Journal of Porous Media, 15 (2): 173–186 (2012)

COMPREHENSIVE MODELING OF COMPLEX PETROLEUM PHENOMENA WITH AN ENGINEERING APPROACH

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Original Manuscript Submitted: 2/24/2010; Final Draft Received: 6/4/2010

It is well established that reservoir simulation studies are very subjective and varies from simulator to simulator. However, reservoir simulation, as practiced in the oil industry, is well recognized and is the standard tool for solving reservoir engineering problems. Unfortunately, almost all the existing simulators and their mathematical models are based on the conventional approach. It is also well-known that this approach comprises inherent assumptions which result the linearization of the model and solution. In contrary, the concept of engineering approach allows to bypassing linearization during the development of model equations. To avoid the hidden misconceptions and uncertainty in formulation, engineering approach is becoming more popular as long as it is more easily understandable. The present study critically reviews the inherent shortcomings of the conventional approach and analyzes the hidden assumptions behind the conventional approach and identifies the limitations of engineering approach. The strengths and weaknesses of the both approaches are outlined. The comprehensive modeling of complex petroleum phenomena will help researcher and industry to rethink and revisit their contribution in reservoir simulation. It will also help to build a new rigorous simulator using the noble concept.

KEY WORDS: *reservoir simulation, mathematical approach, engineering approach, discretization, fluid flow equation, inherent assumptions, grid block, critical review*

1. INTRODUCTION

In this information age, almost all phases of reservoir engineering problems are solved by reservoir simulators, ranging from a simple decision through well testing to prediction of enhanced oil recovery. For every application there is a separate user-friendly and custom-designed simulator. Even though, quite often, "comprehensive," "all-purpose," and other denominations are used to describe a company simulator, every simulation study is a unique process, starting from the reservoir description to the final analysis of the results. Therefore, reservoir simulation is the art of combining science (i.e., physics, chemistry, etc.), mathematics, reservoir engineering, and computer programming to develop a tool for predicting hydrocarbon reservoir performance under various operating strategies. The first step of simulation is to develop a model equation which should be the true representation of the real scenario of the problem. In most cases, it is observed that the model equation is not the true representation of the natural phenomena due to spurious assumptions and some built-in limitations of the conventional mathematical equations. Also there are other immense challenges that lay behind the formulation of the model. Due to the built-in shortcomings of the conventional approach, currently the entire reservoir simulation process is facing significant disagreements. This research captures those mysterious and unrealistic considerations of the conventional simulation approach. On the other hand, the engineering approach shows how to bypass the conventional formulation of the model equations. The model equations are written for a given gridblock in space at a given time level. These equations reflect the flow equations in an algebraic form. The most important features of the model equations are the bypassing scope of the formulation process,

1091–028X/12/$35.00

NOMENCLATURE

$f(t+\Delta t)$	a function of time $(t+\Delta t)$
Ma	Marangoni number
R	universal gas constant, $[kJ/mol-K]$
p	pressure, psia [kPa]
p_i	pressure of gridblock i, psia [kPa]
p_i^n	pressure of gridblock i at time, psia [kPa]
p_i^{n+1}	pressure of gridblock i at time t^{n+1}, psia [kPa]
p_{i-1}	pressure of gridblock $i-1$, psia [kPa]
p_{i-1}^n	pressure of gridblock $i-1$ at time t^n, psia [kPa]
p_{i-1}^{n+1}	pressure of gridblock $i-1$ at time t^{n+1}, psia [kPa]
p_{i+1}	pressure of gridblock $i+1$, psia [kPa]
p_{i+1}^n	pressure of gridblock $i+1$ at time t^n, psia [kPa]
p_{i+1}^{n+1}	pressure of gridblock $i+1$ at time t^{n+1}, psia [kPa]
q_{sc}	well volumetric rate at standard conditions, STB/D or scf/D [std m^3/d]
q_{sc_i}	well volumetric rate at standard conditions in gridblock i, STB/D or scf/D [std m^3/d]
$q_{sc_{i\pm1/2}}$	interblock volumetric flow rates at standard conditions between gridblock i and g^y direction, [m/s/m]

Greek Symbols

α_c	volume conversion factor = 5.614583 for customary units or 1 for SPE preferred SI units
β_c	transmissibility conversion factor = 0.001127 for customary units or 0.0864 for SPE preferred SI units
Δt	time step, [day]
Δx	size of control volume in the x direction, ft [m]
μ	fluid viscosity, cp [Pa s]
σ	surface tension, [N/m]
α_D	thermal diffusivity, [m^2/s]
μ_T	fluid dynamic viscosity at temperature T, [cp]
μ_0	fluid dynamic viscosity at a reference temperature, T_0, [cp]
α	fractional order of differentiation
ϕ	porosity of fluid media, [m^3/m^3]
τ_T	shear stress at temperature T, [Pa]
ξ	dummy variable for time, i.e., real part in the plane of the integral, [s]
ρ_0	density of the fluid, [kg/m^3]
η	ratio of the pseudopermeability of the medium with memory to fluid viscosity, [m^3 s$^{1+\alpha}$/kg]
$\partial\sigma/\partial T$	the derivative of surface tension σ with temperature and can be positive or negative depending on the substance, [N/mk]
$f(t+\Delta t)$	a function of time $(t+\Delta t)$

Acronyms

PDEs	partial differential equations
SPE	Society of Petroleum Engineers
REV	representative elemental volume
1D	one-dimensional system
2D	two-dimensional system
3D	three-dimensional system
LHS	left-hand side
RHS	right-hand side
FVF	formation-volume factor

consideration of fluid and rock properties in a gridblock form, and discretization of those blocks into thousands of blocks. The combination of the independent block equations results in a realistic presentation of the model equation of the reservoir without any spurious assumptions. This is the core strength of the engineering approach.

