Inclusion of the Memory Function in Describing the Flow of Shear-Thinning Fluids in Porous Media

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Abstract

This paper introduces mathematical models with memory to present the complex rheological phenomena combining the in situ shear rate and bulk rheology with fluid memory. The models are numerically solved and compared the results with established experimental data available in the literature. The new models showed excellent agreement with experimental results for the region where most existing models failed. The proposed models can be used in a number of applications, such as enhanced oil recovery (EOR), polymer manufacturing, etc.

Keywords: Non-Newtonian fluid; shear rate; complex fluid rheology; non-Darcy flow; non-linear dynamics; chaos in porous media.

1. INTRODUCTION

The flow of aqueous polymeric fluids through porous media has been studied in the past mainly due to its importance in the areas of EOR. Past investigations focus on the flow of inelastic shear-thinning fluids and more complex viscoelastic polymers. Even when the bulk rheology of the aqueous polymer solution is known, there is still an issue of how this relates to the in situ rheology of the fluid. Clearly, the internal geometry of the pore space in either a packed bed or a porous rock is far more complex than any regular rheumatic flow. Hence the apparent viscosity is really an aggregate or “upscaled” measure of viscous, elastic and extensional flow effects, and the matter of how this should be done has been of recent concern. For polymeric solutions, the apparent viscosity is a function of flow rate through the porous medium, and the flow rate is further correlated with the in situ shear rate within the porous medium. This flow rate may also be interrelated with the fluid memory in the pore network where mineral precipitation or other history of movement may delay the response of the in situ fluid. This delay can lead to restriction of the polymer flow through the pore throat. This paper attempts to eliminate these unsolved complexities of the fluid flow in porous media. To do so, mathematical models with memory are introduced to present the complex rheological phenomena combining the in situ shear rate and bulk rheology with fluid memory. The models are numerically solved and compared the results with existing models by the established experimental data available in the literature. The models can be used in polymer flooding, reservoir simulation and reservoir characterization where pseudoplastic (shear-thinning) fluids are the main concern in porous media.

The Newtonian viscosity model is the most commonly used one for the study of fluid flow in various applications. In principle, the Newtonian model implies that the viscosity is independent of shear rate. Water, the most abundant fluid on earth, has been considered for centuries as an ideal example of a Newtonian fluid. Only recently, Li et al. [1] discovered that when water molecules are forced to move through a small gap (in nanoscale) of two solid surfaces (hydrophilic/wetting), its viscosity increases by a factor of 1000 to 10,000, resulting in a behavior similar to molasses. During their experiment on hydrophobic surfaces, they did not observe such an increase in viscosity. Their findings are in good agreement with the molecular dynamics simulation results that show a dramatically decreased mobility for sub-nanometer thick water films under hydrophilic confinement. They concluded that water has viscous and solid-like properties at its molecular level and is organized into layers. At the nanometer scale, water is a viscous fluid and could be a much better lubricant.

This study received great attention from the general scientific community as well as the general public because of its potential applications to nano technology [2]. However, the fact that any fluid behaves differently under molecular constrains from larger scale has been known for some time. The slow-moving flow of a thin film of a liquid is an ubiquitous phenomenon. This flow exists in nature such as in lava flows, the linings of mammalian lungs, tear films in the eye, and in artificial instances such as microchip fabrication, tertiary oil recovery as well as in many coating processes [3]. Therefore, the natural phenomena, for which viscous fluid flow exists, are normally non-Newtonian type of flow [3-4]. If the most well accepted Newtonian fluid, water exhibit viscous flow, it's a matter of research to say that all fluids are non-Newtonian. As time progresses and our ability to measure with greater accuracy improves, the true nature of fluids is likely to be revealed as non-Newtonian. Even though numerous models for simulating non-Newtonian fluid behavior exist, there is a need to develop a theoretical model that is valid for an entire range of stress-strain relationship.

If viscosity diminishes as shear is increased, the fluid is said to exhibit shear thinning. This occurs in simple liquids as well as in complex mixtures such as foams, micelles, slurries, pastes, gels, polymer solutions, and granular flows [5]. The flow and displacement of non-Newtonian and complex fluids (such as polymer gels and surfactants) in porous media is an important subject. These have a variety of industrial applications. It is well known that polymeric materials, emulsions, foams, and gels exhibit non-Newtonian behavior, even at meso scale. For various industrial applications, ranging from paints, manufacturing and food processing to petroleum production, numerous non-Newtonian models have been proposed.

Kondic et al. [6] studied the non-Newtonian fluid in a Hele-Shaw cell. They have used Darcy's law whose viscosity depends upon the squared pressure gradient to a fluid model with shear-rate dependent viscosity. However, their derivation does not allow strong shear-thinning which is related to the appearance of slip layers in the flow. The influence of shear effects on the adhesive performance of a non-Newtonian fluid under tension has been studied by Miranda [7]. He used the modified Darcy's law in developing a shear-rate model. His results show that, for a relatively small separation, the adhesion strength is considerably reduced if the fluid is shear thinning (thickening). Shear effects become negligible when large plate separation occurs. Afanasiev et al. [8] studied the drag-out problem for shear-thinning liquids at variable inclination angles. They used the power-law, the Ellis and Carreau model as their viscosity model. They considered steady state solution and the dependence on the rheological parameters. However, many applications of polymeric liquids and suspension show nonlinear stress-strain relationships. A distinct viscosity regimes show up during shear stress when most polymer solutes are used under shear-thinning conditions. The type of fluid (e.g. Newtonian or non-Newtonian) can be categorized based on the shear rate. It shows that polymeric fluid behaves as Newtonian at very low shear rates. As the shear rate increases the behavior starts to become nonlinear. When shear rate is further increased it moves into a regime where the viscosity can be modeled by a power-law relation. It is noted that the power-law model is only applicable at large shear rates, Ellis model is for low shear rate [8]. Therefore, their observation further consolidates the need to develop a model that allows for very high shear rates.

Non-Newtonian fluids exhibit another particular phenomenon not observed in Newtonian fluids such as the negative wake behind a bubble are due to memory effects in stress relaxation [9]. Under shearing, a polymer solution may exhibit memory effects during a consequence of stretching of polymeric chains. Huang and Lin I [10] investigated the temporal memory and persistence time correlations of microstructural order fluctuations in quasi-two-dimensional dusty plasma liquids through directly monitoring particle positions. They noticed that persistence length of the ordered and the disordered microstates both follow power-law distribution for the cold liquid. They also identified that increasing thermal noise level deteriorates the memory and leads to the less correlated stretched exponential distribution of the persistent length. Even though the existence has long been recognized (e.g., thixotropic fluid) [11-13], few models have been proposed to include the memory effect. Even fewer models have been developed that is continuous in nature, mostly settling for good agreement within a narrow range of constraints.

