A PROSITY DEPENDENT MODEL FOR A HETEROGENEOUS RESERVOIR

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ABSTRACT

Reservoir simulation is an important tool that gives the information on reservoir prediction. It enhances the management decision during the development, planning, and production optimization. Understanding the system parameters and its alteration with time are very important during the development of a model equation which ultimately leads to predict the behavior of a reservoir. As a result, the objectives of this research is to develop a comprehensive fluid flow model based on porosity alteration with time and compare it with commercial simulator, ECLIPSE. A 1-D, horizontal, and heterogeneous reservoir with time dependent rock and fluid properties is considered during the development of the model equation.

Conservation of mass, equation of state, and various other constitutive equations are used for the specified system. The model equation is solved using explicit formulation. The JAVA programming language is used to solve the model equation for the pressure response with time and space. The results are compared with ECLIPSE. The pressure response using the proposed model is in the same trend with ECLIPSE. However, there is a substantial variation on pressure value which is due to the consideration of porosity alteration with time.

This information leads the researcher to consider the time dependency of rock properties such as porosity with time even if it is nominal. Therefore, the use of proposed model would be suitable in predicting an accurate pressure response within the reservoir.

Keywords: reservoir simulation, diffusivity equation, reservoir modeling, porosity variation, heterogeneous reservoir.

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NOMENCLATURES

A = cross sectional reservoir, ft^3

A_{yz} = cross sectional reservoir in YZ plan, ft^3

B = oil formation volume factor, rev/stb

B_0 = oil formation volume factor at pressure p_0 , rev/stb

c_f = fluid compressibility of the system, $1/psi$

c_s = rock compressibility of the system, $1/psi$

c_t = total compressibility of the system, $1/psi$

k = reservoir permeability at any point, mD

k_x = reservoir permeability in the x-direction, mD

p = pressure of the system, psi

p_o = pressure at a reference point, psi

q = volumetric flow rate out or into the reservoir, ft^3/s

q_m = mass production rate in the reservoir, lb_m/s

q_{prod} $= \frac{q_m}{\rho_0}$ = volumetric production rate, ft^3/s

t = time, days

T = transmissibility factor, $sft^3/D - psi$

u_x = velocity of fluid in x-direction, ft/s

V_b = bulk volume of reservoir, ft^3

V_p = pore volume of the reservoir, ft^3

V_{bi} = bulk volume of reservoir at initial condition, ft^3

$V_{pi} = \varphi_i^o A \Delta x_i$ = pore volume of reservoir at initial condition, ft^3

ρ = density of the fluid at pressure p, lb_m/ft^3

ρ_o = density at a reference pressure, p_o, lb_m/ft^3

ρ_{sc} = density of the fluid at standard condition, lb_m/ft^3

φ_0 = porosity of the grid cell at at reference pressure, p_0

φ = porosity of the reservoir at pressure p, fraction

φ_x = porosity of the reservoir along x-direction at pressure p, fraction

μ = viscosity of the fluid, pois

$\frac{\partial p}{\partial x}$ = pressure gradient, psi/ft

$\frac{\partial \Phi}{\partial x}$ = potential gradient, psi/ft

$\frac{\partial \varphi}{\partial t}$ = porosity derivative with respect to time, t

$\frac{\partial p}{\partial t}$ = pressure derivative with respect to time, t, psi/s

$\frac{\partial \rho}{\partial t}$ = density derivative with respect to time, t, $lb_m/ft^3 s$

Φ = potential at a point, psia

Δx = grid size, ft

Δx_i = grid size for block i, ft

1. INTRODUCTION

Reservoir modeling is a critical component in development planning and production management of oil and gas fields. The ultimate goal of reservoir modeling is to aid the decision making process throughout all stages of field life. During early field development, reservoir models are used to assess the risk and uncertainty in the field performance based on limited data. Once production begins, reservoir models are periodically refined or updated based on reservoir surveillance data. The updated models are then used for making field management decision, such as further drilling decisions. For new fields, accurate reservoir models are required to evaluate opportunities on enhanced oil recovery (Wu X.H. et al 2007). The accuracy of the model equations is very much dependent on proper description of the reservoir, formulation of the mathematical model and discretization of the model (Coats, 1969, Hossain and Islam, 2010 and Hossain, 2012). It is also very important to consider the continuous alteration rock/fluid properties with time (Aziz et al 2002, Hossain et al. 2007, Hossain et al 2008a, Hossain et al. 2008b, Hossain et al. 2009a, Hossain et al. 2009b, Hossain et al. 2009c). Therefore, this research investigates the porosity variation with time and includes its effects on pressure response. A 1-D heterogeneous reservoir is considered to develop a single phase flow model and numerically solved the model equation to investigate its performance comparing with existing models.