While Society of Petroleum Engineers (SPE) benchmarking has helped to accept differences in predicting petroleum reservoir performance, there has been no scientific explanation behind the variability that has frustrated many policymakers, operations managers, and puzzled scientists/engineers. Hossain and Islam (2010a) ex-

plained a new approach, namely, a "knowledge-based" approach where they considered adding the knowledge dimension to the problem. They showed that reservoir simulation equations have embedded variability and multiple solutions that are in line with physics rather than spurious mathematical solutions. With this clear addition of knowledge in reservoir simulation, a fresh perspective in this area is needed. Unlike the majority of reservoir simulation approaches available today, the knowledge-based approach does not stop at questioning the fundamentals of reservoir simulation but offers solutions and demonstrates that proper reservoir simulation should be transparent in order to empower decision makers rather than creating a black box. In this regard, the engineering approach is the proper analytical method to empower the planner's decision because it no longer creates a black box simulator due to its inherent strength in formulation. As mathematical developments of new governing equations occur based on in-depth understanding of the factors, these equations influence fluid flow in porous media under different flow conditions, which is again the strength of the engineering approach. Behavior of the flow-through matrix and fractured systems in the same reservoir, heterogeneity and rock/fluid property interactions, Darcy and non-Darcy flow, and variable rock/fluid properties are among the issues thoroughly needing to be addressed during the development of a commercial simulator.

2. THE FUNDAMENTAL NATURE OF RESERVOIR SIMULATION

It is difficult to capture the natural phenomena and its behavior over time. Sometime it is not possible to explain the features of reality due to the highly nonlinear and chaotic behavior of the natural process. Underground reservoir behavior is not a different case except the true presentation of natural phenomena. In most cases the researchers tried to linearize the natural and chaotic behavior through spurious assumptions. Current literature shows that the formulation of fluid flow equations in discretized form (nonlinear algebraic equations) can be obtained by either the conventional approach or the engineering approach (Ertekin et al., 2001; Abou-Kassem et al., 2006). Both of these approaches make use of the same basic principles and both approaches discretize the reservoir in gridblocks or gridpoints. Both approaches yield the same discretized flow equations for modeling any reservoir-fluid system (multiphase, multicomponent, thermal, heterogeneous reservoir) using any coordinate system in one-, two-, or three-dimensional (1D, 2D, or 3D) reservoirs (Abou-Kassem et al., 2006). Both approaches consider Darcy's law as the constitutive equation that describes the rate of fluid movement into or out of the reservoir element. At this point, the engineering approach considers only the assumptions behind Darcy's law as valid. Recently Hossain et al. (2009a) modified this shortcoming of the engineering approach by using the memory concept along with it. The present study is limited to the conventional and the engineering approach only.

2.1 Conventional Approach

It is worth mentioning that the researchers need to address the core issues and the existing nature of reservoir simulation that lead to spurious and uncertain results and conclusions. The first step is to identify the most inherent shortcomings of the existing developmental features toward the reservoir simulator. Odeh (1982) is probably the researcher who initiated and depicted the major steps involved in the development of a reservoir simulator. To characterize the reservoir simulator, this approach is developed using major steps such as formulation, discretization, well representation, linearization, solution, and validation. In the conventional approach (Fig. 1), the algebraic flow equations are derived in three consecu-

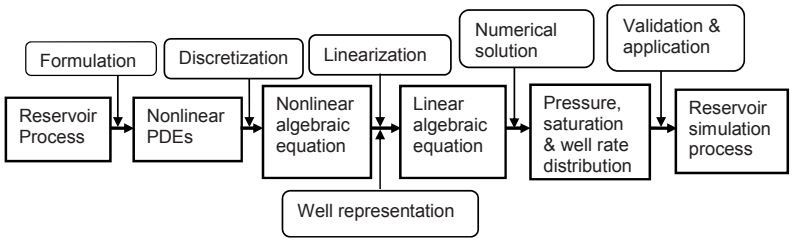

FIG. 1: Major steps used to develop reservoir simulators based on the conventional approach (modified from Hossain et al., 2010b).

tive steps: (i) derivation of the partial differential equations (PDEs) describing fluid flow in the reservoir using the three basic principles (mass conservation, equation of state, constitutive equations), (ii) discretization of the reservoir into gridblocks or gridpoints, and (iii) discretization of the resulting PDE in space and time (Abou-Kassem, 2008). The formulation step outlines the basic assumptions inherent to the simulator. These assumptions in precise mathematical terms apply to a control volume in the reservoir (Fig. 1).

Newton's approximation is used to render these control volume equations into a set of coupled, nonlinear PDEs that describe fluid flow through porous media (Ertekin et al., 2001). These PDEs are then discretized, giving rise to a set of nonlinear algebraic equations. Taylor series expansion is used to discretize the governing PDEs. Even though this procedure has been the standard in the petroleum industry for decades, in 2006 Abou-Kassem et al. pointed out that this is unnecessary. They introduced a new discretization procedure called an "engineering approach." By setting up the algebraic equations directly, one can make the process simple and yet maintain accuracy (Mustafiz and Islam, 2008). The PDEs that are derived during the formulation step, if solved analytically, would give reservoir pressure, fluid saturations, and well flow rates as continuous functions of space and time. Because of the highly nonlinear nature of the PDEs, analytical techniques cannot be used and solutions must be obtained with numerical methods. In contrast to analytical solutions, numerical solutions give the values of pressure and fluid saturations only at discrete points in the reservoir and at discrete times. Discretization is the process of converting PDEs into algebraic equations. Several numerical methods can be used to discretize the PDEs. However, the most common approach in the oil industry today is the finite-difference method. To carry out discretization, a PDE is written for a given point in space at a given time level. The choice of time level (i.e., old time level, current time level, or the intermediate time level) leads to the explicit, implicit, or Crank-Nicolson formulation method. The discretization process results in a system of nonlinear algebraic equations. These equations generally cannot be solved with linear equation solvers, and linearization of such equations becomes a necessary step before solutions can be obtained. Well representation is used to incorporate fluid production/injection into the nonlinear algebraic equations. Linearization involves approximating nonlinear terms in both space and time. Linearization results in a set of linear algebraic equations. Any of the several linear equation solvers can then be used to obtain the solution. The solution comprises pressure and fluid saturation distributions in the reservoir and well flow rates. Validation of a reservoir simulator is the last step in developing a simulator, after which the simulator can be used for practical field applications. The validation step is necessary to make sure that no error was introduced in the various steps of development and in computer programming.