In the case of porous media flow, the problem is accentuated in the consideration of permeability variation with time. The fluid memory can be defined as the change of viscosity and other pressure dependent properties over time whereas the memory of rock can be characterized as the alteration of permeability with time. Therefore, if permeability diminishes with time, the effect of fluid pressure at the boundary on the flow of fluid through the medium is delayed. The information of this effect over time in the fluid flow will continue if the medium (i.e. flowing fluid) holds a memory [14-17]. However, most of the current flow models in porous media have been developed for purely Newtonian and Non-Newtonian fluids where no model represents the fluid memory with the shear-thinning fluid models [18-29]. Most of the researchers have tried to relate the bulk properties of complex fluids to their behavior in a porous medium using a common approach, consisting of representing the medium by a bundle of parallel capillary tubes [27].

The majority of complex fluids used in oil field applications are non-Newtonian polymeric solutions demonstrating shear-thinning (pseudoplastic) behavior in solution. The bulk macroscopic properties of these solutions, mainly their viscosity/shear rate dependency, are well understood and characterized using established models. Existing theoretical models as well as experimental findings are well established in the literature [18-29]. It is rather the numerical modeling that is incomplete [30]. Only recently, Hossain et al. attempted to model shear rate and viscosity of polymeric complex fluid as a function of time and other related bulk properties of fluid and media itself where memory has been incorporated to represent macroscopic and microscopic behavior of fluid and media in a more realistic way.

2. EXISTING MODELS

The two main polymers used in the oil industry for hydrocarbon recovery are synthetic polyacrylamide (in its partially hydrolyzed form, HPAM) and Xanthan biopolymer gum. Bulk properties measurement of polymeric solutions is a standard and reliable experimental procedure. Therefore, researcher's efforts have been made to extend the laws of motion for purely Newtonian fluids (Darcy's law) to rheologically complex ones using easily measurable properties such as the shear rate/viscosity behavior. A bundle of parallel capillary tubes approach has been used to measure the macroscopic and microscopic properties of porous media. This approach leads to the definition of an average radius which is dependent on macroscopic properties of the medium such as porosity, absolute permeability, and some measure of tortuosity. The available mathematical models (such as power law, Carreau, or Cross models) to describe the fluid rheology have been developed to define viscosity and apparent shear rate from the use of the Darcy velocity [19, 20, 23, 24, 31-35]. Experimental results show that the shape of the apparent viscosity curve is similar to that of bulk shear rate. Most of experimental works had been performed by Xanthan biopolymers whose experimental results are available in the literature [19, 20, 24, 31, 33, 34, 35-39] where they tried to find the shape factor, α_{SF}. For the porous media, Chauveteru's form of the definition of porous media wall shear strain rate or in-situ shear rate is [18-23, 25, 26-29, 35, 37]

$$\dot{\gamma}_{pm} = \frac{\alpha_{SF}\, u}{\sqrt{k\,\varphi}} \qquad (1)$$

In the above equation, α_{SF} represents shape factor. In the context of polymer flooding (part of enhanced oil recovery schemes), in-situ rheology depends on polymer type and concentration, residual oil saturation, core material and other related properties is addressed in the available literature [28, 33, 37, 39, 40]. A brief discussion has been outlined by Lopez [35]. The existence of a slip phenomenon in the Newtonian region at ultra low flow rates is confirmed and the degree of shift (the α_{SF} factor) in the non-Newtonian region is quantified. It is shown that the adoption of rigorous and reproducible coreflood procedures is required to yield unambiguous data on in-situ polymer viscosity and polymer retention in real systems. Coombe et al. [41] analyzed the impact of the non-Newtonian flow characteristics of foam, polymers and emulsions. Balhoff and Thompson [42] modeled the flow of shear-thinning fluids, including power-law and Ellis fluids in packed bed using the network model. They also well defined and tried to highlight the existing models used for shear-thinning fluids. They pointed out that Eq. (1) has a generic form which depends on polymer type, medium structure and approach.

There have been developed several constitutive equations in the past that capture the full bulk rheological behavior of pseudoplastic solutions [37]. To model the bulk rheology of the non-Newtonian fluid, Escudier et al. [24] performed a set of experiments on Xanthan gum. Lopez [27, 35] showed that Carreau-Yasuda, Cross and Truncated Power-law models behave almost same when they presents the bulk rheology (e.g. effective viscosity) of shear thinning fluid. Therefore, Carreau-Yasuda model is considered in this study and is used to develop and analyze the memory model in bulk rheology. Carreau-Yasuda model may be written as [20, 21, 24, 27, 35]:

$$\mu_{eff} = \mu_{\infty} + \frac{(\mu_0 - \mu_{\infty})}{[1+(\lambda\,\dot{\gamma})^a]^{\frac{n}{a}}} \qquad (2)$$

3. MATHEMATICAL MODEL WITH FLUID MEMORY

The exact form of the shear stress-shear rate (stress-stain) relationship depends on the nature of the polymeric solution. Therefore, recently, a question is always coming out about the effect of memory on rock/fluids in porous media when predicting oil flow outcomes. Hossain and Islam [30] have reviewed the existing complex fluid flow models with memory available in literature. None of them has focused the shear thinning fluid models which may couple with fluid memory. Recently Hossain et al. [16-17] have developed a model which represents a more realistic rheological behavior of fluid and media. They have developed a stress-strain relation coupling the macroscopic and microscopic properties with memory. They also did not consider the polymeric fluid properties in porous media. However, the conventional practice is to consider the Newtonian fluid flow equations as ideal models for making predictions. Even non-Newtonian models focus on what is immediately present and tangible in regard to fluid properties. This paper argues that the intangible dimension of time and other fluid and media properties can be coupled to demonstrate the more complex behavior of shear thinning fluids in porous media.

