundary and the interior boundary with a constant production rate boundary. Therefore, the outer boundary can be expressed as $p_{i+1}^n = p_{i-1}^n$. The inner boundary can be written as $p_{i+1}^n = p_{i-1}^n + q/e_1$, where $e_1 = -\frac{k\, A_{yz}}{2\mu\, \Delta x}$.

3. RESULTS AND DISCUSSION

Figure 1 presents the variation of pressure with distance from the wellbore towards the outer boundary of the reservoir based on Darcy's diffusivity equation. Pressure response increases towards the boundary with time. The maximum pressure drop is in the wellbore and it gradually increases up to the initial reservoir pressure with distance. Pressure drop reaches at initial pressure at a distance approximately 850 m after 10 month whereas it reaches at around 200 m after 1 month of production.

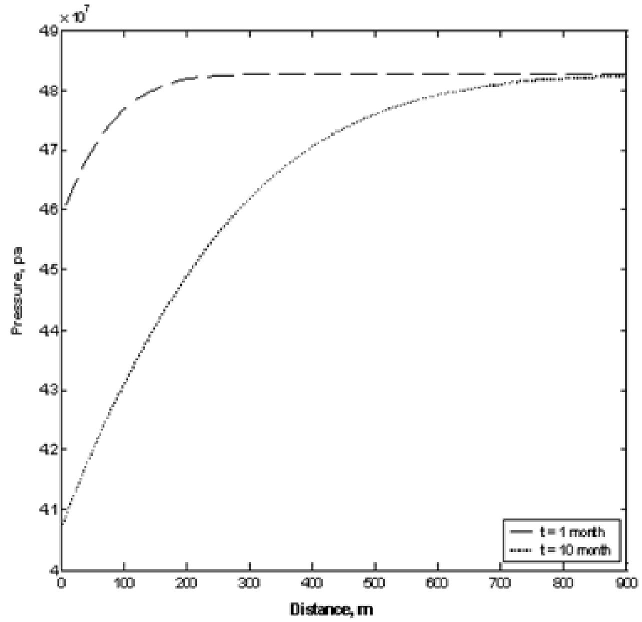

Figure 1. *Pressure variation with distance based on Darcy diffusivity equation*

The effects of Z-values with distance are depicted in Figure 2. The variation is shown for 1 and 10 months where Darcy diffusivity and $\alpha = 0.2$ are considered during computation. Z-values increase very fast around the wellbore and reach its pick at a distance of 120 m and 170 m for 1 and 10 months respectively. The decreasing trend begins after the pick and becomes zero after 350 m and 1200 m. The effects of z-values increase at a wider range of reservoir area from the wellbore with the increase of production life of reservoir. Therefore, it can be concluded that at the beginning of production life, there is a great impact around the wellbore and as time passes, this impact affects throughout the reservoir which becomes 0 at the outer boundary of the reservoir.

3.1 Variation of η with Distance

Figure 3 shows the variation of η with distance from the wellbore towards the outer boundary of the reservoir for $\alpha = 0.1$. This figure compares the Darcy diffusivity equation and proposed diffusivity equation with memory. Figure 3(a) is plotted for 1 month after the start of production and Figure 3(b) is for 2 months. There is no difference of change of η with respect to distance at the initial stage of the reservoir production (Figure 3(a)). However, the difference of changing η becomes significant with production time (Figure 3(b)). η variation is more sensitive with time and around the wellbore of the reservoir which is captured by the notion of "memory". The numerical value of η is substantially reducing when production continues with time and the effects spread through the reservoir with time (Figure 3(b)) which is only capturing by proposed model. So, it can be concluded that there is a strong effects of "memory" in describing fluid flow through porous media.

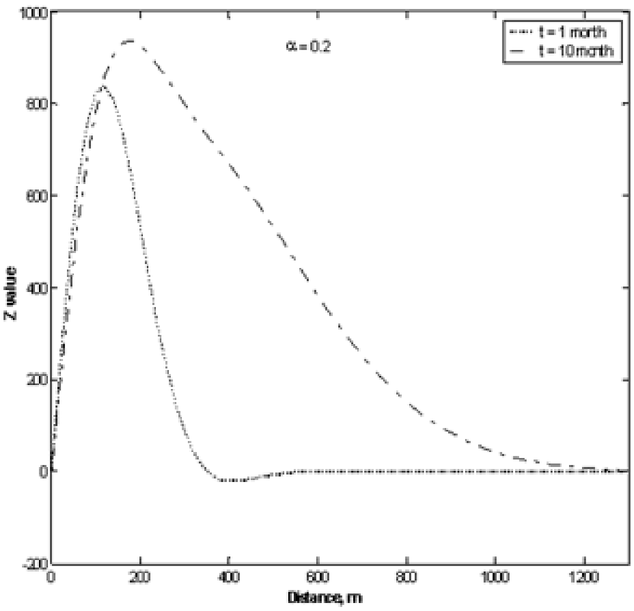

Figure 2. *Z variation with distance based on Darcy diffusivity equation for* $\alpha = 0.1$