2.2 Engineering Approach

The features of the steps force the researchers and developers of reservoir simulators to depend on mathematics in the first two steps of the conventional approach to obtain the third step of finding out the nonlinear algebraic equations or finite-difference equations (Fig. 1). In contrast, it is possible to bypass the step of formulation in the form of PDEs and directly express the fluid flow equation in the form of a nonlinear algebraic equation, as pointed out by Abou-Kassem (2008). In fact, by setting up the algebraic equations directly, one can make the process simple and yet maintain accuracy through the engineering approach. In the engineering approach, the finite-difference equations are derived without going through the severity of PDEs and discretization (Fig. 2). In this approach the derivation of the nonlinear algebraic flow equation is straightforward. It is accomplished in two consecutive steps up to the derivation of nonlinear algebraic flow equations where the formulation process of the conventional approach is absent. Figure 2 shows these steps as a discretization of the reservoir into gridblocks or gridpoints to remove the effect of the space variable. The next step is then the derivation of the algebraic flow equation for gridblock i (or gridpoint i) using the three basic principles, taking into consideration the variation of interblock flow terms and the source/sink term with time within a time step. The approximation of the time integrals is then considered in the resulting flow equation to produce a finite nonlinear algebraic flow equation for time t. The basic difference between these two approaches is that one has room for the time approximation (engineering) and the other (conventional) does not have that option. Once a nonlinear algebraic equation is derived, the well presentation is added to include production/injection wells. Using the approximation of time integral, the equation truncates to an intermediate stage where one can solve the equation numerically using different numerical schemes. The solution can be validated using an experimental procedure. It is also possible to compare it with other existing natural phenomena which has reasonable results. Finally, the validated model is being used to develop a new simulator.

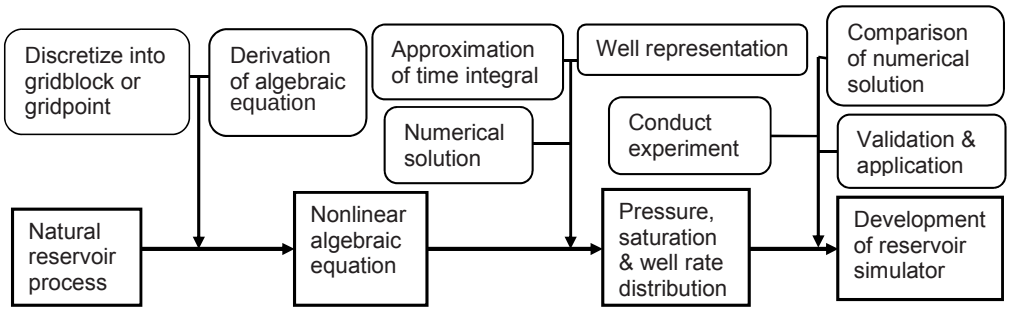

FIG. 2: Major steps used to develop reservoir simulators based on the engineering approach.

2.3 The Outlook of Both Approaches

There are three methods available for the discretization of any PDE: (i) the Taylor series method, (ii) the integral method, and (iii) the variational method (Aziz and Settari, 1979). The first two methods result in the finite-difference method, whereas the third results in the variational method. The "mathematical approach" refers to the methods that obtain the nonlinear algebraic equations through deriving and discretizing the PDEs. Developers of simulators rely heavily on mathematics in the conventional approach to obtain the nonlinear algebraic equations or the finite-difference equations. However, sometimes they do not realize the inherent assumptions behind this approach. Abou-Kassem et al. (2006) introduced a new approach which is used to derive the finite-difference equations without going through the rigor of PDEs and discretization. This approach utilizes fictitious wells to represent boundary conditions. They describe this approach as an "engineering approach" because it is closer to the engineer's thinking and to the physical meaning of the terms in the flow equations. Both approaches treat boundary conditions with the same accuracy if the conventional approach uses second-order approximations. The engineering approach is simple and yet general and rigorous—another strength of the engineering approach. In addition, it results in the same finite-difference equations for any hydrocarbon recovery or reservoir process. The engineering approach is independent of the mathematical approach. It reconfirms the use of the central-difference approximation in space discretization and highlights the assumptions involved in choosing a time level in the mathematical approach. Therefore the engineering approach does not need those approximations, which is the most significant contribution of this approach.

Abou-Kassem (2007) pointed out that both approaches give identical equations for specified flow rate, pressure gradients, and specified pressure boundary conditions for both the point-distributed and block-centered grids. This treatment of boundary conditions is of second-order accuracy. The engineering approach gives a more accurate treatment than the conventional approach if first-order approximation is used in the treatment of specified pressure boundary conditions in the block-centered grid (Abou-Kassem and Osman, 2008). Using the engineering approach, Abou-Kassem et al. (2006) derived the flow equations and presented the treatment of boundary conditions for the cases of single-well simulation in radial-cylindrical coordinates and multidimensional, multiphase flow in black-oil models. It gives the same finite-difference equations for any formation process due to the scope of bypassing the whole formulation process (Fig. 2). Since the engineering approach is free from the conventional approach, it provides justification for use of the central-difference approximation in space and gives allusions of the estimates. On the other hand, these approximations usually use the conventional approach in time discretization.

