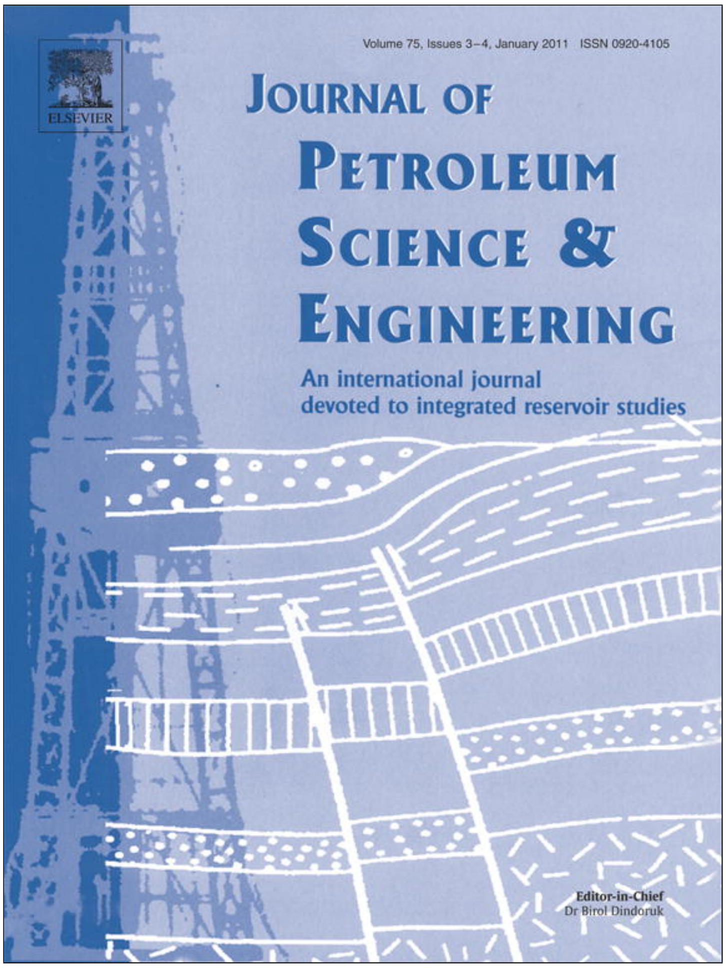
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Modified Kozeny–Carmen correlation for enhanced hydraulic flow unit characterization

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ABSTRACT

Although several techniques have been proposed to predict permeability using porosity–permeability relationships, the Kozeny–Carmen (K–C) correlation is the most widely acceptable methodology in the oil industry. Amaefule et al. (1993) modified that correlation introducing the concept of Reservoir Quality Index (RQI) and Flow Zone Indicator (FZI) to enhance its capability to capture the various reservoir flow behavior based on its respective characters. Yet, there are challenges in using the original correlation due to its inherent limitations and over simplified assumptions that prevent accurate Hydraulic Flow Unit (HFU) definitions. This research addresses some of those shortcomings and proposes a modified K–C correlation by handling the tortuosity term in a more robust manner. Core data from major carbonate reservoirs in Saudi Arabia is used to test the model. Additional data sets obtained from literature on sandstone reservoirs are used as well to demonstrate the global applicability of the proposed model. Results show that more permeability variations are to be expected within a given HFU. Moreover, the conventional model underestimates permeability values within a specific HFU significantly in comparison with the new model.

1. Introduction

Permeability is one of the most important parameters to quantify in any reservoir rock. Its importance arises due to the major role it plays during the development phase of any reservoir. For many years, various techniques have been proposed to measure permeability. Literature shows that permeability can be measured by three major techniques; (1) well testing, (2) routine core analysis, and (3) formation testers (Ahmed et al., 1991). Ahmed et al. (1991) provided a critical and detailed review of permeability measurement techniques and their interrelationships.

During any reservoir simulation study, permeability perdition is a very critical and perhaps the most challenging task. In the early stage of the industry, simple permeability–porosity transformations were generated to estimate permeability at un-cored wells. However, such simple relationships were unreliable and results were not in good agreement with field data. Hence, many models have been proposed to predict permeability by incorporating many parameters other than effective porosity.

Nelson (1994) made an extensive review of most permeability models available two decades ago. Haro (2004) also made a detailed comparison of four permeability models (Windland, Kozeny–Carmen, Civan and Lucia). He concluded that the K–C model is the most practical correlation that has good theoretical bases. However, The K–C correlation has inherent limitations since it was derived based on the assumption that porous media can be represented as a bundle of unconnected capillary tubes having identical radius and constant cross-sectional area (Civan, 2002).

In 1993, Amaefule et al. (1993) introduced for the first time the concept of reservoir quality index (*RQI*) and flow zone indicator (*FZI*) to identify HFU based on the K–C model. In this regard, Amaefule's technique is recognized as a very simple, practical, and widely used established technique (Amaefule et al., 1993; Davies and Vessell, 1996; Shenawi et al., 2007). However, the developed technique suffers from the same limitations of the original K–C model that prevent accurate HFU identification.

In this article, a modification of the K–C correlation is proposed by handling the tortuosity term in a more representative manner. Core data from major carbonate reservoirs in Saudi Arabia is used to test the model. Additional data sets obtained from literature are used as well to show the global applicability of the proposed model.

2. Kozeny–Carmen (K–C) correlation

Many models have been proposed to estimate permeability from effective porosity and other relevant parameters. One of the earliest is the Kozeny model (Kozeny, 1927). Its correlation expresses the permeability as a function of effective porosity, tortuosity and specific surface area. It was able to derive correlation by considering the

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porous medium as a bundle of tortuous capillary tubes with the same radius. By combining Poiseuille's equation with Darcy's law and solving for permeability (k), Kozeny obtained the following relationship:

$$k = \frac{\phi}{8\tau} r^2 \tag{1}$$

where (k) is permeability in μm^2, (τ) is the tortuosity, (ϕ) is the effective porosity in fraction and (r) is the radius of the capillary tubes in μm.