In practical macroscopic point of view, several authors [29, 43, 44] have reported that the apparent viscosity of polymer solutions within various porous media are usually bead packs, sand packs and outcrop sandstone rocks. A detail overview of the earlier works have been presented by the researchers [18, 19, 20, 26, 27] and summarized in the earlier section. Referring to them, the apparent viscosity for the polymeric solutions can be defined from Darcy's law as follows:

$$\mu_{app} = \frac{k\ A}{Q} \frac{\Delta p}{L} \tag{3}$$

For polymeric solutions, the apparent viscosity μ_{app} is a function of flow rate through the porous medium [29, 40], and the flow rate is further correlated with $\dot{\gamma}_{pm}$, the in situ shear rate within the porous medium, which is expressed as Eq. (1). In this simple equation, however, it is not obvious why $\dot{\gamma}_{pm}$ should vary linearly with the Darcy velocity, due to the complexity of the internal flows within the porous medium. The central theoretical problem of in situ rheology is therefore to establish clearly how the local (microscopic or pore-scale) rheology in a single pore relates to the aggregated or average bulk property as expressed by the apparent viscosity discussed above. Hence it is the effect of this local behavior, mediated through the interconnected network of pores in the porous medium to the macroscopic scale that must be found. Therefore it is necessary to clarify how the local (microscopic or pore-scale) rheology relates to the average bulk rheology as expressed by the apparent viscosity. Some researchers [20, 27, 45] considered the local bulk rheology (e.g. Carreau model) for shear-thinning fluids and calculate the "upscaled" macroscopic apparent viscosity using a connected 2D network of cylindrical capillaries as an idealized porous media. Using these concept, Perrin et al. [29] showed that the average in situ shear rate in the network (which corresponds to $\dot{\gamma}_{pm}$) actually varied linearly with the flow velocity, u. We know that the complexity of the internal flows within the porous media leads to a non-linear behavior of shear rate with Darcy velocity [16, 17, 30].

To investigate the local phenomenology, we may introduce memory effect for the fluid complex rheological properties. In this regard, fluid precipitation of minerals and temperature may be considered. However, some fluids carry solid particles that may impede some of the pores. The precipitation and obstruction may reduce the pore size and thus decrease the permeability in time. Some fluid may have chemically reacting behavior with the medium which may enlarge the pore size. These phenomena let the researcher to think about the local mineralization and permeability changes which lead for a spatially variable pattern. Therefore, if permeability diminishes with time, the effect of fluid pressure at the boundary on the flow of fluid through the medium is delayed. The information of this effect over time in the fluid flow will continue when the medium (i.e. flowing fluid) holds a memory.

Auriault and Boutin [46] pointed out that the macroscopic description is sensitive to the ratios between the different scales of the characteristic lengths of the pores, the fractures, and the macroscopic medium, respectively. They described the memory effect can exhibit in case of the ratio the characteristic lengths of the pores and the macroscopic medium. This memory is due to the seepage through the micropores. Under transient excitations, they concluded that the harmonic quasi-static excitations, with complex and frequency dependent effective coefficients in porous medium therefore display memory effects. They referred to this mechanism as the 'viscoelastic property' of fluid saturated, cracked solids. Savins [43, 44] also acknowledged the effects of memory as a 'viscoelastic effects'. He mentioned the previous researchers who worked on the phenomenological theories of incompressible memory fluids.

Hossain et al. [16] initiated a rate equation model to study the flow of these fluids with memory. Caputo [14-15] modified Darcy's law by introducing the memory represented by a derivative of fractional order of differentiation which simulates the effect of a decrease of the permeability in time. If the fluid flows in x-direction, the mass flow rate equation may be written as (14-17, 30];

$$q_x = -\eta\,\rho_0\,\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right] \tag{4}$$

where

$$\frac{\partial^\alpha}{\partial t^\alpha}[p(x,t)] = \frac{1}{(1-)}\int_0^t (t-\xi)^{-\alpha}\,\frac{\partial}{\partial \xi}[p(x,t)]d\xi, \text{ with } 0 \le \alpha < 1$$

It is clear that the memory introduced in Eq. (4) to describe the flow of the fluid implies the use of two parameters, namely α and η. These two parameters are used instead of the permeability and viscosity in conventional Darcy's law. In Eq. (4), pressure gradient is negative and this has a decreasing slop [16-17]. Therefore, Eq. (4) can be written for fluid velocity which is related to pressure gradient as;

$$u_x = \eta\,\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial x}\right)\right] \tag{5}$$

where

$$\frac{\partial^\alpha}{\partial t^\alpha}[p(x,t)] = \frac{1}{(1-)}\int_0^t (t-\xi)^{-\alpha}\,\frac{\partial}{\partial \xi}[p(x,t)]d\xi, \text{ with } 0 \le \alpha < 1$$

Eq. (5) may be written as,

$$u = \frac{\eta}{(1-)}\int_0^t (t-\xi)^{-\alpha}\,\frac{\partial^2 p}{\partial \xi \partial x}d\xi \tag{6}$$

Substituting Eq. (6) in Eq. (1)

$$\dot{\gamma}_{pm} = \frac{\alpha_{SF}}{\sqrt{k\,\varphi}}\,\frac{\eta}{(1-)}\int_0^t (t-\xi)^{-\alpha}\,\frac{\partial^2 p}{\partial \xi \partial x}d\xi \tag{7}$$

The above mathematical model provides the effects of the polymer fluid and formation properties in one dimensional fluid flow with memory and this model may be extended to a more general case of 3-Dimensional flow for a heterogeneous and anisotropic formation. It should be mentioned here that the first part of the Eq. (7) is the apparent core properties; second part is the effects of fluid memory with time and the pressure gradient. The second part is in a form of convolution integral that shows the effect of the fluid memory during the flow process. This integral has two variable functions of $(t-\xi)^{-\alpha}$ and $\partial^2 p/\partial\xi\partial x$ where the first one is a continuous changing function and second one is a fixed function. This means that $(t-\xi)^{-\alpha}$ is an overlapping function on the other function, $\partial^2 p/\partial\xi\partial x$ in the mathematical point of view. These two functions depend on the space, time, pressure, and a dummy variable.

Several workers [36, 47] used almost same expression to represent the apparent shear rate as an effective shear. Moreover, Savins [43, 44] refers this problem as "scale up". Sorbie et al. [47] review some of the alternative approaches to this problem. In this study, we ignore this shifting or scale up problems due to a constant factor involvement. This will not affect the big picture of the effective or apparent viscosity model. Therefore, to analyze the memory effect in the shear-thinning fluid viscosity, the model presented in Eq. (7) is applied in Eq. (2). Substituting Eq. (7) in Eq. (2)

$$\mu_{eff} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{\left[1+\left(\frac{\lambda\,\eta\,\alpha_{SF}}{\sqrt{k\,\varphi}}\frac{\int_0^t (t-\xi)^{-\,\alpha}\,\frac{\partial^2 p}{\partial\xi\partial x}d\xi}{(1-\,)}\right)^{a}\right]^{\frac{n}{a}}} \tag{8}$$

4. NUMERICAL SIMULATION

To solve the convolution integral of Eqs. (7) and (8), a reservoir of length (L = 5000.0 m), width ($W = 100.0\,m$) and height, ($H = 50.0\,m$) have been considered. The porosity and permeability of the reservoir are 30% and 1×10^{-14} to $13.5\times10^{-14}\,m^2$ (10 md to 135 md), respectively. The reservoir is completely sealed and produces at a constant rate where the initial pressure is $p_i = 27579028\,pa$ (4000 psia). The fluid is assumed to be Xanthan gum, with the properties $c = 1.2473\times10^{-9}\ 1/pa,\ \ \mu_0 = 13.2\,pa\,s.$ The initial production rate is $q_i = 8.4\times10^{-9}\ m^3/s$ and the initial fluid velocity in the formation is $u_i = 1.2\ \times10^{-5}\ m/s$. The fractional order of differentiation, $\alpha = 0.2 - 0.8,\ \Delta x = 1000\,m,$ and $\Delta t = 7.2\ \times10^4\ \mathrm{s}$ have also been considered. The computations are carried out for $\mathrm{Time} = 40\,months$. The same input data has been used by Hossain et al. [16-17, 30] except the fluid properties. In solving this convolution integral with memory, trapezoidal method is used. All computation is carried out by Matlab 6.5.