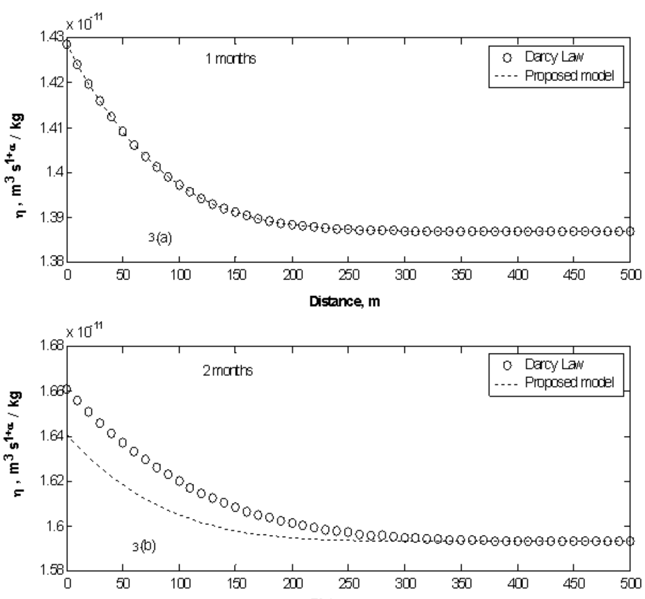

Figure 3. η *variations with distance for* $\alpha = 0.1$

3.2 Variation of η with Reservoir Pressure

The variation of η with reservoir pressure is depicted by Figure 4 for the proposed model. The trend of the curve is highly non-linear and it is an elliptical shape. At the beginning of pressure, η variation is very high and it starts to decrease with the increase of pressure. The variation of η becomes stable at very high pressure where there is no change of η value.

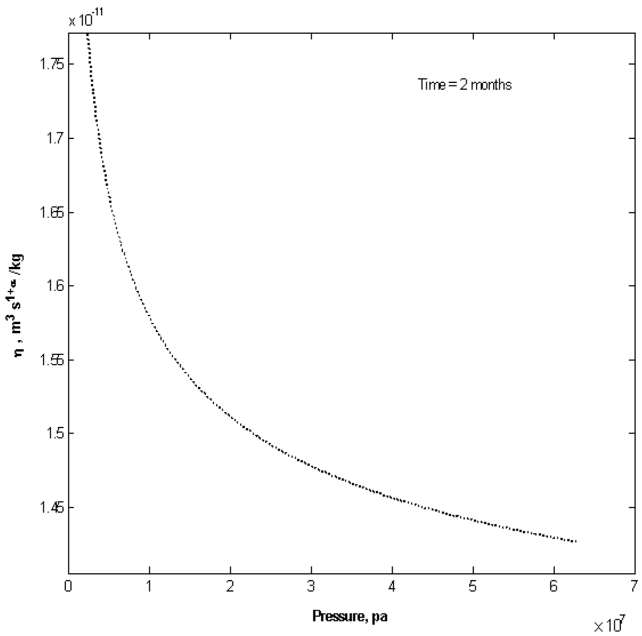

Figure 4. *Variation of* η *with pressure using proposed model with memory for* $\alpha = 0.1$.

The pressure response with respect to the variation of η is compared for both Darcy model and proposed model in Figure 5(a) when $\alpha = 0.1$. Figure 5(a) is plotted when reservoir production time is considered as 1 month and Figure 5(b) is shown the comparison for 2 months of production life. It is very interesting that after 1 month of production, there is no pressure variation over time or over η when proposed model is used to calculate the pressure variation throughout the reservoir (Figure 5(a)). However, the variation of pressure starts to decline with η after one month. This only due to the memory effects on fluid and rock which is not possible to capture by conventional diffusivity equation. The declining pressure variation response is dominant with time. As production time of the reservoir increases, the effect of memory becomes dominant and gives a substantial difference in pressure response (Figure 5(b)).

3.3 Porosity Change with Distance

The variation of porosity over distance from the wellbore towards the outer boundary of the reservoir is shown in Figure 6 using the conventional diffusivity equation and proposed model where $\alpha = 0.1$ is considered. At the beginning of the production, there is no substantial difference of porosity change with the initial porosity of 0.30 **(**Figure 6(a)). Use of Darcy diffusivity equation, does not give any significant variation of porosity. However, use of proposed model gives a difference of porosity value of 0.292 which represent a contribution of memory effect on rock property. This change is almost same throughout the reservoir. The variation of porosity becomes significant with the hydrocarbon production life. Figure 6(b) shows the variation of porosity over time at different reservoir position.