The other investigates conducted series of researches to model the reservoir under different scenario. However, there are very limited literatures that consider the continuous alteration of rock/fluid properties. Das (1998) noticed that the use of the steady state Darcy's equation and its equivalent velocity term is affected by mainly three factors which are a source/sink term, heterogeneity such as spatial porosity variation, and unsteady nature of flow. He mentioned that these factors cause large pressure prediction errors ranging as high as 1000 psi for a typical time step of 10 days. Therefore, by the use of the analytical Navier-Stokes equations, he was able to derive new model equations utilizing a new concept called "the pore average velocity" instead of the Darcian velocity. HoJeen (2004) derived a new diffusivity equation that takes into account the non-Darcy behavior of the fluid using a certain non-Darcy flow correction factor. He utilized the Forchheimer's equation which is similar to Darcy's equation but with a non-Darcian flow coefficient. These coefficients can be obtained from oil flow tests, or from gas deliverability tests in case of gas flow since the coefficients differ from gas to oil. Belhaj et al. (2004) were able to derive a new diffusivity equation for the fluid flow under Darcian and non-Darcian behavior. Their derivation was based on the basic Darcy's equation, Forchheimer's equation, and Brinkman equation. The numerical results show closer to the experimental results, especially if comparing the flow velocity vs. pressure gradient profiles. Moreover, they found that in the high velocity region which is called the non-Darcian region there were some deviations where the model predicts higher pressure drops than the old Darcian model because of inertial forces, and it predicts lower pressure drops due to frictional forces. But, the net deviation from the Darcian was upward due to the stronger effects of inertial forces, and this made it very close to the experimental results. However, none of them consider the porosity variation with time and investigate its affect on reservoir pressure response.

Furthermore, in the work of Hossain et al. (2008b) did not take into account the Darcian and the non-Darcian flow issue; however they were focusing on proposing a model that

considers the change of permeability and viscosity with time. Since the viscosity is a function of pressure and the pressure is changing with time in the system, they considered the change of viscosity with time in their model. Also, they considered the change of permeability over time in their model, where the variation of permeability becomes significant when there is mineralization in the pore network and this network is noticeably affected during the fluid extraction and pressure changes. They were able to come up with a model that considers these changes by the use of the concept of memory which was applied on the momentum balance equation during the development of the model equation.

There are lots sources of uncertainty cloud in reservoir simulation (Hossain et al. 2010). These clouds directly affect the prediction of reservoir performance. To avoid the hidden uncertainty and risk during the discretization and solution, several researchers tried to address the issue in several ways. Busswell et al. (2006) dispensed the use of reservoir simulation to model the performance of the reservoir. They utilized the analytical solution as their tool and presented a set of new analytical solution of single layer reservoir both in real time and Laplace space. The solution is derived by using the method of integral transforms. They were able to use this method to calculate the pressure response in time and space by using any continuous function to describe the production rate of a point source. In terms of accuracy and speed, they found that this method if compared with the solution of a commercial finite difference simulator, the accuracy is pretty good and there is a decrease of the CPU times by factors not less than 300. In case of single phase volumetric gas reservoirs, particularly, there are some methods that are considered more accurate than the conventional approach. Therefore, Lee et al. (1998) introduced a new method where they used the finite element approach rather than the finite difference approach. However, in case of bounded reservoir it shows a pseudo steady behavior after a small period of time, which is the infinite acting flow period. Therefore, they were able to introduce a solution that considers the pseudo steady state behavior by solving the linearized forms of the diffusivity equation. Also, the use of the finite element method was to be able to handle complex reservoir geometries, dependent properties, and multiple wells. As a result, it is very important to keep the model equation nonlinearized during the discretization process which is another aim of this research.

In this research, Darcy's law is used as the rate equation during the development of the model equation. Heterogeneity of the reservoir is also considered where rock properties change with space and time. Therefore, the present research investigates the alteration of porosity with respect to space and time and its effects on pressure response. This consideration is very important because reservoir pressure changes with time and space while porosity is pressure dependent. Here, permeability is only a function of space due to the single phase (black oil) reservoir. Permeability variation is dominant for multiphase (i.e. existence of gas phase) where Klinkenberg effect exists (1941). Thereofore, this research did not consider permeability variation with time.

2. THE PROBLEM STATEMENT

A 1-D, horizontal, heterogeneous reservoir is considered. The top of the reservoir is 8,500 ft below the surface. A reservoir of $6450' \times 5000' \times 80'$ is considered (Figure 1). The reservoir is divided into 5 major grid blocks horizontally. Each grid has its own length, pressure value, and rock properties.
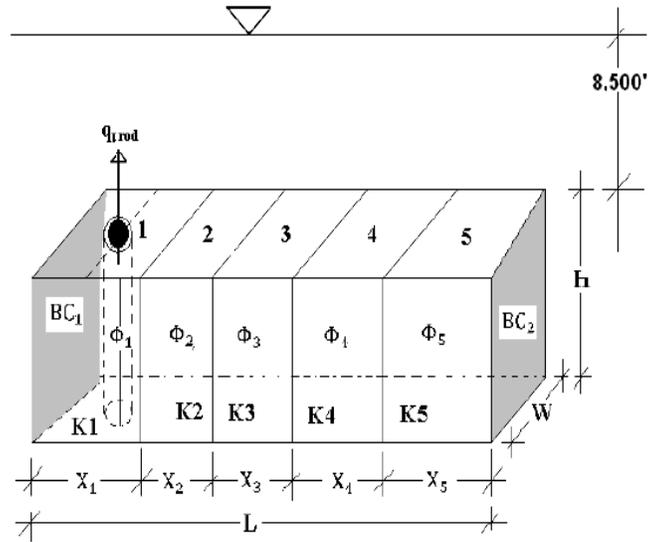
Figure 1. A 1D heterogeneous reservoir.