3. INHERENT MYSTERIES IN THE SIMULATION

The whole petroleum industry is like a reservoir of risk and uncertainty. At present, investors, planners, and executives are in a situation where they do not feel secure to invest their capital investment in the petroleum industry due to the probability of success: 8–12% in any exploration activity (Hossain et al., 2010b). Unstable energy pricing is the other factor which mitigates the decision of the investor. All this uncertainty and risk is directly or indirectly related to the reservoir simulation. Therefore it is important to identify the big challenges and uncertainty

clouding the petroleum industry. Recently, Hossain et al. (2010b) demonstrated the chronological steps and major sources of uncertainty and risks of the entire petroleum industry, ranging from exploration through production to end user. This research discusses some of the fundamental assumptions behind the modeling approaches and their mathematics.

3.1 Assumptions Behind Various Modeling Approaches

Reservoir performance is traditionally predicted using three methods—(i) analogical, (ii) experimental, and (iii) mathematical. The following critical review is made based on their assumptions and ability to forecast reservoir performance.

3.1.1 Analogical Method

The analogical method consists of using mature reservoir properties that are similar to the target reservoir to predict the behavior of the reservoir. This method is especially useful when there is a limited available data. The data from the reservoir in the same geological basin or province may be applied to predict the performance of the target reservoir. In this method the following two approaches are taken to predict the future of the reservoir.

Statistical approach.—In the statistical approach, the past performance of numerous reservoirs is statistically accounted for by deriving the empirical correlations. In general, these correlations are used for future performance predictions of the reservoir. This approach may be described as a "formal extension of the analogical method." Statistical methods have several assumptions that are listed in Table 1. Moreover, Zatzman and Islam (2007) questioned the fundamental point of why the assumption of randomness anywhere is an antinature phenomenon. The problem lurks in the character of the very definition of randomness, that any of a (potentially infinite) set of possible outcomes is equally likely. Since outcomes of any natural process depend on the state of nature at some given points in space and time, some possible outcomes will be much more likely and some much less likely at that same point in space and time. Hence nature can never be "random" in the sense required for applying the theory of mathematical probability to compute outcomes. Mathematical randomness is just a special binary-outcome case of any function or equation that produces a unique result, whether that result takes the form of a single number or the form of a set of values occupying a uniqueness (i.e., fixed range). The question then revolves around how nature works, since a reservoir is all about natural phenomena. Zatzman and Islam (2007) address the issue very well. In addition, they point out a more subtle, yet far more important shortcoming of the statistical method. Practically all statistical methods assume that two or more objects based on a limited number of tangible expressions makes it legitimate to comment on the underlying science. It is equivalent to stating that if effects show a reasonable correlation, the causes can also be correlated. According to them, this poses a serious problem, as in the absence of a time-space correlation (a pathway rather than end result), anything can be correlated with anything, making the whole process of scientific investigation spurious. They make their point by showing the correlation between global warming increases with a decrease in the number of pirates. The absurdity of the statistical process becomes evident by drawing this analogy.

Decline curve analysis.—The rate of oil production decline generally follows one of the following mathematical forms—exponential, hyperbolic, or harmonic. The following assumptions apply to the decline curve analysis (Mustafiz and Islam, 2008): (i) past processes continue to occur in the future and (ii) operational practices are assumed to remain the same.

3.1.2 Experimental Method

The need for well-designed experimental work cannot be overemphasized in order to improve the quality of reservoir simulators. Experimental methods measure the reservoir characteristics in the laboratory models and scale these results to the entire hydrocarbon accumulation. The most significant challenges in experimental design arise from the determination of rock/fluid properties. Even though progress has been made in terms of specialized core analysis and pressure volume and temperature (PVT) measurements, numerous problems persist due to difficulties associated with sampling techniques and core integrity. In a reservoir simulation study, all relevant thermal properties, including coefficient of thermal expansion, porosity variation with temperature, and thermal conductivity, need to be measured in case such information is not available. Experimental facilities such as double-diffusive measurements, transient rock properties, and point permeability measurements can be very important in fulfilling the task. In this regard, the work of Belhaj et al. (2006) is noteworthy, where they used a 3D spot gas permeameter to measure permeability at any spot on the

TABLE 1: Assumptions behind different equations and techniques.

Equations	Assumptions
Statistical method	1) Reservoir properties are within the limit of the database. 2) Reservoir symmetry exists. 3) Ultimate recovery is independent of the rate of production.
Material balance equation	1) Rock and fluid properties do not change in space. 2) Hydrodynamics of the fluid flow in the porous media is adequately described by Darcy's law. 3) Fluid segregation is spontaneous and complete. 4) Geometrical configuration of the reservoir is known and exact. 5) PVT data obtained in the laboratory with the same gas-liberation process (flash vs differential) are valid in the field. 6) Sensitive to inaccuracies in measured reservoir pressure. The model breaks down when no appreciable decline occurs in reservoir pressure, as in pressure maintenance operations.
Momentum balance equation	1) The fluid is homogenous, single-phase, and Newtonian. 2) No chemical reaction takes place between the fluid and the porous medium. 3) Laminar flow condition prevails. 4) Permeability is a property of the porous medium, which is independent of pressure, temperature, and the flowing fluid. 5) There is no slippage effect, e.g., Klinkenberg phenomenon. 6) There is no electrokinetic effect. 7) It is valid for the slow flow of a Newtonian fluid with rigid solid matrix (i.e., inertial effects are neglected and most of the porous media fluids are non-Newtonian in nature (Hossain et al., 2007). 8) No-slip boundary conditions are assumed at the fluid–solid boundary on the microscopic level.
Taylor series expansion	1) All points of the function are assumed continuous on an interval containing x and x_0 for limited numbers and the resultant product exists on this interval. 2) The plotting of function [e.g., $f(x)$] in a certain interval of f, truncate this to a polynomial $P(x)$, and plot this polynomial with the underlying assumption that $f(x) \approx p(x)$ in this interval. 3) Taylor series expansion based on the mean value theorem, i.e., $f'(c) = [f(x) - f(x_0)/x - x_0]$, where c is a number between x and x_0. 4) The validity of the formula rests on the assumption that the function [e.g., f] is differentiable to all orders and is showing that the remainder term tends to zero as the denominator of the series [i.e., k] tends to infinity for c in a suitable interval.