The equation was later modified by Carmen (Carman, 1937) and the popular form is given by the following formula:

$$k = \left(\frac{1}{f_g \tau S_{V_{gr}}^2}\right) \frac{\phi^3}{(1-\phi)^2} \tag{2}$$

where (k) is permeability in μm^2, (f_g) is the shape factor, (τ) is the tortuosity, ($S_{V_{gr}}$) is the specific surface area of the grain in μm^{-1} and (ϕ) is the effective porosity in fraction.

The K–C correlation was developed based on the concept of average pore throat size. It was found that this correlation works best for synthetic porous media where pore systems are homogenous and easy to quantify. However, this equation does not work properly in heterogeneous and complex pore systems (Ahmed et al., 1991; Babadagli and Al-Salmi, 2004; Francisco et al., 2009).

3. Conventional HFU characterization technique

Amaefule et al. (1993) developed a technique to characterize HFU using the K–C model based on the concept of mean hydraulic radius and flow units. Tiab (2000) defined a hydraulic flow unit as "a continuous body over a specific reservoir volume that practically possesses consistent petrophysical and fluid properties, which uniquely characterize its static and dynamic communication with the wellbore".

Theoretically, RQI versus the ratio of pore volume to grain volume (ϕ_z) plotting should yield a straight line on log–log plot with a unit slope line. Rock samples with similar FZI values will be positioned on a unit slope line forming a HFU. Other rock samples with different FZI values will lie on other parallel lines. Unfortunately, this is not always the case. In fact, Civan (2002) and Haro (2004) showed that natural rock systems tend to show various slopes rather than having a fixed slope as suggested by Amaefule et al. (1993) and the K–C-model.

4. Proposed modification to the K–C model

A modified K–C model is developed by handling the tortuosity term in more robust approach. The tortuosity can be approximated accurately from electrical property measurements and effective porosity. The following sections illustrate the theoretical development of the proposed model and its application in characterizing HFU.

4.1. Tortuosity role in the K–C model

Tortuosity (τ) is defined as the squared ratio of the path traveled by a fluid particle through a porous medium (L_a) to the actual length of the porous medium, L (Rose and Bruce, 1949; Wyllie and Rose, 1950) which can be expressed mathematically as:

$$\tau = \left(\frac{L_a}{L}\right)^2 \tag{3}$$

A relationship between tortuosity, formation resistivity factor (F_R) and cementation exponent (m) has been derived using theoretical approaches (Wyllie and Rose, 1950; Winsauer et al., 1952). Wyllie and Rose (1950) were able to develop the following relationship:

$$\tau = (F_R * \phi)^2 \tag{4}$$

Since (F_R) can be approximated using Archie's equation (Archie, 1942) as:

$$F_R = \frac{a}{\phi^m} \tag{5}$$

where, (a) is the lithology factor and (m) is the cementation exponent. Eq. (4) then can be written as:

$$\tau = \left(\frac{a}{\phi^{m-1}}\right)^2 \tag{6}$$

Eq. (6) demonstrates the nonlinear relationship between tortuosity and porosity. Theoretically, a bundle of capillary tubes would have (a) and (m) equal to one. In that case, tortuosity would also be one. Similarly, as porosity approaches a hypothetical value of 100% (the common range of porosity in petroleum reservoirs is between 10% and 20%, Tiab and Donaldson, 2004), tortuosity would also approach one, as expected. Fig. 1 shows the tortuosity variation with porosity for different m values where the above mentioned phenomena can be explained. Moreover, if we increase the m value, the tortuosity–porosity relationship becomes more nonlinear.

4.2. Verification of tortuosity model using experimental data

Hagiwara (1986) data set was used to validate the approximation of tortuosity using Eq. (6). Hagiwara proposed a model to estimate permeability using a theoretical approach which is given by the following relation:

$$k = c\,\phi^m \langle R^2 \rangle \tag{7}$$

where, ($\langle R^2 \rangle$) is the average pore throat radius squared in μm^2 and (c) is a constant. Eq. (7) can be rewritten as:

$$k = c\frac{\phi}{(1/\phi^{m-1})}\langle R^2 \rangle = c\frac{\phi}{\tau_H}\langle R^2 \rangle \tag{8}$$

One can notice that Eq. (8) is similar to Kozeny's equation (Eq. (1)) but with different tortuosity definition (i.e. c is equivalent to 1/8 and τ is equivalent to $(\tau_H)^2$). Consequently, the tortuosity model in Eq. (6) (assuming a = 1 for simplicity) is employed in

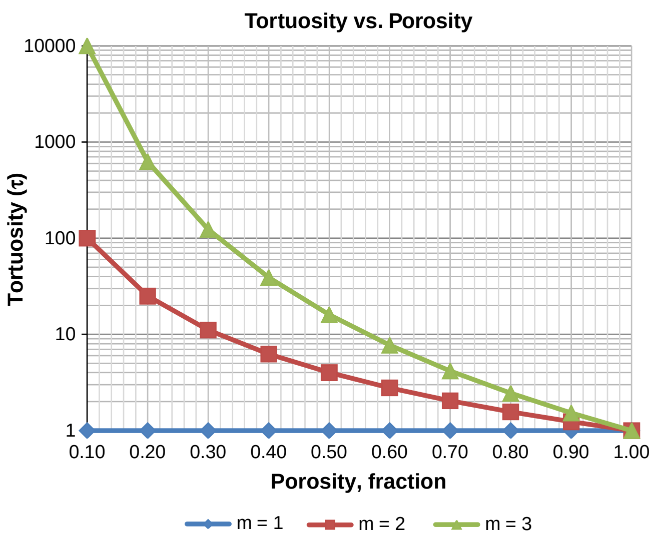

Fig. 1. Tortuosity variation with porosity at various cementation exponent values.

Eq. (8) to get the following relationship:

$$k = c\,\phi^{2m-1} \langle R^2 \rangle \tag{9}$$

A comparison is made using Hagiwara data set between Hagiwara's equation (Eq. (7)) and the modified equation (Eq. (9)). Hagiwara measured porosity, cementation exponent, permeability and pore throat radius for 24 rock samples. The data is shown in Table 1.