Table 1. All necessary data used to solve the problem

Set of Data			
Discretized dimension (ft)		X_1	1200
		X_2	1300
		X_3	1400
		X_4	1250
		X_5	1300
Rock Properties	Φ	1	0.20
		2	0.16
		3	0.22
		4	0.25
		5	0.17
	K (mD)	1	230
		2	190
		3	270
		4	300
		5	150
Fluid Properties	B (rb/STB)		1.11
	μ (cp)		1.55
	$C \times 10^{-5}$ (psi^{-1})		2.60
BC	BC1		NF
	BC2		NF
Pressure	P_i (psi)		4500
	P_{wf} (psi)		1900
Temperature	T_i (°F)		275
Life, N (yrs)			15
q_{prod} (STB/D)			700
Production well diameter (inch)			6.5

Also, it is assumed that there are constant fluid properties for the reservoir. Production per day is given as 700 stb/day, and the diameter of production well is 6.5 inch. After production, the well is switched to a constant formation bottom hole pressure (Pwf) if the reservoir cannot sustain the specified production rate. The life of the reservoir is considered as N years. Table 1 shows the necessary data that are used in solving the problem numerically. This research leads to show the pressure response in space and time where porosity alteration exists.

3. THEORETICAL DEVELOPMENT OF YHE MODEL

The mass conservation can be written as

$$-\frac{\partial}{\partial x}(\rho\,u_x) = \frac{\partial}{\partial t}(\varphi\rho) + \frac{q_m}{V_b} \tag{1}$$

The conservation of momentum is obtained from Darcy's law as

$$u_x = -\frac{k_x}{\mu}\frac{\partial \Phi}{\partial x} \;\rightarrow\; u_x = -\frac{k_x}{\mu}\frac{\partial p}{\partial x} \tag{2}$$

By substituting Eq. (2) in Eq. (1):

$$-\frac{\partial}{\partial x}\left[\rho \times \left(-\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right)\right] = \frac{\partial(\varphi\rho)}{\partial t} + \frac{q_m}{V_b} \tag{3}$$

The equation of state, $\rho = \rho_o e^{c_f(p-p_o)}$ can be written as Aziz et al (2002):

$$\frac{\partial p}{\partial x} = \frac{1}{c_f\rho} \times \frac{\partial \rho}{\partial x} \tag{4}$$

In Eq. (4), fluid compressibility is considered as constant. Now, porosity needs to be defined to consider its change with the time, and this is what has not been done by previous researches as mentioned above. The following partial differential equation can be obtained from the relationship $\varphi(p) = \varphi_0[1 + c_R(p - p_o)]$ as Ertiken et al (2001):

$$\frac{\partial \varphi}{\partial t} = \varphi_0 c_R \times \frac{\partial p}{\partial t} \tag{5}$$

where φ_0 is the porosity at any reference pressure, p and rock compressibility is assumed to be constant. Now substituting Eq. (4) and Eq. (5) in Eq. (3), the final form of the equation can be written as:

$$\frac{\partial}{\partial x}\left[\frac{k_x\,\rho}{\mu}\frac{\partial p}{\partial x}\right] = \varphi_0[1 + c_R(p - p_o)]\frac{\partial \rho}{\partial t} + \rho\,\varphi_0 c_R\frac{\partial p}{\partial t} + \frac{q_m}{V_b} \tag{6}$$

Again equation of state can be written with respect to time derivative as $\frac{\partial \rho}{\partial t} = c_f \rho \frac{\partial p}{\partial t}$. Therefore, Eq. (6) can be written as Hossain et al (2008a):

$$\frac{\partial}{\partial x}\left[\frac{k_x \rho}{\mu} \frac{\partial p}{\partial x}\right] = \rho \varphi_0 c_R \frac{\partial p}{\partial t} + \varphi_0 c_f \rho[1 + c_R(p - p_o)]\frac{\partial p}{\partial t} + \frac{q_m}{V_b} \tag{7}$$

Dividing the Eq. (7) by ρ_0, and substituting $\frac{\rho}{\rho_0} = \frac{B_0}{B}$ using Aziz and Settari (2002), the equation becomes as

$$\frac{\partial}{\partial x}\left[\frac{k_x}{B} \frac{\partial p}{\partial x}\right] = \frac{\mu}{B} \varphi_0\{c_R + c_f + c_f c_R(p - p_o)\}\frac{\partial p}{\partial t} + \frac{\mu}{B_0 \rho_0} \frac{q_m}{V_b} \tag{8}$$

Using Aziz et al (2002), substitute $\frac{1 + c_f(p - p_o)}{B_0} = \frac{1}{B}$ the final form of the equation becomes as

$$\frac{\partial}{\partial x}\left\{k_x\left[1 + c_f(p - p_o)\right]\frac{\partial p}{\partial x}\right\}V_b - \mu\, q_{prod}$$

$$= \mu\, \varphi_0 V_b\left[1 + c_f(p - p_o)\right]\{c_R + c_f + c_f c_R(p - p_o)\}\frac{\partial p}{\partial t} \tag{9}$$

Since $V_b = A\, \Delta x\ and\ c_t = c_R + c_f$, Eq. (9) becomes as:

$$\frac{\partial}{\partial x}\left\{k_x\left[1 + c_f(p - p_o)\right]\frac{\partial p}{\partial x}\right\}A\, \Delta x - \mu\, q_{prod}$$

$$= \mu\, \varphi_0 V_b\left[1 + c_f(p - p_o)\right]\{c_t + c_f c_R(p - p_o)\}\frac{\partial p}{\partial t} \tag{10}$$

Equation (10) represents the final form of fluid flow equation where porosity is a function of space and time interms of reference porosity, pressure and compressibilities. However, permeability is a function of space only. Equation (10) is called as diffusivity equation where there is an option for the consideration of production or the injection wells from the grid cell if there is any. Ertekin et al. (2001) is also developed a model where porosity alteration is considered as a function time in terms of formation formation volume factor. Their model is well stated in the reference.