Sources: Bear (1972); Hassanizadeh and Gray (1979a, 1979b, 1980); Whitaker (1986a, 1986b); Mustafiz and Islam (2008); Zatzman and Islam (2007).

surface of the sample, regardless of shape and size. Despite making great advances (Tharanivasan et al., 2004; Yang and Gu, 2005) in the experimental method, proper characterization of complex phenomena continues to be a formidable challenge.

3.1.3 Mathematical Method

In the mathematical method, basic conservation laws and constitutive equations are applied to formulate the behavior of fluid flow and other characteristics in mathematical notations and formulations. The two basic equations are the material balance and momentum balance equations. The material balance equation is one of the most widely used mathematical representations for the reservoir. This equation describes the fundamental physics of the production scheme of the reservoir. It is well known that Darcy's law has been used in the petroleum industry for centuries. As a result, practically all reservoir simulation studies involve the use of this law as a momentum balance equation. However, these equations have several inherent assumptions that might raise questions in the researcher's mind. Therefore it is important to understand the assumptions behind this equation. The assumptions are listed in Table 1.

These two equations are expressed for different phases of fluid flow in the reservoir and are combined to obtain a single equation for each phase of the flow. However, it

is necessary to apply other equations/laws for modeling to enhance oil recovery. As an example, the energy balance equation is necessary to analyze the reservoir behavior for steam injection or *in situ* combustion reservoirs. The mathematical model traditionally includes material balance equations, decline curves, statistical approaches, and also analytical methods. In most cases, the mathematical models for fluid flow through the porous media are extremely difficult to solve analytically. Analytical methods can only be applied to linear equations. These methods can apply to some simplified models. However, this solution can be applied as the benchmark solution to validate the numerical approaches. The numerical computations of the derived mathematical model are mostly based on the finite-difference method. All these models and approaches are based on several assumptions and approximations that may cause erroneous results and predictions. The following sections describe the inherent assumptions behind those equations and techniques.

Taylor series expansion.—Taylor series expansion is a very important tool in numerical analysis. If we use this expansion technique, the most well-behaved functions are converted to simple polynomials. When the Taylor series expansion is carried out for a finite number of terms and if the remainder is ignored, the series becomes an approximation of the function. This approximation generates the truncation error during numerical computation. Furthermore, it is a direct effect of the local truncation error or local discretization error. The assumptions behind the Taylor series expansion are listed in Table 1. Therefore it is important to look for some improvement in simulation.

Finite-difference approximation.—Finite-difference calculus is used to approximate values of functions and their derivatives at unknown discrete points. Newton's formula suffers from the approximation that the magnitude and direction change independently with each other. There is no problem in having separate derivatives for each component of the vector or in superimposing their effects separately and regardless of order. That is what mathematicians mean when they describe or discuss Newton's derivative being used as a "linear operator." Following this, it comes on Newton's difference-quotient formula. This method relies implicitly on the notion of approximating instantaneous moments of curvature, or infinitely small segments, by means of straight lines. This alone should have tipped everyone off that this derivative is a linear operator precisely because, and to the extent that, it examines change over time/distance within an already established function (Islam, 2006). This function is applicable to an infinitely small domain, making it nonexistent. When integration is performed, however, this nonexistent domain is assumed to be extended to a finite and realistic domain, making the entire process questionable. During these mathematical operations, all confusions arise due to the assumptions behind the finite-difference approximation which are listed in Table 2.

By examining the first assumptions involved, Zatzman and Islam (2007) were able to characterize Newton's law as aphenomenal for three reasons: (1) it removes time consciousness, (2) it recognizes the role of "external force"; and (3) it does not include the role of first premise. In brief, Newton's law ignores, albeit implicitly, all intangibles from natural science. Zatzman and Islam (2007) identified the most significant contribution of Newton in mathematics and showed how the derivative works with its limitations. Indeed, it took another century following Newton's death before mathematicians would work out the conditions. These conditions are the requirements for continuity of the function to be differentiated within the domain of values. In such a domain, its derivative (the name given to the ratio quotient generated by the limit formula) could be applied and yield reliable results. Kline (1972) detailed the problems involving this breakthrough formulation of Newton's. However, no one in the past has proposed an alternative to this differential formulation, at least not explicitly.