Eqs. (7) and (9) are used to generate permeability values. The estimated values were then compared with the measured permeability values. The results are shown in Figs. 2 and 3. Using the linear regression method, the best fit of Eqs. (7) and (9) with data in Table 1 showed the following correlations, respectively:

$$k = 15.4\left(\phi^{m} \langle R^2 \rangle\right)^{1.15} \tag{10}$$

$$k = 78.7\left(\phi^{2m-1} \langle R^2 \rangle\right)^{1.00} \tag{11}$$

Eq. (10) shows a slope of 1.15 (Fig. 2), which is higher than anticipated by the model while Eq. (11) gives a perfect unit slope line and higher correlation coefficient (Fig. 3). This result strongly confirms the validity of the tortuosity estimation using Eq. (6).

4.3. Development of proposed model

Since the tortuosity model based on theoretical approach has been verified experimentally, Eq. (6) can now be incorporated into the original K–C model (Eq. (2)). The proposed model can then be written as:

$$k = \left(\frac{1}{f_g a^2 S_{V_{gr}}^2}\right) \frac{\phi^{2m+1}}{(1-\phi)^2} \tag{12}$$

Eq. (12) represents the proposed model in estimating the permeability and demonstrates the impact of tortuosity on the K–C correlation. For a bundle of capillary tubes, where m, a and τ are equal to one, the proposed model is identical to the K–C model. This explains why the K–C model is so successful for homogenous rocks and synthetic porous media. The nonlinear nature of tortuosity results in a power-law

Table 1
Hagiwara (1986) data set.

φ (%)	k (mD)	m	R (μ)
13.0	15.3	1.89	8.60
26.2	4005.0	1.64	41.25
31.1	4133.0	1.60	26.63
22.9	1170.0	1.70	12.38
24.6	355.0	1.77	9.80
21.2	796.0	1.78	12.28
23.7	990.0	1.75	22.50
19.2	224.0	1.80	19.50
17.8	255.0	1.78	18.40
10.1	8.1	1.74	5.15
13.1	150.0	1.82	18.90
19.0	434.0	1.76	10.40
14.9	6.8	2.04	8.00
30.1	468.0	1.68	16.88
23.5	73.8	2.05	6.38
28.1	550.0	1.96	16.38
28.7	1.0	2.30	3.43
11.0	7.0	1.77	4.10
18.3	65.0	1.91	9.25
18.1	50.7	1.92	10.75
11.5	12.0	1.76	3.63
23.2	35.1	2.06	7.38
16.8	23.8	1.88	7.03
17.2	110.0	1.72	11.25

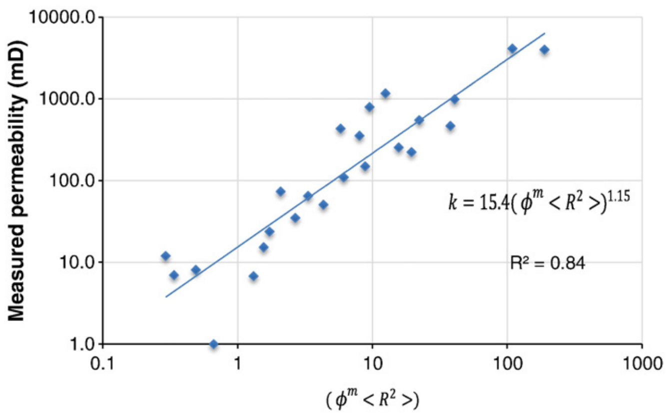

Fig. 2. Permeability cross-plot using Hagiwara equation (Eq. (7)).

model that is strongly impacted by cementation exponent. The common range of (m) values has been reported in literature to vary from 1 to more than 3 (Salem, 1993). The cementation exponent (m) is a very important parameter that reflects many petrophysical and geometrical properties of a certain porous medium, including: degree of cementation, the shape and the size of particles, packing and sorting, pore type, and grain type (Towle, 1962; Helander and Campbell, 1966; Salem, 1993). Consequently, the incorporation of m into the proposed model is certainly essential for better permeability estimation.

4.4. HFU characterization technique using proposed model

Rearranging and taking the square root of Eq. (12) results in the following form:

$$0.0314\sqrt{\frac{k}{\phi}} = \left(\frac{1}{\sqrt{f_g}\,a\ S_{V_{gr}}}\right) \frac{\phi^{m}}{(1-\phi)} \tag{13}$$

The left hand side of Eq. (13) is the reservoir quality index (RQI) where permeability (k) is in mD. The first part of RHS $\left(1/\sqrt{f_g}\,a\,S_{V_{gr}}\right)$ is the modified flow zone indicator (FZI_m). Since the normalized porosity index (ϕ_z) equals to ($\phi/(1-\phi)$), rearrangement of Eq. (13) yields:

$$RQI = FZI_m * \phi_z * \phi^{m-1} \tag{14}$$

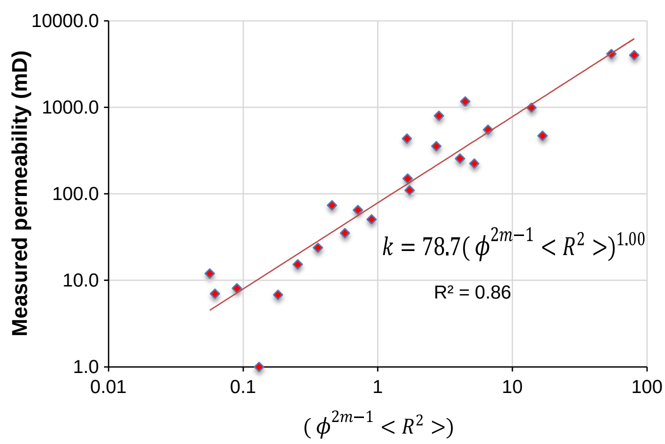

Fig. 3. Permeability cross-plot using modified equation. (Eq. (9)).

Table 2
Winsauer et al. (1952) data set.