4. NUMERICAL SIMULATIONS:

To solve Eq. (10) numerically, the field data of Table 1 and the following initial and boundary conditions are used.

Initial condition: The initial pressures for all the grid cells are constant and they are equal i.e. $p(x, 0) = p_i$

Boundary condition: The interior boundary is considered as a constant production rate at the wellbore. The external boundaries are considered as no-flow boundary at the external boundary of the grid block 1 and 5.

The inner boundary. According to Darcy's law,

$$q_{x=0} = A\,u_x = -\frac{k_x\,A_{yz}}{\mu}\frac{\partial p}{\partial x}$$

$$q = \left(-\frac{k_x\,A_{yz}}{\mu}\right)\left(\frac{p_{i+1}^n - p_i^n}{\Delta x}\right) = \left(-\frac{k_x\,A_{yz}}{\mu\,\Delta x}\right)(p_{i+1}^n - p_i^n)$$

$$p_{i+1}^n = p_i^n + q\mu/e_1 \tag{11}$$

where $e_1 = -\frac{k_x\,A_{yz}}{\Delta x}$.

The outer boundary. According to Darcy's law,

$$u_{x=L} = -\frac{k}{\mu}\frac{\partial p}{\partial x} = 0, \Rightarrow \frac{k}{\mu}\frac{\partial p}{\partial x}\bigg|_{x=L} = 0, \Rightarrow \frac{\partial p}{\partial x}\bigg|_{x=L} = 0$$

$$\frac{p_{i+1}^n - p_i^n}{\Delta x} = 0, \Rightarrow p_{i+1}^n = p_i^n \tag{12}$$

Equation (11) and Eq. (12) are used in solving the Eq. (10). Explicit scheme is used to solve Eq. (10) numerically. Here permeability variation is only considered for individual grid cell. So, the final form of Eq. (10) can be expanded as follows:

$$p_i^{n+1} = p_i^n + \frac{\frac{\Delta t\,A\,k_x}{\Delta x}\Big\{[1+c_f(p_i^n - p_o^n)](p_{i+1}^n - 2\,p_i^n + p_{i-1}^n) + c_f(p_{i+1}^n - p_i^n)^2\Big\} - \Delta t\,\mu\,q_{prod}}{\mu\,\varphi_0 V_b[1+c_f(p_i^n - p_o^n)]\{c_t + c_f\,c_R(p_i^n - p_o^n)\}} \tag{13}$$

4. RESULTS AND DISCUSSIONS

In this section, there will be comparison of the results obtained by solving the above proposed model (Eq. (10)) with the results obtained by ECPLISE results. First of all, there are two figures showing the results of the new model equation that has been developed in this article, one is for the pressure vs. distance for two different times in Figure 2, and the other one is for the pressure vs. time at different distance in Figure 3. Figures 4 − 6 show the comparisons between proposed model, and ECLIPSE in three different ways for the pressure responses with respect to time and space. Results of proposed model show a reasonable difference comparing with ECLIPSE because of the different pressure response due to the porosity alteration with time. It means that to get a good reservoir prediction, one should not ignore the continuous alteration of any property of rock/fluid with time. As porosity alteration is incorporated, a substantial difference in pressure response is observed in this research which needs to be considered. Finally, Figure 7 depicts the porosity changes with time at different distance.

3.1. Variation of Pressure with Space

Figure 2 and 4 show the pressure vs distance plotting after one and two years of production when proposed model and ECLIPSE are considered respectively. For both cases,

the pressure response trend is same. However, the decline rate of pressure for proposed model is higher than that of ECLIPSE results.
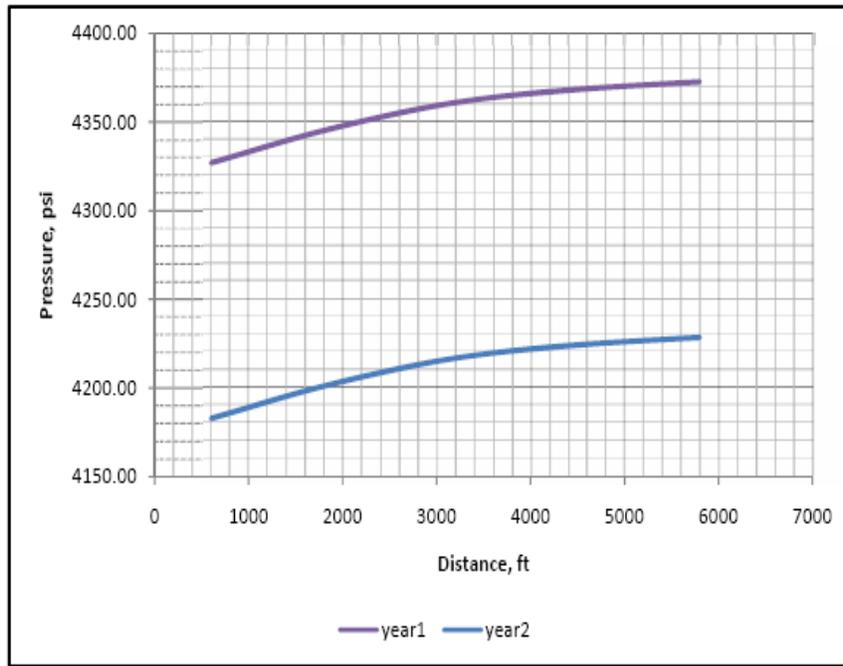