5. RESULTS AND DISCUSSION

The results of the in situ or apparent shear rate and effective viscosity based on the model presented in Eqs. (7) and (8) can be obtained by solving these equations. These models are solved numerically. In this paper, we focused on the dependence of the shear rate on porosity, permeability, shape factor and flow velocity which is related to the effect of fluid memory and finding a numerical description for a sample reservoir.

To solve the Eq. (7), $\eta = 0.343249$ and $\alpha_{SF} = 1.25$ are used in numerical computation [16, 17, 20, 22, 27, 36, 45, 47]. In solving the proposed modified Carreau-Yasuda model presented in Eq. (8), the Xanthan gum fluid viscosities at low (μ_0) and high shear rate (μ_∞) are considered as 13.2 pa-s and 0.00212 pa-s respectively [24]. The power-law index, *n* and the time constant, λ are taken as 0.689 and 60.7 s respectively [24]. Here, the diminishing permeability in the pore space and throat of the reservoir formation are varied within the range as mentioned before to considered fluid memory of the flowing shear thinning fluid.

5.1 Shear rate dependency on different parameters

5.1.1 Shear rate dependency on permeability for different α values

Figure 1 presents the variation of in situ or apparent shear rate with permeability for different α values. These are the nonlinear profiles for different α values. These curves show that as permeability increases, the in-situ shear rate increases slightly for a very tight reservoir. This trend reduces when α value increases. This indicates that when memory effect is dominant in a tight reservoir, it tries to restrict the fluid movement. However, as permeability increases, shear rate start to decrease up to a certain range of permeability values. The range is 45 mD – 55 mD. The fixed value depends on α value. After this transient permeability level, shear rate increases with the increase of permeability value. The whole characteristics and trend of the curves are same for all the α values. The memory of fluid has a great influence on in situ shear rate. For the same permeability value, the in situ shear rate decreases with the increase of α value at a range of up to 0.4 and after that it increases with the increase of α value. This behavior is a special characteristic of the fluid memory [16-17]. The magnitudes of the in situ shear rate variation are more dominant when fluid memory (α value) increases. Moreover, we already focused earlier that the variable permeability is one of the cause of fluid memory. The initial decrease and then increase of shear rate variation with permeability indicates that fluid take some time or delay to move after feeling pressure or force to move. This delaying of fluid movement is nothing but a microscopic property of the viscoelastic fluid. This phenomenon is defined by fluid memory [14-17, 30]. Therefore the fluid movement can be characterized by the memory effects.

5.1.2 Shear rate dependency on shape factor for different permeability

The available literature [20, 22, 24, 27, 36] reported that the value of α_{SF} lies in the range of 1.0 – 14.1. This variation depends on the type of porous media. This may vary from 1.0 in regular unconsolidated packs up to 10.0 in consolidated sandstone [22]. The experimental results show that α_{SF} is in the range of 1.1 – 2.5 for ballotini beads and 1.9 – 9.1 for sandstone cores [20]. Therefore, to analyze the dependency of in situ shear rate with α_{SF}, it has been considered the value of α_{SF} in the range of 1.0 – 15.0.

Figure 2 shows the variation of apparent shear rate with shape factor, α_{SF} for $k = 70\ md, 80\ md \text{ and } 90\ md$ at a α value of 0.2. The in situ shear rate verses shape factor cure shows a linear relation that has an increasing trend with the increase of shape factor. The affects of shape factor on apparent shear rate is more sensitive at higher permeability where the range of variation of $\dot{\gamma}_{pm}$ is more. On the other hand, it is less sensitive at lower permeability.

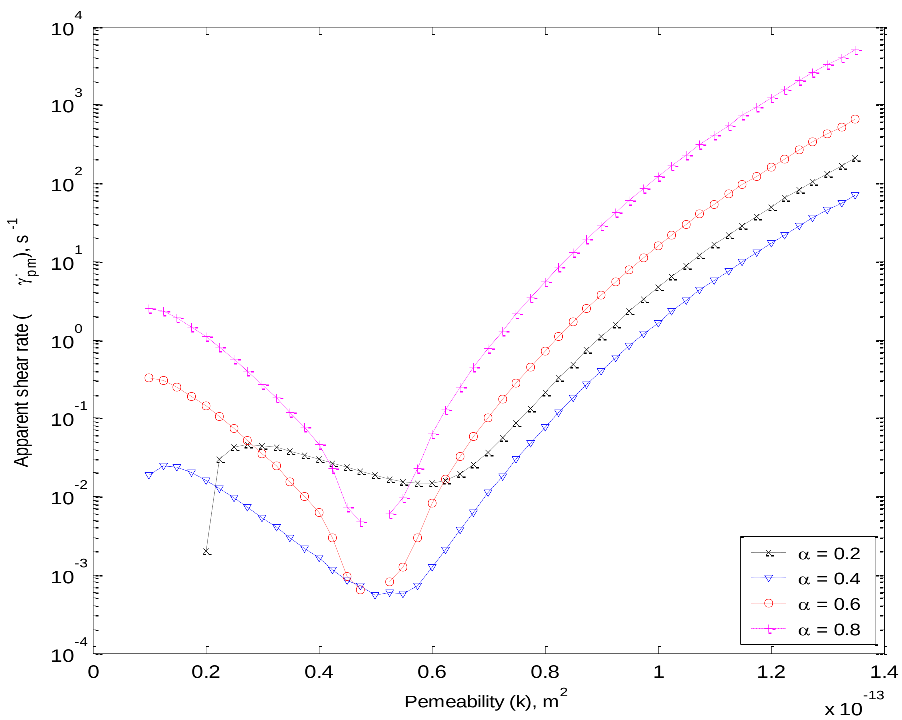

FIGURE 1: Variation of apparent shear rate with permeability of the porous media for $\alpha = 0.2 \text{ to } 0.8$

5.1.3 Shear rate dependency on shape factor for different porosity

Figure 3 shows the variation of apparent shear rate with shape factor, α_{SF} for different porosity of the formation at a α value of 0.2 in a semi log plotting. The present plotting trend of the cure shows a non-linear type however this is a complete linear plotting in a Cartesian graph paper. For a tight reservoir (low porosity), shear rate is much higher comparing with medium or highly porous media for the same shape factor. Shear rate increases with the increase of shape factor for all porosity.