φ (%)	k (mD)	F	m
30.7	70	8.40	1.80
19.4	8	24.00	1.94
16.1	3	42.00	2.05
20.6	36	16.60	1.78
19.7	18	20.80	1.87
27.3	88	12.40	1.94
19.3	19	24.40	1.94
19.1	36	17.20	1.72
18.6	25	22.90	1.86
14.7	7	51.00	2.05
15.0	9	41.00	1.96
22.1	200	13.10	1.70
25.1	370	11.60	1.77
18.8	98	19.30	1.77
29.8	1180	8.40	1.76
17.0	90	23.30	1.78
6.7	4	67.00	1.56
18.4	130	19.00	1.74
31.5	2200	6.90	1.67
17.6	220	16.60	1.62
26.3	1920	8.60	1.61
24.8	1560	10.80	1.71
28.2	2100	10.90	1.89
15.0	115	37.30	1.91
18.8	410	18.60	1.75
16.4	330	21.10	1.69
13.9	145	33.00	1.77
25.6	4400	9.40	1.64
27.1	3200	11.70	1.88
39.5	–	4.65	1.65

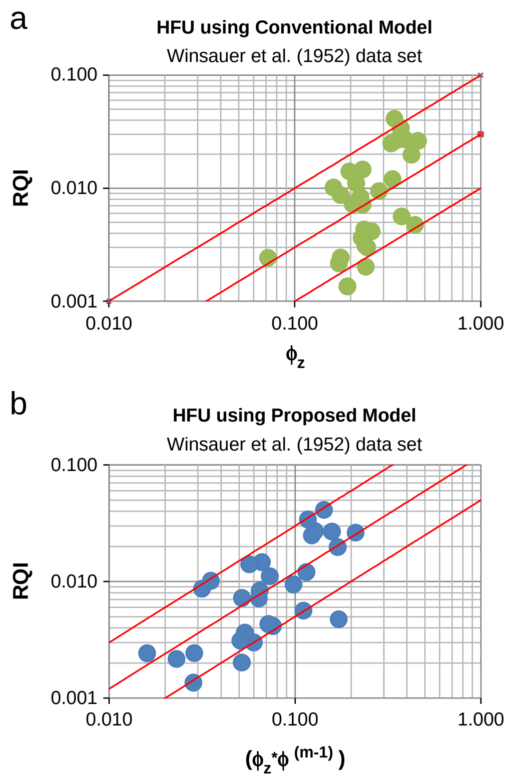

Fig. 4. RQI vs. (φ_z) for Winsauer et al. (1952) data set using conventional model (a) and RQI vs. ($\varphi z{*}\varphi^{m-1}$) using proposed model (b). All red lines represent unit slope lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Taking the logarithm of both sides of Eq. (14) results in the following relationship:

$$\log(RQI) = \log(FZI_m) + \log(\phi_z) + (m-1)\log(\phi) \quad (15)$$

It can be noticed that if the cementation exponent (m) equals to one, then Eq. (15) becomes identical to Amaefule's et al. (1993) model. As (m) increases, the plot of RQI versus ($\phi_z{*}\phi^{m-1}$) on log–log scale gives higher slope lines. Each group of rocks having similar FZI_m will constitute a HFU.

5. Applications and verification of proposed model

5.1. Field examples on sandstone reservoirs

5.1.1. Example I: Winsauer et al. (1952) data set

Winsauer et al. (1952) made a detailed description on more than 40 sandstone samples including permeability, porosity, formation resistivity and other properties. Table 2 lists all samples with their measured properties. Fig. 4(a) and (b) depicts the log–log plot of [RQI vs. ϕ_z] and [RQI vs. $\phi_z{*}\phi^{m-1}$], respectively. All red lines represent unit slope lines. The figures show distinct HFU clusters between the proposed model and the conventional model. To classify the core data into discrete HFU, the following mathematical relation is used (Shenawi et al., 2007):

$$HFU = Round\,[2\ln(FZI) + 10.6] \quad (16)$$

Several clustering techniques have been reported in literature (Abbaszadeh et al., 1996; Lawal and Onyekonwu, 2005). However, Eq. (16) is preferred in this research for comparison purposes. Fig. 5(a) and (b) depicts the permeability–porosity transformations per HFU for the conventional and proposed models, respectively. The conventional model (Fig. 6a) shows similar trend for all HFU. On the other hand, the proposed model (Fig. 6b) shows different trending, especially for HFU-1 and HFU-2. This is mainly to the impact of cementation exponent (m). The average m value for HFU-1 and HFU-2 is 1.9 while the other HFUs have an average of 1.7. This result highlights the strong and important impact of m on HFU identification and how it is being missed by the conventional model. HFU using the proposed model (HFU-1 and HFU-2, in particular) would give higher permeability values at high porosities compared to the permeability values from the conventional model.

5.1.2. Example II: Wyllie and Spangler (1952) data set

Wyllie and Spangler (1952) made porosity, permeability and formation resistivity measurements on 6 samples from Pennsylvanian age consolidated sedimentary rock. Table 3 lists all data. Fig. 6 shows the log–log plot of [RQI vs.ϕ_z] for the conventional model and [RQI vs. ($\phi_z{*}\phi^{m-1}$)] for the modified model. Eq. (16) is used to classify the data into discrete HFUs and it is shown that all sample points are within a single HFU. Since all samples are within the same HFU, RQI vs. (ϕ_z) or ($\phi_z{*}\phi^{m-1}$) should give a straight line with unit slope. Yet, Fig. 6 shows that the conventional model has higher slope of 1.65, and the proposed model has a slope very close to unity (0.98). Using the regression analysis technique, the best fit of both models gave the following correlations:

$$RQI = 13.5 * (\phi_z)^{1.65} \quad (17)$$

$$RQI = 14.6 * \left(\phi_z * \phi^{m-1}\right)^{0.98} \quad (18)$$

where the first part of the RHS in Eqs. (17) and (18) represents FZI and FZI_m, respectively. Eqs. (17) and (18) demonstrate the robustness and the validity of the proposed model.