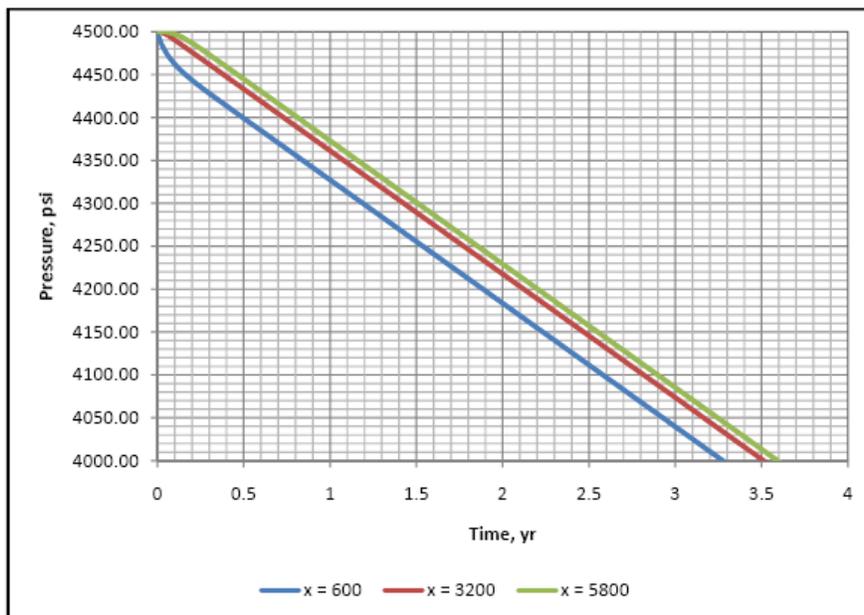Figure 2. Variation of pressure with distance by the new model.

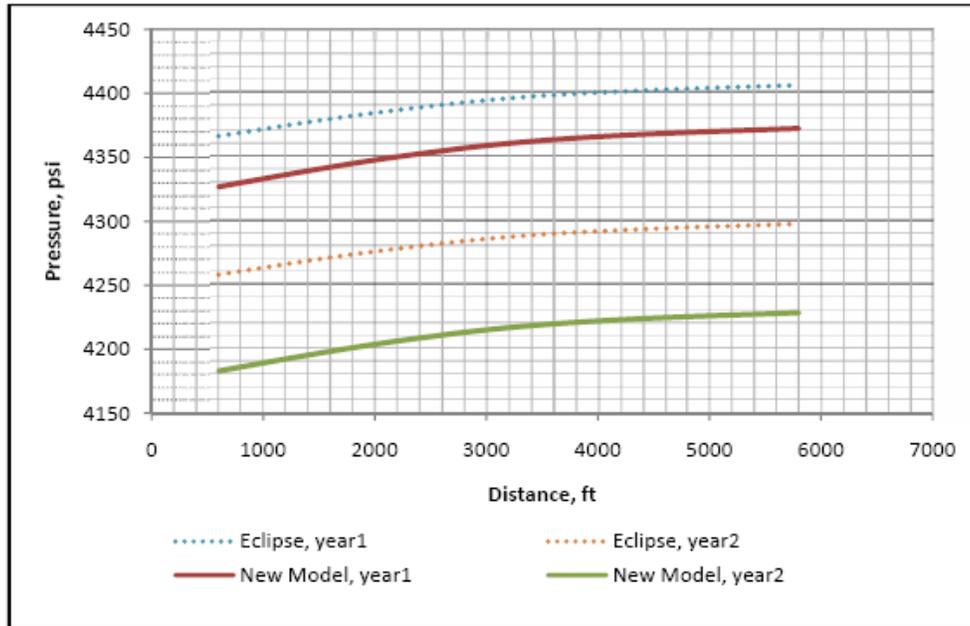Figure 3. Variation of pressure with time by the new model.

Figure 4. Comparison of pressure variation with distance for the three cases – new model, and ECLIPSE.

It is due to the consideration of porosity alteration with time during the production period. Figures 4 shows the comparison between the proposed model and ECLIPSE for different time of one and two year. A clear separate pressure response scenario exists for both model. The pressure value is showing lower in the proposed model and predicts lower pressure decline rate comparing with ECLIPSE. The difference becomes larger with the increase of production life. ECLIPSE predicts much higher pressure than the proposed model while ECLIPSE ignores the consideration of porosity alteration with pressure which is well established in the literature (Hossain 2012, Mousavizadegan et al. 2008). However, it shows better results since it considers the change of the PVT properties with pressure, thus with time, while those properties are considered constant in this research. When hydrocarbon production starts, the pressure decline begins and due to this continuous change of pressure within the formation, rocks try to expand. As a result, porosity starts to decrease with time which ultimately helps to the alteration of pressure in the formation. So, this phenomena can be well described and captured by the proposed model.