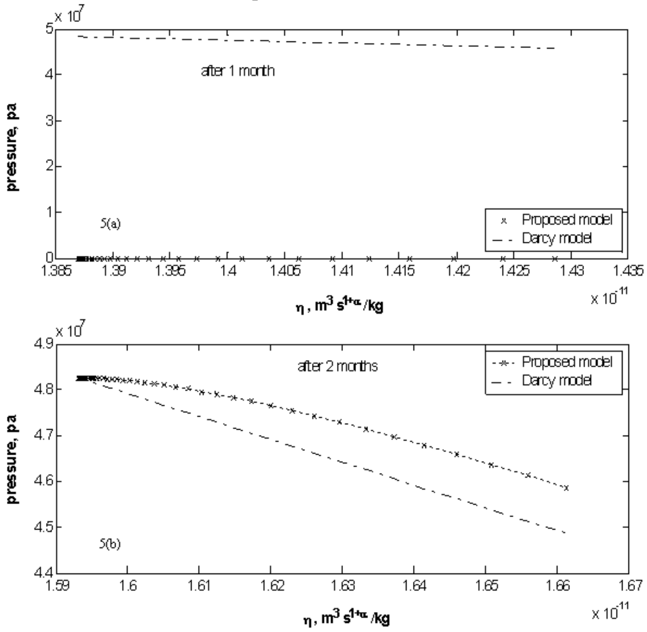

Figure 5. *Variation of pressure with η based on Darcy diffusivity equation and proposed diffusivity equation with memory for* $\alpha = 0.1$.

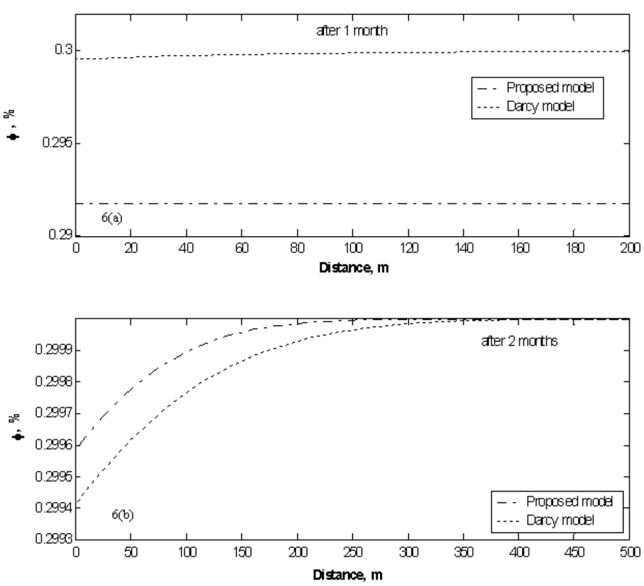

Figure 6. *Variation of porosity and distance using Darcy diffusivity equation and proposed diffusivity equation with memory for* $\alpha = 0.1$.

4. CONCLUSIONS

A diffusivity equation with rock/fluid memory has been developed by invoking time-dependence to permeability and viscosity. The resulting highly non-linear theoretical model has an option of triggering the memory variable, depending on the applicability. These equations were solved in their non-linear form, showing the difference in prediction between the conventional diffusivity equation and the new memory-induced diffusivity. The results indicate that this model can be used for a wide range of applications. The variation of η for different reservoir parameters is identified and the effects of "memory" is shown using both Darcy model and proposed model. The findings of this research establish the contribution of memory in reservoir fluid flow through porous media.

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6. NOMENCLATURE

A_{yz} = Cross sectional area of rock perpendicular to the flow of flowing fluid, m^2
c_f = $c_o + c_w$ = total fluid compressibility of the system, $1/pa$
c_s = formation rock compressibility of the system, $1/pa$
c_t = $c_f + c_s$ = total compressibility of the system, $1/pa$
c_w = formation water compressibility of the system, $1/pa$
k = initial reservoir permeability, m^2
L = distance between production well and outer boundary along x direction, m
p = pressure of the system, N/m^2
p_i = initial pressure of the system, N/m^2
p_o = a reference pressure of the system, N/m^2
q_i = Au = initial volume production rate, m^3/s
q_x = fluid mass flow rate per unit area in *x*-direction, kg/m^2s
t = time, s
u = filtration velocity in *x* direction, m/s
u_x = fluid velocity in porous media in the direction of *x*, *y* and *z* axis, m/s
x = flow direction for 1-D for a particular point, *x* from wellbore, m
α = fractional order of differentiation, dimensionless
ρ = density at pressure *p*, kg/m^3
ρ_o = density at a reference pressure p_o , kg/m^3
ϕ = porosity of fluid media at pressure *p*, m^3/m^3
ϕ_o = porosity of fluid media at reference pressure p_o, m^3/m^3
μ = fluid dynamic viscosity, pas
η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3s^{1+\alpha}/kg$

ξ = a dummy variable for time i.e. real part in the plane of the integral, *s*

Γ = a gamma function

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