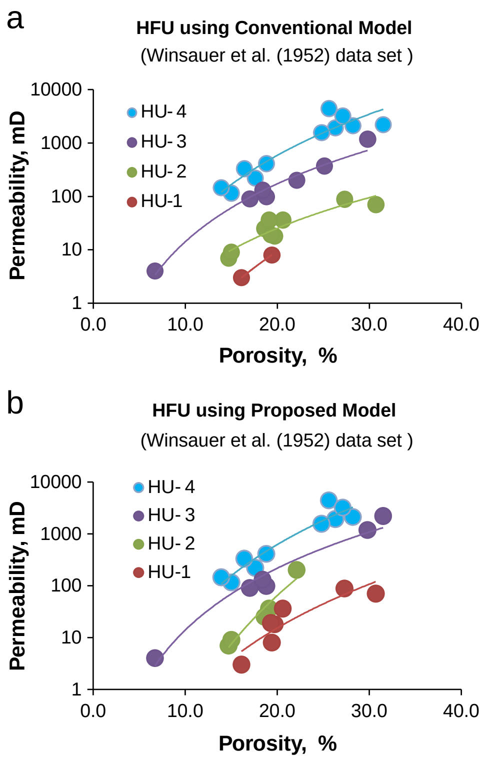

Fig. 5. Permeability–porosity transformations per HFU for conventional (a) and proposed (b) models using Winsauer et al. (1952) data set.

5.1.3. Example III: Ehrlich et al. (1991) data set

Ehrlich et al. (1991) published some measurements on 11 samples from Benoist Sandstone. The measurements included porosity, permeability and formation resistivity (Table 4). The data is used to generate HFUs. Fig. 7(a) and (b) shows the log–log plot of [RQI vs.ϕ_z] for the conventional model and [RQI vs. $(\phi_z{*}\phi^{m-1})$] for the modified model, respectively. The proposed model (Fig. 7b) shows the data to be in better alignment with the unit slope lines (in red). It also shows the presence of two HFUs very clearly compared to the conventional model (Fig. 7a).

Similar to other examples, Eq. (16) was used to classify the data into discrete HFUs. Fig. 8(a) and (b) shows the porosity–permeability

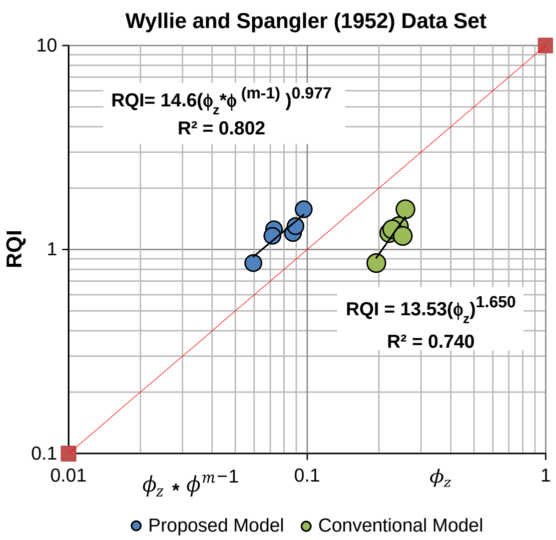

Fig. 6. RQI vs. (φ_z) for conventional model and RQI vs. $(\varphi_z{*}\varphi^{m-1})$ for proposed model using the Wyllie and Spangler (1952) data set.

Table 3
Wyllie and Spangler (1952) data set.

φ (%)	k (mD)	F	m
16.3	122	20.1	1.654
18.1	267	14	1.544
19.5	336	13.9	1.610
20.5	516	13	1.619
18.5	296	16.9	1.676
20.1	278	17.5	1.784

transformations for each HFU, respectively. If those plots are used to predict permeability at high porosity values, the permeability using the conventional model would be underestimated especially for HFU-2.

5.2. Field examples on carbonate reservoirs

5.2.1. Example I: carbonate reservoir-A from Saudi Arabia

A total of 36 samples were obtained having porosity, permeability and formation resistivity measurements. The data were used to generate log–log plots of [RQI vs.ϕ_z] for the conventional model (Fig. 9a) and [RQI vs. $(\phi_z{*}\phi^{m-1})$] for the modified model (Fig. 9b), respectively. Permeability–porosity transformations are shown in Fig. 10(a) and (b) for conventional and proposed models, respectively. The proposed model (Fig. 10b) shows less number of HFUs. It also shows different trend behavior for HFU–1 and HFU-2.

5.2.2. Example II: carbonate reservoir-B from Saudi Arabia

A total of 20 samples were acquired having porosity, permeability and formation resistivity measurements. The data is used to generate log–log plots of [RQI vs.ϕ_z] for the conventional model (Fig. 11b) and [RQI vs. $(\phi_z{*}\phi^{m-1})$] for the proposed model (Fig. 11a), respectively. Permeability–porosity transformations are shown in Fig. 12(a) and (b) for conventional and proposed models, respectively. HFU-4 using the proposed model shows different trends than the other HFUs.

5.2.3. Example III: field-C from Saudi Arabia

More than 5000 routine porosity–permeability data points are used in this example. It is found from the electrical properties measured on a number of core samples that the average cementation exponent for this field is approximately two. The data is used to generate log–log plots of [RQI vs.ϕ_z] for the conventional model (Fig. 13a) and [RQI vs. $(\phi_z{*}\phi^{m-1})$] for the proposed model (Fig. 13b), respectively. Using the conventional model, Fig. 13a shows the whole data points deviate from the unit slope lines very clearly. However, the proposed model shows much better alignment of data points with the unit slope lines (Fig. 13b). This result confirms the validity of the proposed model even when using an average m value for the whole data set.

Table 4
Ehrlich et al. (1991) data set.