3.2. Variation of Pressure with Time

Figure 3 and 5 depict the pressure response with time at a distance of $x = 600', 3,200'$ and $5,800'$ respectively using the proposed model and ECLIPSE. For both cases, there is a sharp pressure decrease with time. However, the slope of the pressure response with time is less toward the outer boundary of the rerservoir. The similar trend exists for both cases except the magnitude of the pressure value. Figures 5 shows the comparison of

pressure vs time plotting between the proposed model and ECLIPSE at two different distance, $x = 600'$, and $3,200'$ respectively.
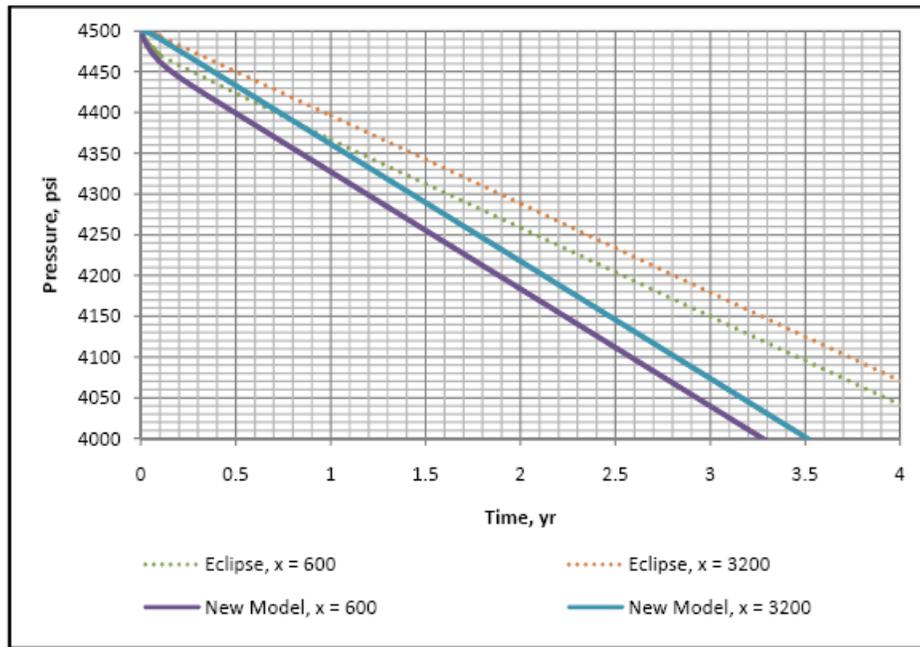

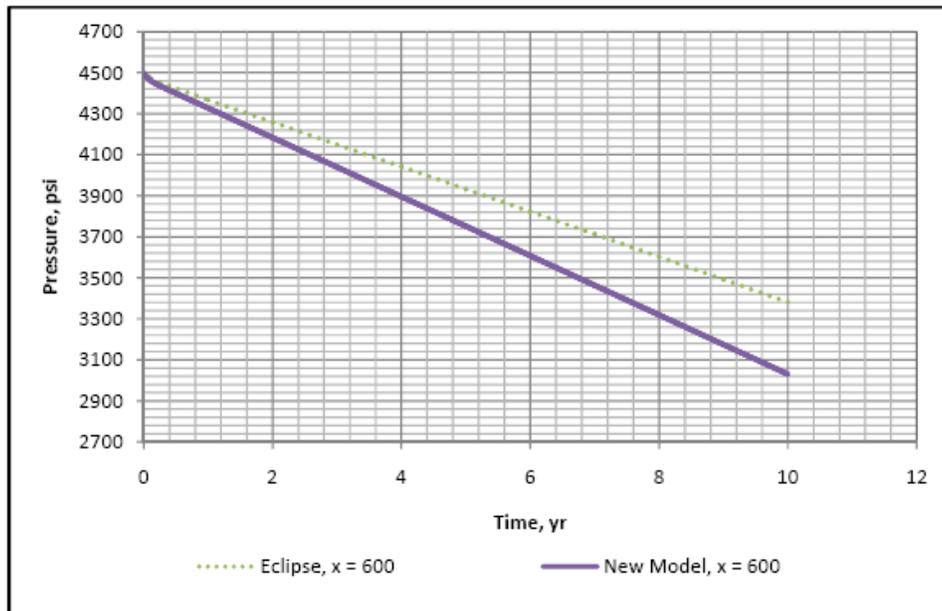Figure 5. Comparison of pressure variation with time between the new model and ECLIPSE.