Figure 3 illustrates the above-mentioned difficulty. In this figure, the economic index (it may be one of many indicators) is plotted as a function of time. In nature, all functions are very similar. They do have local trends as well as global trends (in time). One can imagine how the slope of this graph, on a very small timeframe, would be quite arbitrary and how devastating it would be to take that slope to a long term. One can easily show that the trend emerging from Newton's differential quotient would

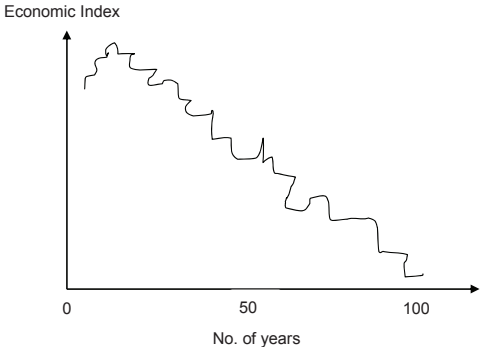

FIG. 3: Economic wellbeing is known to fluctuate with time (adapted from Zatzman et al., 2009).

TABLE 2: Assumptions behind the finite-difference approximation.

Equation	Assumptions
Finite-difference methods	1) The relationship between derivative and the finite-difference operators is established through the Taylor series expansion. 2) The relationship involves truncation of the Taylor series of the unknown variables after few terms. Such truncation leads to accumulation of error. a. The forward difference and the backward difference approximations are the first-order approximations to the first derivative. b. Although the approximation to the second derivative by a central-difference operator increases accuracy because of a second-order approximation, it still suffers from the truncation problem. c. As the spacing size reduces, the truncation error approaches zero more rapidly. Therefore, a higher order approximation will eliminate the need of same number of measurements or discrete points. It might maintain the same level of accuracy; however, less information at discrete points might be risky as well. 3) The solutions of the finite-difference equations are obtained only at the discrete points. 4) The solutions obtained for gridpoints are in contrast to the solutions of the continuous equations. 5) The local truncation error or local discretization error is not readily quantifiable because the calculation involves both continuous and discrete forms. 6) The computational operation increases, which eventually increases the round-off error.

Sources: Zatzman and Islam (2007); Mustafiz and Islam (2008).

be diametrically opposite to the real trend. Zatzman and Islam (2007) provided a basis for determining the real gradient rather than the local gradient that emerges from Newton's differential quotient. In that formulation it is shown that the actual value of Δt, over which a reliable gradient has to be observed, needs to be several times greater than the characteristic time of a system. The notion of REV, as first promoted by Bear (1972), is useful in determining a reasonable value for this characteristic time. The second principle is that at no time can Δt be allowed to approach 0 (Newton's approximation), even when the characteristic value is very small (e.g., phenomena at nanoscale). According to Abou-Kassem (2008), use of the engineering approach turns out such an approximation because this approach bypasses the recasting of governing equations into Taylor series expansion instead of relying on directly transforming governing equations into a set of algebraic equations. Finally, the initial analysis should involve the extension Δt to ∞ in order to determine the direction, which is related to the sustainability of a process (Zatzman et al., 2008).

4. AN EXAMPLE OF COMPLEXITY

Finally, an example can be drawn for which the elimination of one assumption helps capture natural phenomena. Hossain et al. (2007) developed a fluid rheological model that had eliminated Newton's assumption regarding viscosity and shear stress. Instead of resorting to a so-called nonlinear relationship, a memory function was introduced as a continuous function of time. The model equations are shown in Eqs. (1) and (2). With the inclusion of the memory function, new governing equations become far more complex than those of the conventional approach. One immediate consequence is the possibility of having multiple solutions. Because the exact form of the memory is never known, this depiction would give one an opportunity to refine the prediction envelope rather than putting too much emphasis on a single solution. This mode of recasting the governing equation opens up opportunities for finding solutions that are closer to the real solutions of a natural phenomenon:

$$\tau = \mu[(du_x)/(dy)], \tag{1}$$

$$\tau = (-1)^{0.5}\left(\frac{\partial\sigma}{\partial \mathrm{T}}\frac{\Delta T}{\alpha_D M_a}\right)\left[\frac{\int_0^t (t-\xi)^{-\alpha}\left\{\frac{\partial^2 p}{\partial\xi\partial x}\right\}d\xi}{\Gamma(1-\alpha)}\right] \times a^{0.5}\left[\frac{6K\mu_0\eta}{(\partial p)/(\partial x)}\right]^{0.5} e^{E/(RT)}\frac{du_x}{dy}. \tag{2}$$

5. STRENGTHS AND WEAKNESSES OF DERIVED FLOW EQUATIONS

The nonlinear algebraic equations for fluid flow through porous media can be obtained by either the traditional approach or the engineering approach. The same basic principles are used for both these approaches. Both approaches discretize the reservoir into gridblocks/gridpoints and yield the same discretized flow equations for modeling any reservoir-fluid system (multiphase, multicomponent, thermal, heterogeneous reservoir) using any coordinate system (Cartesian, cylindrical, spherical) in 1D, 2D, or 3D reservoirs (Abou-Kassem et al., 2006). Therefore the presentation here is limited to only single-phase, compressible fluid in a horizontal, 1D reservoir using irregular block size distribution in rectangular coordinates. This research takes advantage of this simple case to demonstrate the strength of the engineering approach.

5.1 Conventional Approach

The conventional approach follows the steps outlined in Fig. 1 during the development of the algebraic flow equations. These equations are derived in three consecutive steps: (i) derivation of the PDE describing fluid flow in the reservoir using the three basic principles mentioned earlier; (ii) discretization of the reservoir into gridblocks/gridpoints; and (iii) discretization of the resulting PDE in space and time. Finally, the flow equation can be obtained by combining the continuity equation, the equation of state, and Darcy's law as (Abou-Kassem, 2007)

$$\frac{\partial}{\partial x}\left(\beta_c \frac{k_x}{\mu B}\frac{\partial p}{\partial x}\right) + \frac{q_{sc}}{V_b} = \frac{1}{\alpha_c}\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) \tag{3}$$