φ (%)	k (mD)	F	m
15.6	28.5	26.6	1.766
17.2	91.9	21.77	1.750
14.9	39.9	26.74	1.726
14.9	64.6	28.57	1.761
16.7	190.4	21.5	1.714
18.3	270.3	17.03	1.669
17.2	200.5	20.05	1.703
18.6	424.9	22.5	1.851
16.3	137.3	26.99	1.817
19.6	669.5	14.58	1.644
18.3	540.5	15.87	1.628

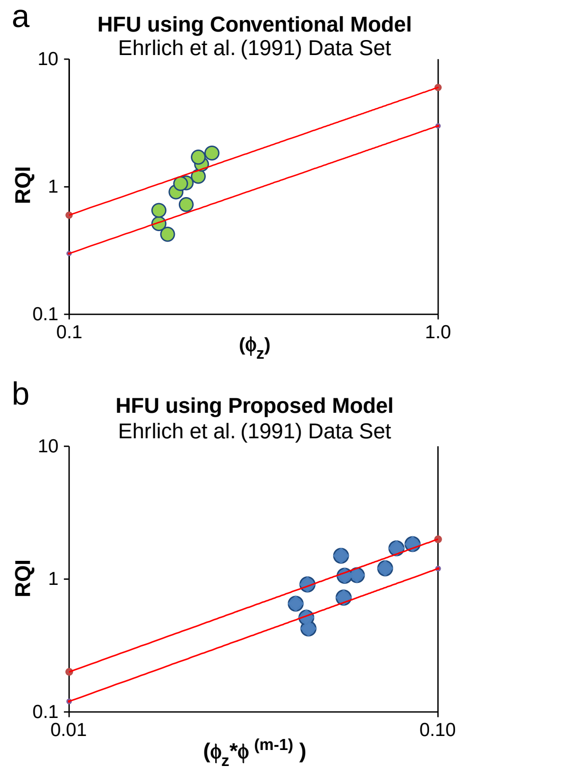

Fig. 7. RQI vs. (φ_z) for Ehrlich et al. (1991) data set using the conventional model (a) and RQI vs. (φ_z*φ^{m-1}) using the proposed model (b) . All red lines represent unit slope lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

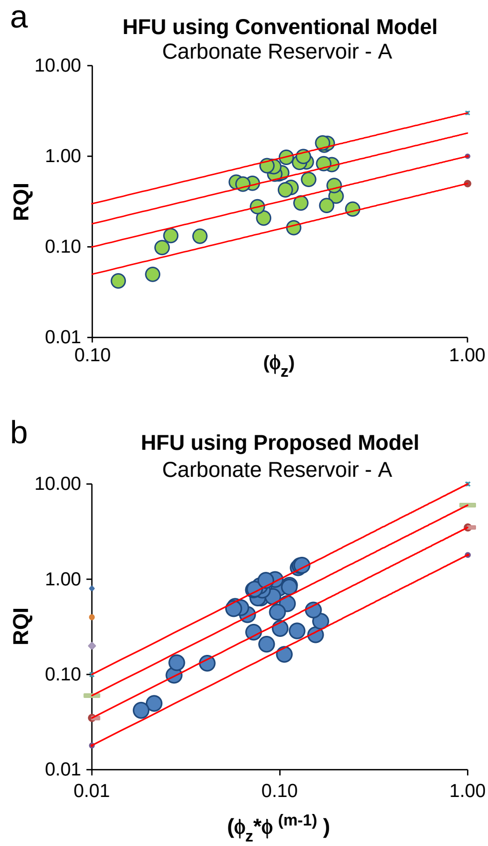

Fig. 9. RQI vs. (φ_z) for a carbonate reservoir (A) using the conventional model (a) and RQI vs. (φ_z*φ^{m-1}) using the proposed model (b). All red lines represent unit slope lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 14(a) and (b) shows the porosity–permeability transformations on semi-log plots using the conventional and the proposed models, respectively. It is observed from the figures that porosity–permeability transformations using the proposed model have stepper slopes compared to the conventional model. This indicates that the proposed model generates HFUs that are more sensitive to porosity. Accordingly, permeability has higher variation within a single HFU with the proposed model.

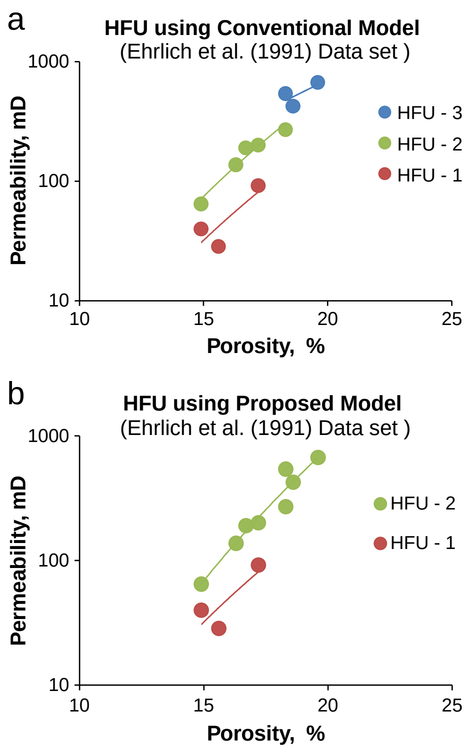

Fig. 8. Permeability–porosity transformations per HFU for the conventional (a) and proposed (b) models using Ehrlich et al. (1991) data set.

6. Future work

In this research, the characterization of HFU is based only on core data. In normal practices, HFUs generated from cores are linked to well logs through certain correlations to allow for continuous HFU identification along logged intervals including un-cored wells. Permeability values can then be estimated from porosity–permeability transformations per HFU (such as Fig. 14).

To extend the usage of the proposed model efficiently to the uncored wells, a proper link between well logs and HFU based on core data should be found. All literatures that deal with this issue can be applied to the proposed model. The only additional parameter that needs to be estimated is the cementation exponent (m). Fortunately, ways have been proposed in literature to estimate (m) from well logs (Salem, 1993). Average (m) can also be used as an alternative way based on laboratory experiments or previous knowledge of the field.