Figure 6. Comparison of pressure variation with time for new model, and Eclipse after 10 years of production.
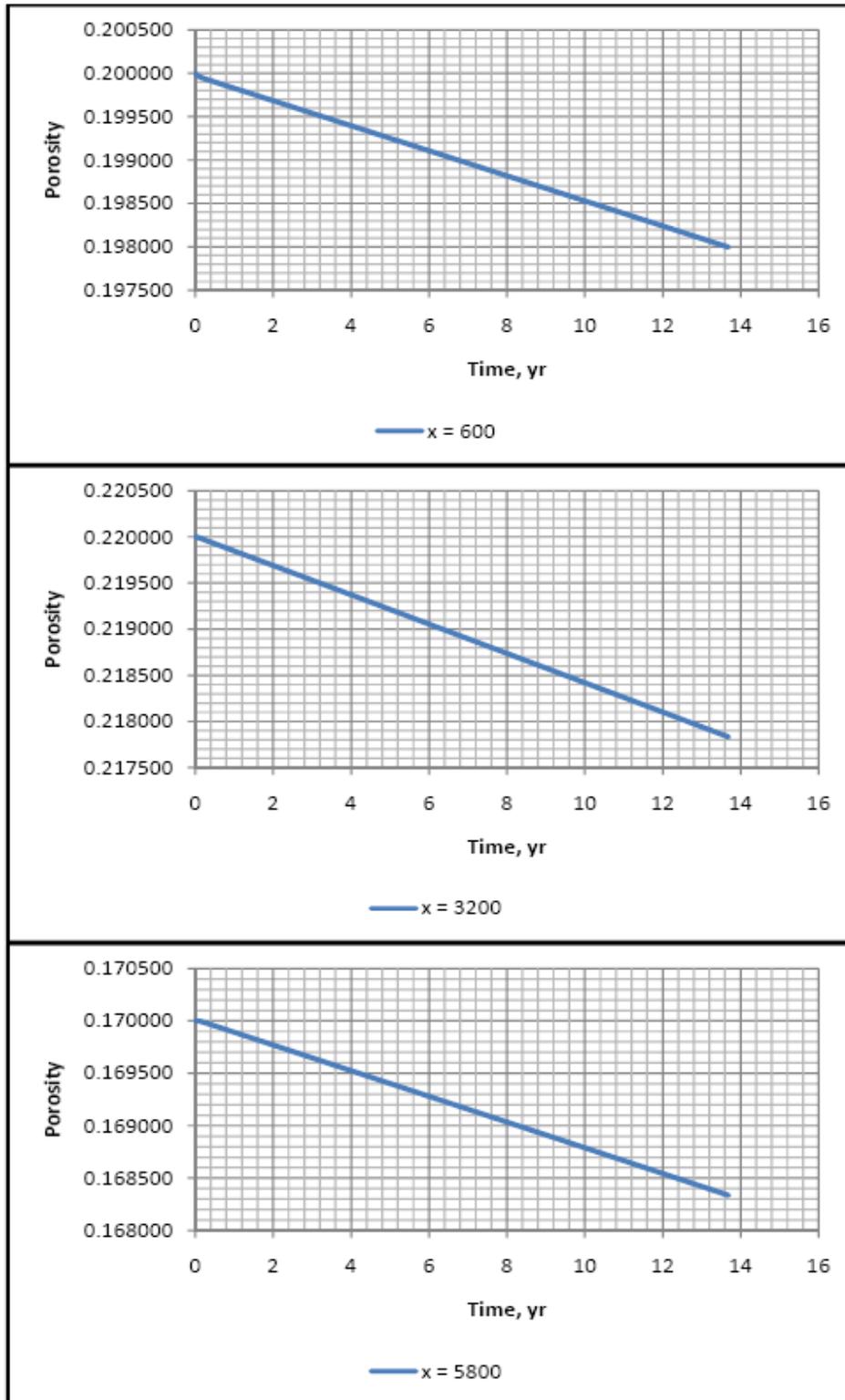
Figure 7. Variation of porosity with time at different distance.

The figure shows that pressure decline rate with time for proposed model is higher than that of ECLIPSE. This difference becomes larger with time and toward the outer boundary of the reservoir. It is also noticed that a sudden decrease of pressure around the wellbore within short period of time. This is due to the beginning of the production. Furthermore, Figure 6 depicts the pressure vs. time plotting for proposed model, and ECLIPSE simulator at $x = 600'$ same as Fig. 5 but with a longer period of time, 10 years. This is to show how serious is this consideration and to what extent it affects the results after long period. Since hydrocarbons are being withdrawn from the reservoir with no supporting aquifer to keep the pressure constant, the pressure response trend is decreasing. By comparing the results obtained from the proposed model developed above and the results from ECLIPSE, it is observed that they have the same trend of decrease but as mentioned previously that the ECLIPSE has highest pressures than the proposed model.

3.3. Variation of Porosity with Time

As mentioned earlier, the proposed model only differs the consideration of continuous alteration of porosity with time comparing with ECLIPSE. Figure 7 presents how porosity changes with time at different distance of $x = 600', 3,200' and 5,800'$ respectively. The porosity calculation is based on the proposed model. The graphs show that porosity decreases with time due to the decline of pressure with time. However, the magnitude of porosity change is very low and the change of magnitude increases with time and distance. Equation (5) shows that porosity is a function of pressure and rock compressibility which is usually very small in the order of 10^{-6}. This compressibility factor makes the change very small.

CONCLUSION

A model equation has been developed for a 1-D, heterogeneous, and horizontal reservoir where porosity alteration with time is considered. Results show that porosity change has reasonable effects on pressure response. The proposed model predicts lower pressure than ECLIPSE. So, pressure is over estimated if one considers the ECPLISE. This interpretation of pressure response gives a misleading information to the decision maker who evaluate the reservoir performance.

On the other hand, consideration of porosity alteration with time is important because in reality every rock/fluid property changes with time even if the changes are very small. The proposed model shows that magnitude of porosity change increases with time. As a result, the proposed model predict a different pressure value comparing with the simulator.