5.1.4 Shear rate dependency on flow velocity for different α values

To investigate the effects of flow rate on shear rate with memory, porosity and permeability are considered as 30% and 70 mD respectively. Flow velocity, u is varied in the range of 1.0×10^{-5} to $13.5 \times 10^{-5}\ m/s$.

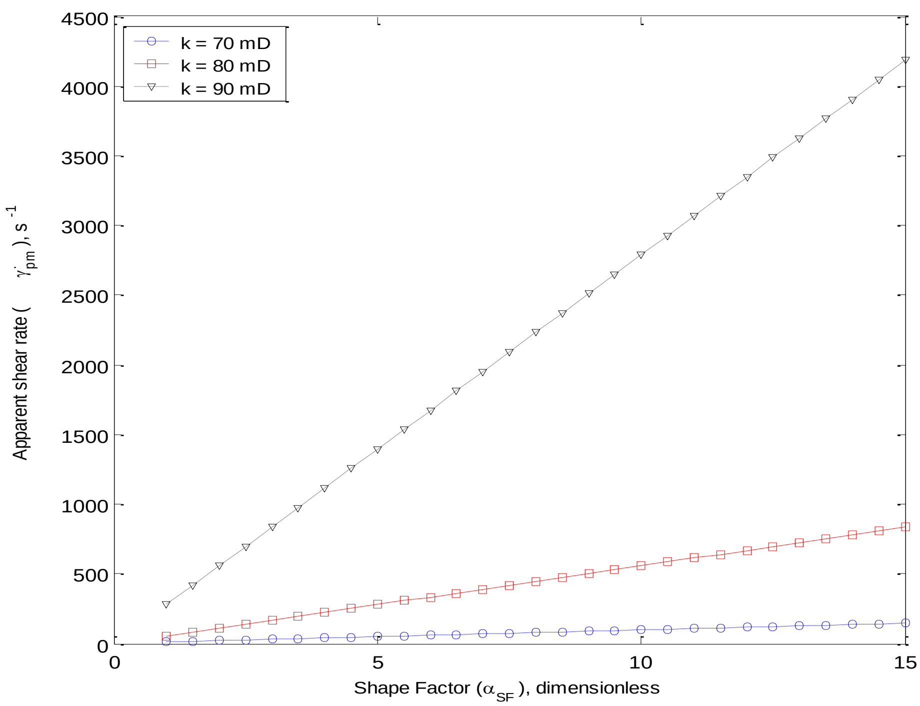

FIGURE 2: Variation of apparent shear rate with shape factor for $k = 20\ mD$ and $60\ mD$

Figures 4-7 present the variation of in situ shear rate with flow velocity for different α values. Shear rate decreases with the increase of flow rate at low velocity range. There is slide faster decrease in the range of velocity increment of 1.5×10^{-5} to $2.0 \times 10^{-5}\ m/s$. After this velocity range, shear rate increases faster with the increase of flow velocity which is an asymptotic variation. This trend continues until the velocity of $5.5 \times 10^{-5}\ m/s$ and again it starts to reduce after this velocity. The relationship between in situ shear rate and flow rate is nonlinear trend whereas Perrin et al. [29] showed that the average in situ shear rate varied linearly with the flow velocity. This non-linearity is only due to the memory effects on fluid flow behavior. Moreover, the shape and trend of the cures are similar for all the α values except the magnitude of the shear rate values which also leads to the existence of fluid memory on shear-thinning fluid. Here, shear rate increases with the increase of α values.

Figure 8 presents the conventional in situ shear rate model (Eq. (1)) available in the literature. The nonlinear trend of the $\dot{\gamma}_{pm}$ curves (Figs. 4-7) are due to the memory dependency of flowing fluid which did not captured by the Perrin et al. [29] and other researchers. So, it can be concluded that fluid memory has an influence on the shear thinning fluid and microscopic rheological behavior of the fluid may be characterized by this memory effect.

5.2 Comparison of proposed model with the conventional model based on flow velocity

Figure 9 presents a semi log plotting for the variation of in situ shear rate with flow velocity for different α values to compare the proposed model and the conventional model. Here porosity and permeability are considered as 30% and 70 mD respectively for both the model. It is clear that memory has a potential influence on the shear thinning fluid which may be characterized by the microscopic rheological property of the media and fluid. If we consider the memory effect, $\dot{\gamma}_{pm}$ increases with the increase of α value. This indicates that at a very low α value, the memory effect is not very significant at higher flow rate.

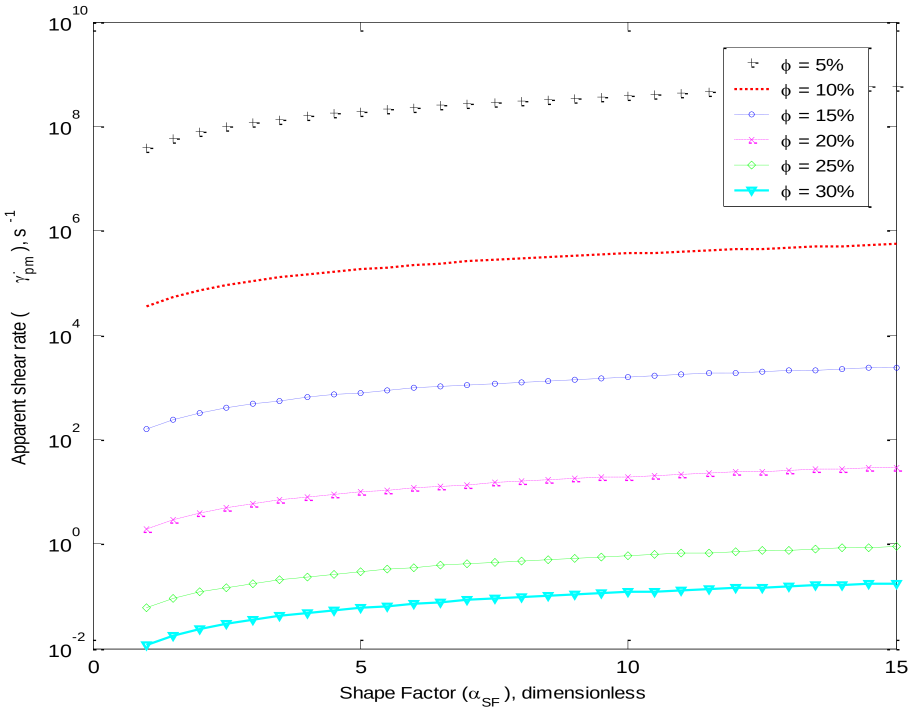

FIGURE 3: Variation of apparent shear rate with shape factor for $\varphi = 20\%\ and\ 40\%$