7. Conclusion

In this study, a modified K–C model is developed based on an accurate theoretical approach. The modified model incorporates the tortuosity term in a more representative manner. It is shown that the tortuosity term can be approximated accurately using theoretical and experimental approaches based on effective porosity and

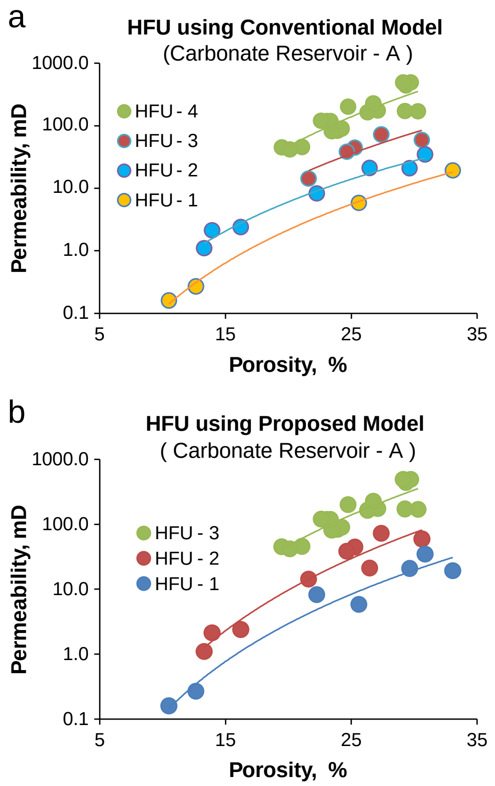

Fig. 10. Permeability–porosity transformations per HFU conventional (a) proposed (b) model for a carbonate reservoir (A).

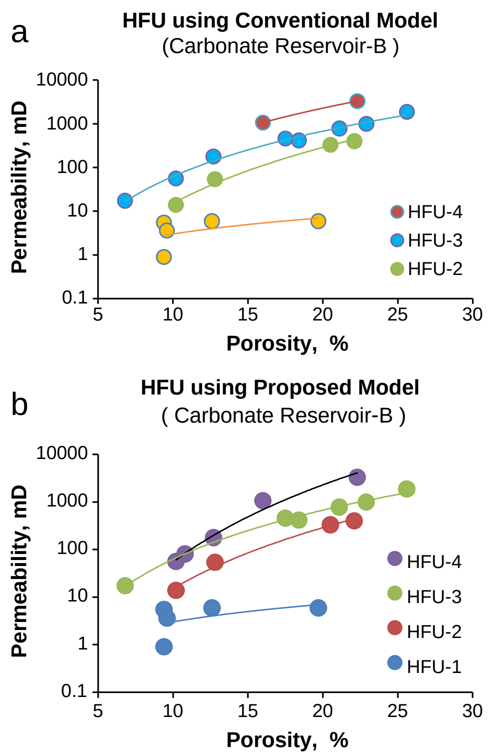

Fig. 12. Permeability–porosity transformations per HFU for conventional (a) and proposed model for a carbonate reservoir (b).

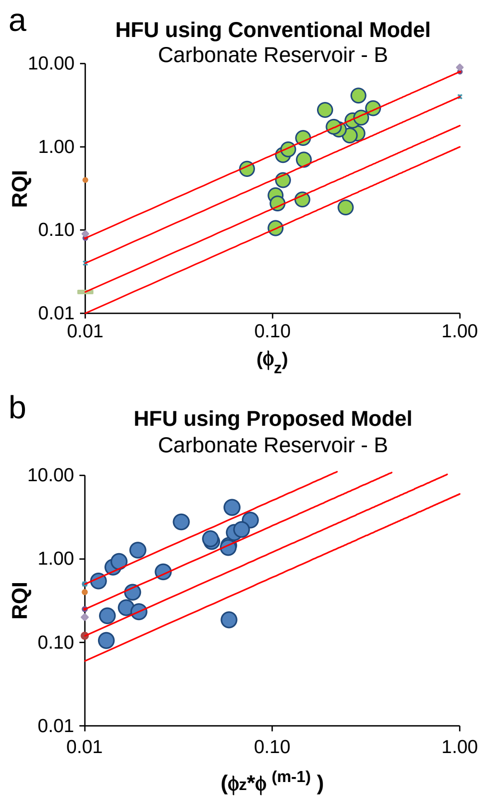

Fig. 11. RQI vs. (φ_z) for a carbonate reservoir (B) using the conventional model (a) and RQI vs. ($\varphi z^*\varphi^{m-1}$) using the proposed model (b). All red lines represent unit slope lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

cementation exponent. The incorporation of the validated tortuosity model into the original K–C model leads to a variable power on the porosity term as a function of cementation exponent.

A total of six data sets from sandstone and carbonate reservoirs are used to verify and validate the proposed model and to confirm its global applicability in identifying HFU. Comparisons between the proposed and convention models are also included. Results demonstrate significant improvement in HFU identification using the proposed model that closely matches with field data sets in comparison with the conventional approach. Results also show that HFU obtained using the proposed model is more sensitive to porosity values. Consequently, for a given HFU, the proposed model can capture a wider range of permeability values for a particular range of porosities. This leads to the shortcomings of the classical model where underestimation of permeability values especially at higher porosities can occur.

Nomenclature

a	Lithology factor, dimensionless
c	Least-squares fit constant
f_g	Shape factor, dimensionless
F_R	Formation resistivity factor, dimensionless
FZI	Flow zone indicator, μm
FZI_m	Modified flow zone indicator, μm
HFU	Hydraulic flow unit
k	Permeability, mD
L_a	Actual length of a fluid particle through porous media, unit length
L	Length of a porous medium, unit length
m	Cementation exponent, dimensionless
r	Radius of capillary tubes, μm
$\langle R^2 \rangle$	Average squared pore throat radius squared, μm^2
RQI	Reservoir quality index, μm
$S_{V_{gr}}$	Specific surface area of the grain, μm^{-1}