Equation (3) is a PDE that describes single-phase flow in a 1D rectangular coordinate system. The above equation can be discretized using forward-difference discretization and obtained as (Abou-Kassem, 2007)

$$\begin{aligned} &T^n_{x_{i-1/2}}\left(p^n_{i-1}-p^n_i\right) + T^n_{x_{i+1/2}}\left(p^n_{i+1}-p^n_i\right) + q^n_{sc_i} \\ &\cong \frac{V_{bi}}{\alpha_c \Delta t}\left(\frac{\phi}{B}\right)'_i \left(p^{n+1}_i - p^n_i\right), \end{aligned} \tag{4}$$

where

$$T^n_{x_{i\pm1/2}} = \left(\beta_c \frac{k_x A_x}{\mu B \Delta x}\right)_{i\pm1/2} \quad \text{and}$$

$$\left(\frac{\phi}{B}\right)'_i = \frac{(\phi/B)^{n+1}_i - (\phi/B)^n_i}{\left(p^{n+1}_i - p^n_i\right)}.$$

5.1.1 Limitations and Observation of Equation (4)

The interblock flow terms and production/injection rates on the LHS of the equation is dated at time level n for the explicit flow equation. On the other hand, the RHS of the flow equation represents an accumulation over time step Δt. This means that the accumulation term in the RHS of the flow equation does not take into consideration the variation of interblock flow terms and the production/injection rate (source/sink term) within a time step.

A close inspection of the flow terms on the LHS of the discretized flow equation reveals that these terms are nothing but Darcy's law describing volumetric flow rates at standard conditions $q_{sc_{i\pm1/2}}$ between gridblock i and its neighboring gridblock $(i-1)$ or $(i+1)$ in the x direction, which means that all the assumptions behind Darcy's law are being incorporated here.

The interblock geometric factor $[\beta_c(k_x A_x)/(\Delta x)]$ is constant, independent of space and time.

The pressure-dependent term $(\mu B)_{i\pm1/2}$ of transmissibility uses some average viscosity and FVF of the fluid contained in block i and neighboring block $(i+1)$ or some weight at any instant of time t. In other words, the term $(\mu B)_{i\pm1/2}$ is not a function of space but a function of time as block pressure changes with time.

The transmissibility factor $T_{x_{i\pm1/2}}$ between block i and neighboring block $(i+1)$ is a function of time only; it does not depend on space at any instant of time.

5.2 Engineering Approach

The engineering approach follows the steps outlined in Fig. 2 during the development of the algebraic flow equations. These equations are developed in three successive steps: (i) discretization of the reservoir into gridblocks/gridpoints to remove the effect of the space variable as mentioned in observations 3, 4, and 5 above; (ii) derivation of the algebraic flow equation for gridblock i or gridpoint i using the three basic principles mentioned earlier, taking into consideration the variation of the interblock flow terms and source/sink term with time within a time step; and (iii) approximation of the time integrals in the resulting flow equation to produce the nonlinear algebraic flow equations. Therefore the flow equation can be written as (Abou-Kassem, 2007)

$$\begin{aligned} &\int_{t^n}^{t^{n+1}} \left[T_{x_{i-1/2}}\left(p_{i-1}-p_i\right)\right] dt + \int_{t^n}^{t^{n+1}} \left[T_{x_{i+1/2}}\left(p_{i+1}-p_i\right)\right] dt \\ &+ \int_{t^n}^{t^{n+1}} q_{sc_i} dt = \frac{V_{bi}}{\alpha_c}\left(\frac{\phi}{B}\right)'_i \left(p^{n+1}_i - p^n_i\right). \end{aligned} \tag{5}$$

This equation is rigorous and involves no assumptions other than the validity of Darcy's law to estimate the fluid volumetric velocity between gridblock i and its neighboring gridblock $(i-1)$ and $(i+1)$.

5.2.1 Strengths/Weaknesses and Observation of Equation (5)

The accumulation term in the RHS of the flow equation does take into consideration the variation of interblock flow terms and production/injection rates (source/sink term) within a time step.

A close inspection of the flow terms on the LHS of the discretized flow equation reveals that these terms are nothing but Darcy's law describing volumetric flow rates at standard conditions $q_{sc_{i\pm1/2}}$ between gridblock i and its neighboring gridblock $(i-1)$ or $(i+1)$ in the x direction; i.e., all the assumptions behind Darcy's law are being incorporated here. However, it is a time-dependent function within the time integral t^{n+1} and t^n.

The interblock geometric factor $[\beta_c(k_xA_x)/(\Delta x)]$ is a time-dependent function within the time integrals t^{n+1} and t^n.

The pressure-dependent term $(\mu B)_{i\pm1/2}$ of transmissibility uses some average viscosity and FVF of the fluid contained in block i and neighboring block $(i+1)$ or some weight at time integral t^{n+1} and t^n. In other words, the term $(\mu B)_{i\pm1/2}$ is not a function of space, but it is a function of time as block pressure changes with time.

The transmissibility factor $T_{x_{i\pm1/2}}$ between block i and its neighboring block $(i+1)$ is a function of time only. However, it depends on the time integral t^{n+1} and t^n.

The main strength of the engineering approach lies in being close to the engineer's thinking. It gives the physical interpretation of the approximations involved in the forward, backward, and central difference of the first-order time derivative used in the conventional approach. In addition, the algebraic equations can be easily attained without going through the severity of the conventional approach. In reality, the development of a reservoir simulator requires the appropriate nonlinear algebraic equations for the process being simulated. The majority of the available commercial reservoir simulators were developed without even looking at analysis of truncation errors, consistency, convergence, or stability. On the other hand, the scope of bypassing the PDEs makes the engineering approach stronger than the present approach where the above limitations are absent.

The nonlinear algebraic equations of any process in any coordinate system can be obtained in a rigorous way by the engineering approach without going through the severity of obtaining the PDEs describing the process in space and time for reservoir simulation (Abou-Kassem et al., 2006).