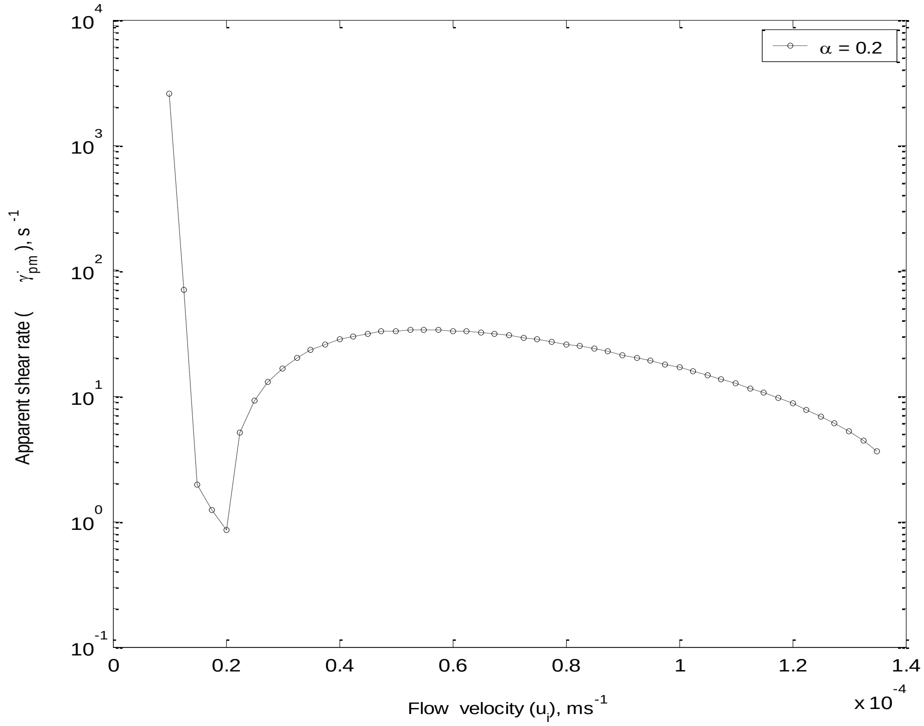

FIGURE 4: Variation of apparent shear rate with flow velocity for $\alpha = 0.2$

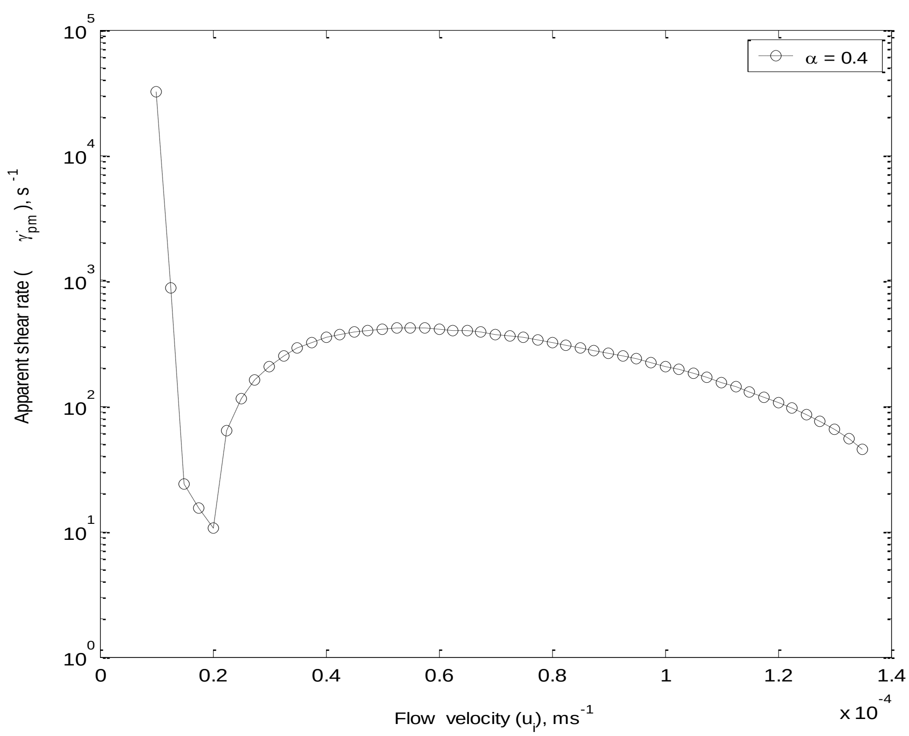

FIGURE 5: Variation of apparent shear rate with flow velocity for $\alpha = 0.4$

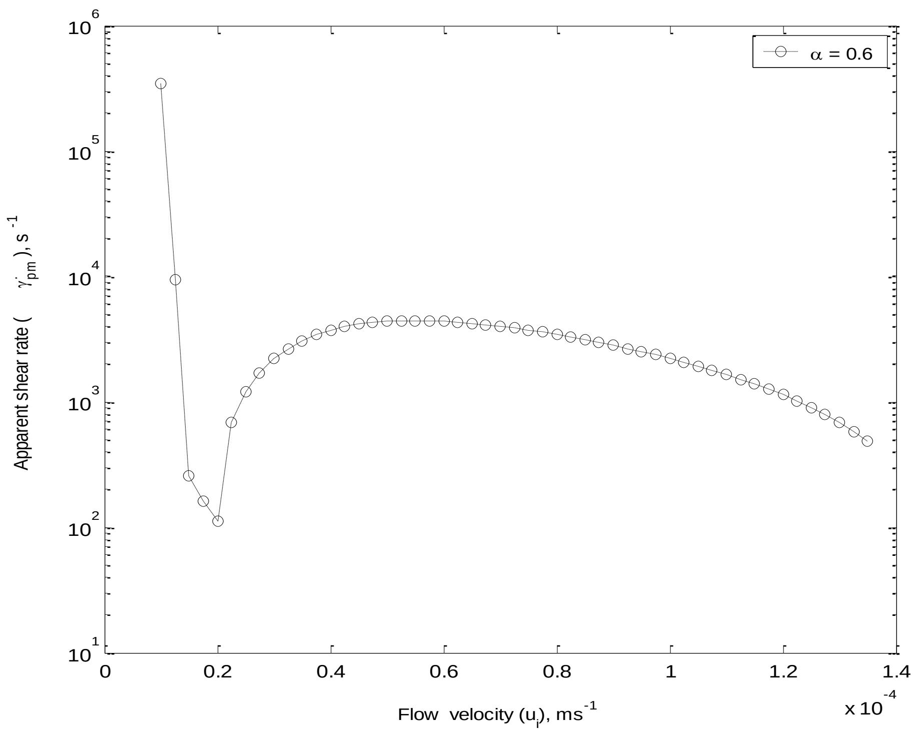

FIGURE 6: Variation of apparent shear rate with flow velocity for $\alpha = 0.6$

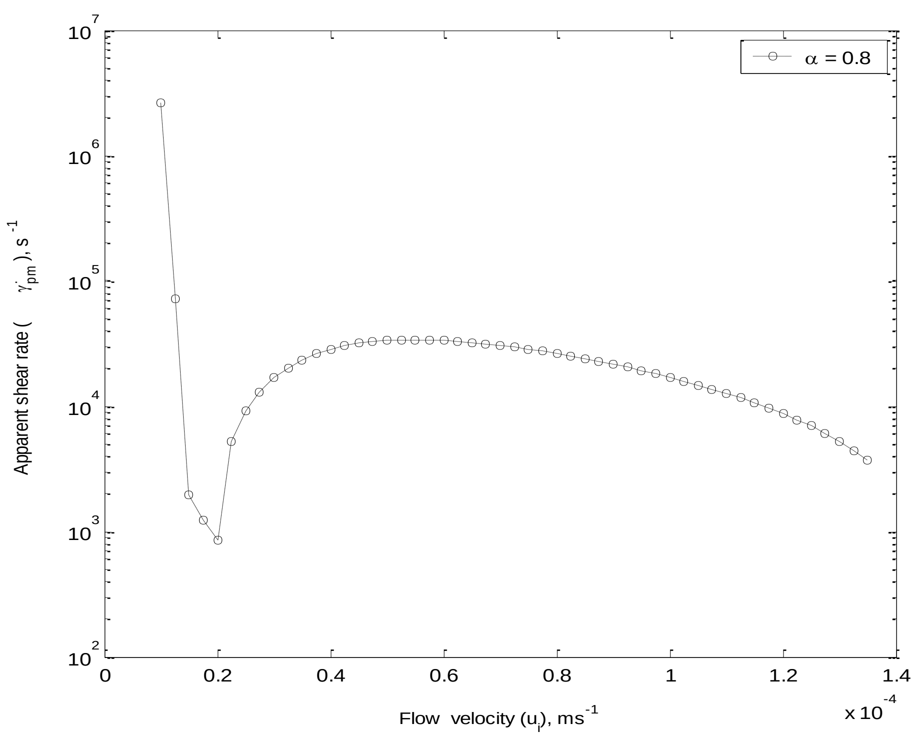