The results using the proposed model is reasonable because it considers the naturally occurring phenomena. It gives a good prediction of pressure value at different time and space. So, this result will not give any misleading information to the senior executive. Therefore, the proposed model will help in enhancing the reservoir management interms of more accurate information. However, the future work should include the alteration of PVT data with time which will predict even more accurate and better results.

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REFERENCES

Aziz, K. and Settari, A. , "*Petroleum Reservoir Simulation*". Blitzprint Ltd., Calgary, Alberta, p.476, pp. 13 – 15, 2002.

Busswell G., Banerjee R., *Thambyanayagan* R. K. M., Spath J., "Generalized Analytical Solution for Reservoir Problems with Multiple Wells and Boundary Conditions", SPE-99288, Schlumberger Oilfield Services, (2006).

Coats K.H., "*Use and Misuse of Reservoir Simulation Models*", The University of Texas at Austen, SPE-2367, 1969.

Das A. K., "*Generalized Darcy's Law with Source and Heterogeneity* Effects", Indian School of Mines, (1998).

Ertekin T., Abou-Kassem J.H., King G.R., "Basic Applied Reservoir Simulation", Duke Engineering and Services Co., Los Angeles, *SPE book series* Vol. 7, (2001).

H.A. Belhaj, K.R. Agha, S.D. Butt, M.R. Islam, Dalhousie University, "Simulation of Non-Darcy Flow in Porous Media Including Viscous, Inertial, and Frictional Effects", SPE International Improved Oil Recovery Conference in Asia Pacific, Kuala Lumpur, Malaysia 20-21 October (2003).

HoJeen Su, "A Three-Phase Non-Darcy Flow Formulation in Reservoir Simulation", *SPE Asia Pacific Oil and Gas Conference and Exhibition*, 18-20 October 2004, Perth, Australia. (2004).

Hossain, M.E., Mousavizadegan, S.H. Ketata, C. and Islam, M.R. "A Novel Memory Based Stress-Strain Model for Reservoir Characterization". *Journal Nature Science and Sustainable Technology,* (2007). 1(4), pp. 653 – 678.

Hossain, M.E., Mousavizadegan, S.H. and Islam, M.R. "The Effects of Thermal Alterations on Formation Permeability and Porosity". *Petroleum Science and Technology*, 26(10-11), pp. 1282 – 1302, (2008a).

Hossain, M.E., Mousavizadegan, S.H. and Islam, M.R. "A New Porous Media Diffusivity Equation with the Inclusion of Rock and Fluid Memories". E-Library, *Society of Petroleum Engineers*, SPE– 114287-MS, (2008b).

Hossain, M.E., Mousavizadegan, S.H. and Islam, M.R. "Effects of Memory on the Complex Rock-Fluid Properties of a Reservoir Stress-Strain Model". *Petroleum Science and Technology,* 27 (10), pp. 1109 – 1123. (2009a).

Hossain, M.E., Liu, L. and Islam, M. R. "Inclusion of the Memory Function in Describing the Flow of Shear-Thinning Fluids in Porous Media". *International Journal of Engineering* (IJE), 3(5), pp. 458 – 477 (2009b).

Hossain, M.E. and Islam, M.R. "A Comprehensive Material Balance Equation with the Inclusion of Memory during Rock-Fluid Deformation". *Advances in Sustainable Petroleum Engineering Science*, 1(2), 141 – 162, (2009c).

Hossain, M.E., Mousavizadegan, S.H. and Islam, M.R. "Inclusion of Memory Concept to describe Rock/Fluid Rheology and Heat Transfer during Thermal Operations". Paper ID: 208, *Proc. at 4*th *International Conference on Thermal Engineering Theory and applications*, January 12 – 14, 2009, Abu Dhabi, United Arab Emirates (2009d).

Hossain, M.E. and Islam, M.R., "Knowledge – Based Reservoir Simulation – A Novel Approach", *International Journal of Engineering*, 3(6), pp. 622 – 638, 2010.

Hossain, M.E., Mousavizadegan, S.H. and Islam, M.R., The Mystery and Uncertainty Cloud during Reservoir Simulation in Petroleum Industry, *Advances in Sustainable Petroleum Engineering Science,* 2(4), 2010.

Hossain, M.E. "Comprehensive Modeling of Complex Petroleum Phenomena with Engineering Approach", *Journal of Porous Media*, 2012, 15, in press.

Klinkenberg, L.J. "The Permeability of Porous Media to Liquids and Gases," *Drill. and Prod*. *Prac.,* (1941).

Lee K.S., Miller M.A., and Kamy Sepehrnoori, "Succession-of-States Model for Calculating Long-Time Performance of Depletion Reservoirs", *SPE Journal*, University of Texas, (1998).

Mousavizadegan, S.H., Mustafiz, S., Zaman, M.S., and Islam, M.R., "The Effects of Linearization in Multi-Phase Flow Simulation in Petroleum Reservoirs". *Journal of Nature Science and Sustainable Technology*, 2(3), 379-398, (2008).

Wu X.H., Stone M.T., Parashkevov R.R., Stern D., and Lyons S.L., "*Reservoir Modeling with Global Scaleup*", SPE-105237, ExxonMobil Upstream Research Co. 2007.