The conventional approach derives the nonlinear algebraic equations by first deriving the PDEs, followed by discretizing the reservoir and finally discretizing the PDEs. On the other hand, the engineering approach first discretizes the reservoir, then derives the algebraic equations with time integrals, and finally approximates the time integrals to obtain the nonlinear algebraic equations.

6. GUIDELINES TO MEET THE RESEACH CHALLENGES

The most important aspect of eliminating the spurious assumptions and consideration of the appropriate approach is that it leaves open the choice of multiple solutions, generating a set of cloud points rather than a single-point solution. In addition, a more accurate range of predicted values will reduce the uncertainty to a great extent. In the past, multiple solutions were observed decades ago but not with nonlinear solvers (Islam and Nandakumar, 1986; 1990). Without nonlinear solvers, this amounts to reducing the issue into an initial-value problem. For petroleum engineering applications, multiple solutions were observed as early as the 1950s and were correctly dubbed "spurious" by the groundbreaking work of Buckley and Leverett (1942). These multiple solutions occurred because the capillary pressure term was dropped from the governing equation. In order to avoid this problem, the notion of shock was introduced in place of a realistic transition of the saturation profile. Even though it was recognized that the need for introduction of shock is eliminated with the introduction of the capillary term decades ago, only recently has it been solved with a nonlinear solver (Mustafiz et al., 2008b). As elaborated by Islam et al. (2009), these solutions without linearization revealed a number of key observations, such as (i) there is a wide range of operating parameters for which the nonlinear solvers predict results remarkably different from those predicted by linear solvers, (ii) the possibility of multiple solutions is inherent to the reservoir simulation problems, and (iii) linearization of governing equations is likely to pervert subsequent results, biasing the decision-making process irreversibly.

Recent studies show that linearized and nonlinearized equations produce different results. If this linearization

is eliminated, the accuracy of the results can improve as much as 30% (Hossain and Islam, 2010a). Future research will explore the possibility of introducing nonlinear solvers in reservoir simulations. The existing simulators (such as Eclipse, etc.) and linear solvers are unable to handle the nonlinearity and chaotic behavior of the natural reservoir phenomena. Most importantly, recently the memory concept was introduced for the first time in the petroleum industry to capture the continuous alteration of reservoir rock and fluid properties (Hossain et al., 2007; Hossain et al., 2008; Hossain and Islam, 2009; Hossain et al., 2009b, 2009c). This pioneering concept can also be incorporated to develop a new knowledge-based model which will capture the natural phenomena of formation. The development of new governing flow equations thoroughly addresses in-depth understanding of fluid flow in a heterogeneous reservoir and interaction of rock/fluid properties. Superior mathematical and numerical techniques have been recently developed that allow one to systematically track multiple solutions inherent to nonlinear equations (Mousavizadegan et al., 2007, 2008; Mustafiz et al., 2008a; Mustafiz et al., 2008b; Islam et al., 2009). This will help develop a knowledge-based technique for producing the numerical simulation results in the form of a cluster of points, rather than a set of single-point solutions. With this approach, the risk assessment will be based on science that depends on the dynamics of the reservoir and not an arbitrarily set coefficient that is misleading at the very least. The solution scheme is accurate and simple to implement but is computationally time-consuming. That is why this solution scheme forms an ideal case for parallel computing, which is both necessary and complimentary. With parallel computing, a protocol is needed to establish tracking for multiple solutions with the speed of conventional reservoir simulators but with unprecedented accuracy. In summary, the combination of memory or continuous time function and tracking of multiple solutions will develop the protocol of a truly knowledge-based simulation (Hossain and Islam, 2010a).

As stated earlier, the results and successes reported by Mousavizadegan et al. (2007, 2008) and Mustafiz et al. (2008a, 2008b) in solving equations without linearization promise the future success of the engineering approach. Even Hossain et al. (2009a) showed the future research trend on reservoir simulation, where the engineering approach has been incorporated to bypass the assumptions. The knowledge-based approach developed by Hossain and Islam (2010a) can be coupled with the engineering approach to develop new models. The benefits of the proposed research are twofold. For a specific study, if the results show significant differences between the solutions of the linearized and nonlinearized models, the stage will be set to seriously consider the new approach in reservoir simulation. If, however, the results show insignificant differences for a given range of parametric values, then the proposed research reaffirms that the current method of linearization of model equations is appropriate for the given range and, therefore, delineates the range for which fine tuning of the current techniques is necessary. This line of computing has tremendous implications on petroleum engineering practices in particular, and far-reaching impact on other engineering disciplines.

7. CONCLUSIONS

In this study, a critical review and detailed comparison are made between conventional and engineering approaches for the development of a new reservoir simulator. The strengths and weaknesses of the engineering approach are also recognized. The current misconceptions and inherent assumptions are identified with elaborate discussions that are directly or indirectly related to reservoir simulation. Such distinction was not possible with conventional modeling techniques. Proposals are made to overcome a number of challenges encountered during modeling of petroleum reservoirs based on the engineering approach. It is shown that with the engineering approach, results would be significantly different for most of the solution regime. This finding would help to determine a more accurate range of risk factors in petroleum reservoir management. This new era of reservoir simulators will open a door for describing the natural phenomena in a better way and with greater understanding.

ACKNOWLEDGMENTS

The author gratefully acknowledges the support of King Fahd University of Petroleum and Minerals during the preparation of the manuscript. Special thanks to Professor M. R. Islam, Dalhousie University, for his support and valuable comments during the accomplishment of this research. Thanks to Professor Ahmad Dogan, Earth Science Department of King Fahd University of Petroleum and Minerals, for his comments.

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