FIGURE 7: Variation of apparent shear rate with flow velocity for $\alpha = 0.8$

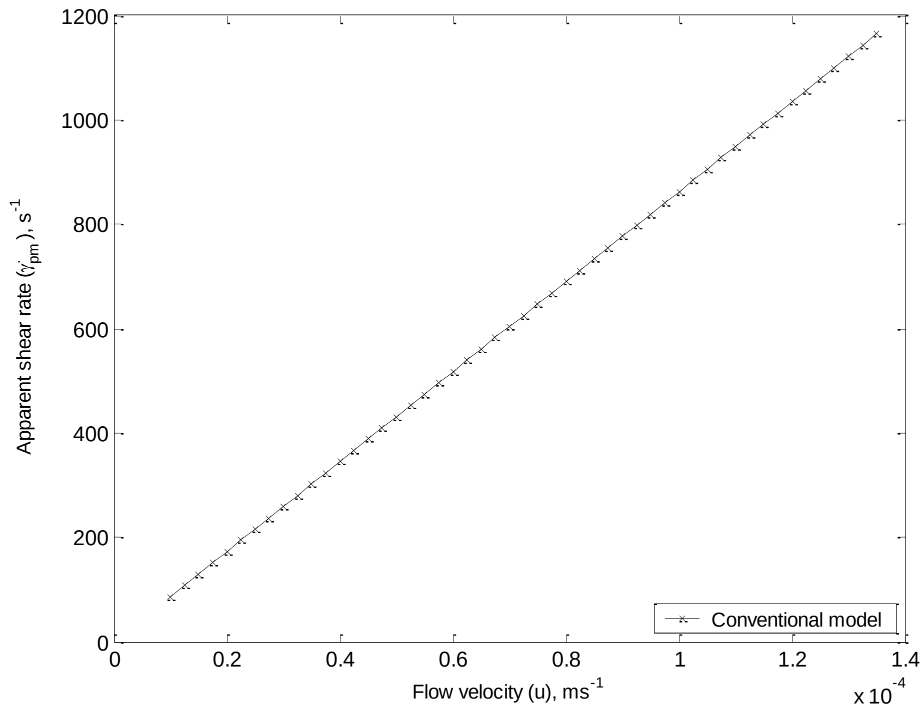

FIGURE 8: Variation of shear rate with flow velocity for the model presented in Eq. (1).

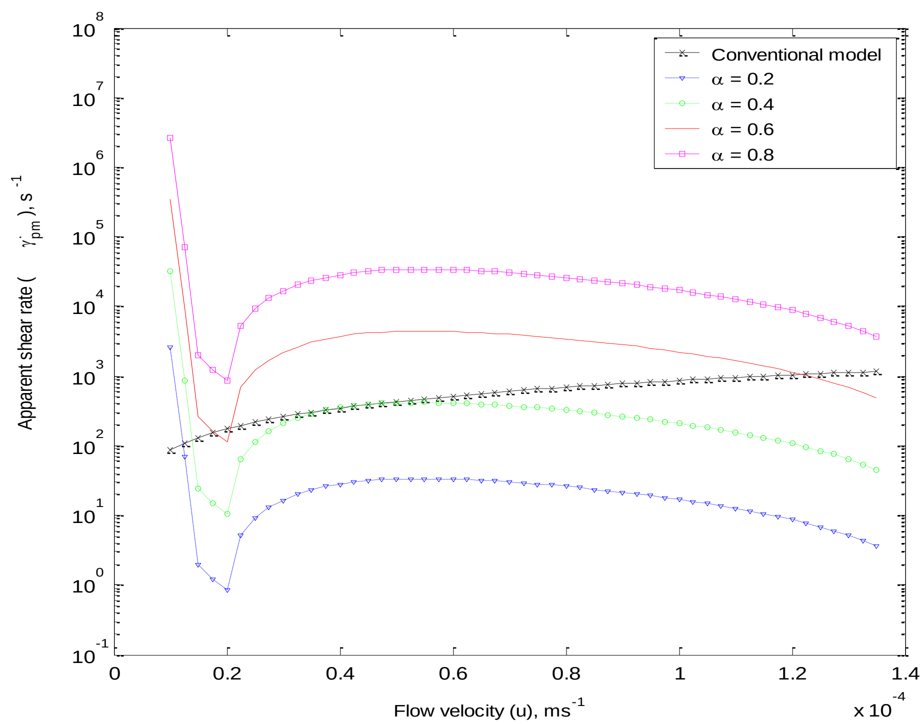

FIGURE 9: Variation of apparent shear rate with flow velocity for comparing the proposed model (Eq. (6)) and the model presented in (Eq. (1)).