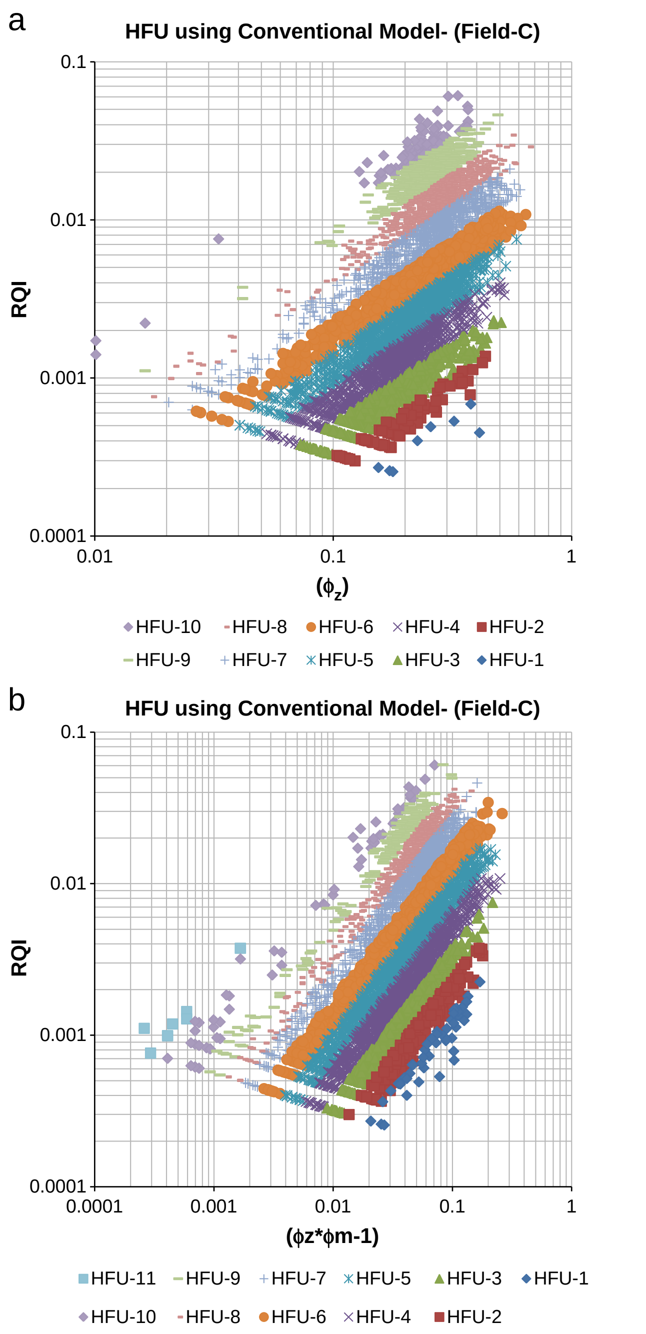

Fig. 13. (a): RQI vs. (φ_z) for Field-C samples using the conventional model. (b): RQI vs. ($\varphi_z{}^*\varphi^{m-1}$) for Field-C samples using the proposed model. HFUs are in better alignment with the general trend of the data points.

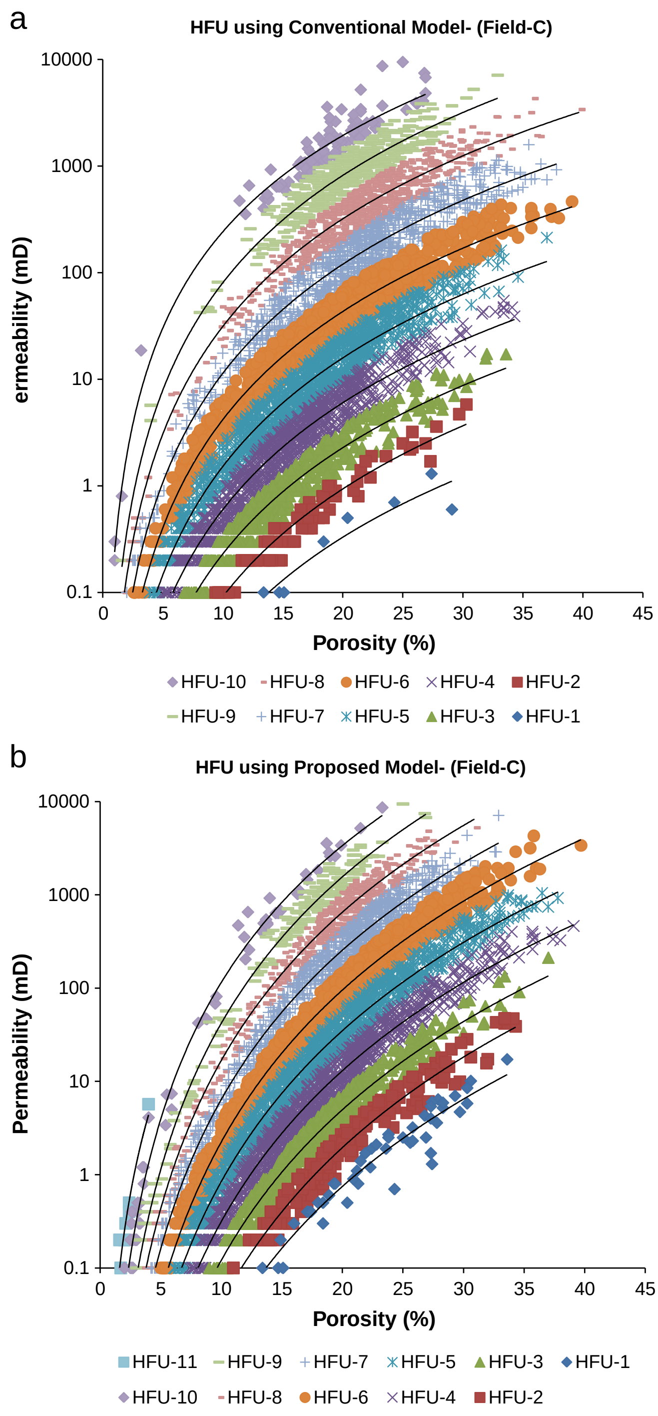

Fig. 14. (a): Permeability vs. porosity transformations for Field-C using the conventional model. (b): Permeability vs. porosity transformations for Field-C using the proposed model.

Greek Symbols

ϕ	Porosity, fraction
τ	Tortuosity, dimensionless
ϕ_z	Normalized porosity index, fraction

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