5.3 Effective viscosity dependency on shear rate for different α values

Figures 10 – 13 present separately the variation of effective viscosity with shear rate in a log-log plotting for different α values. The trend and shape of the curves generated by the proposed model (Eq. (8)) are almost same except range of data towards upper-lower level and side by side in the direction of viscosity and shear rate respectively. It is interesting to note that the original shape and trend of the Carreau-Yasuda model (Eq. (2)) [20, 27, 36] is similar to that of the proposed modified model except the range and shifting of the data variation. This shifting may be characterized by adjusting the apparent and effective rheological properties or by "scaled up" as stated by Savins [43-44]. The data range and shifting range are different for different α values (Figs. 10 – 13). As α increases, the range of data expands in both for viscosity and shear rate direction. This simply means that the zero shear region, transition region, power law region and infinite shear region as stated by Lopez [27, 35] for a viscosity-shear rate cure are more visible as α increases. The ranges of these different regions are dependent fluid types used in polymer flooding. Therefore it may be concluded that the viscosity-shear rate curve regions are dependent on fluid memory and the regions are more dominant at higher fluid memory e.g. higher α value. The cures show that the viscosity variation is low at low shear rate and it turns to reduce with the increase of shear rate. As α increases, viscosity reduction turns to reduce with the increase of shear rate. This is the behavior of a viscoelastic fluid which is captured by fluid memory. Therefore, it may be concluded that the shear-thinning fluid has the memory when it tries to start move in porous media.

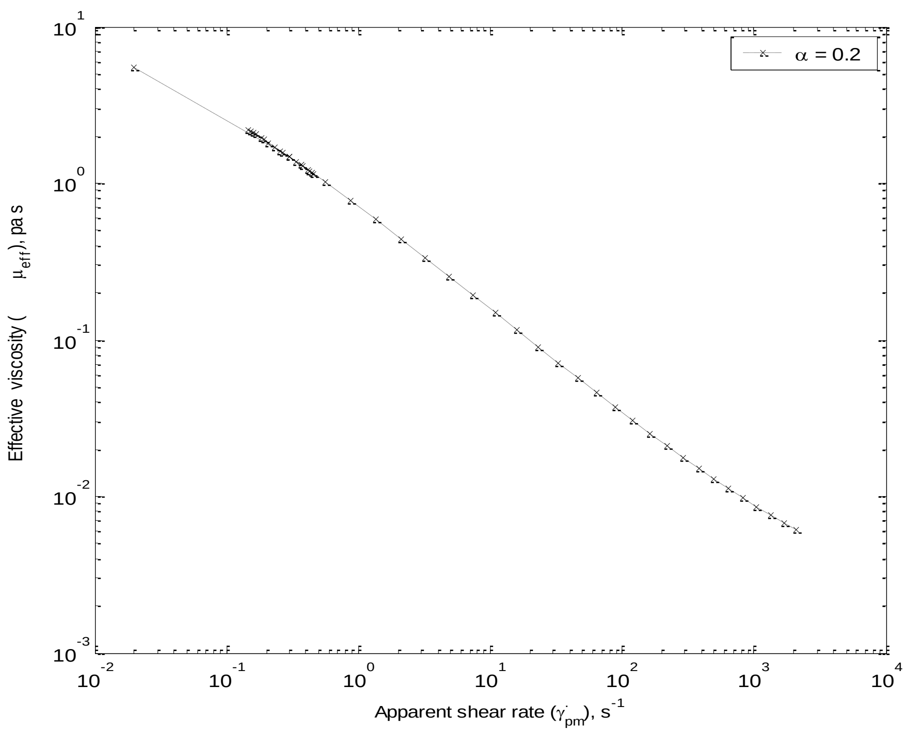

FIGURE 10: Variation of Effective viscosity with apparent shear rate for $\alpha = 0.2$

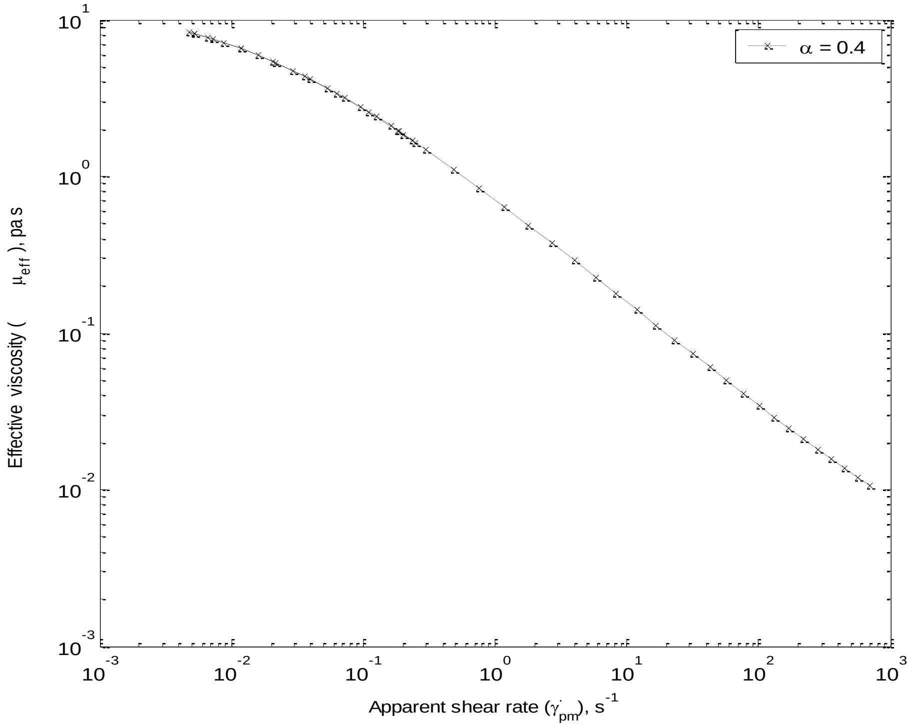

FIGURE 11: Variation of Effective viscosity with apparent shear rate for $\alpha = 0.4$

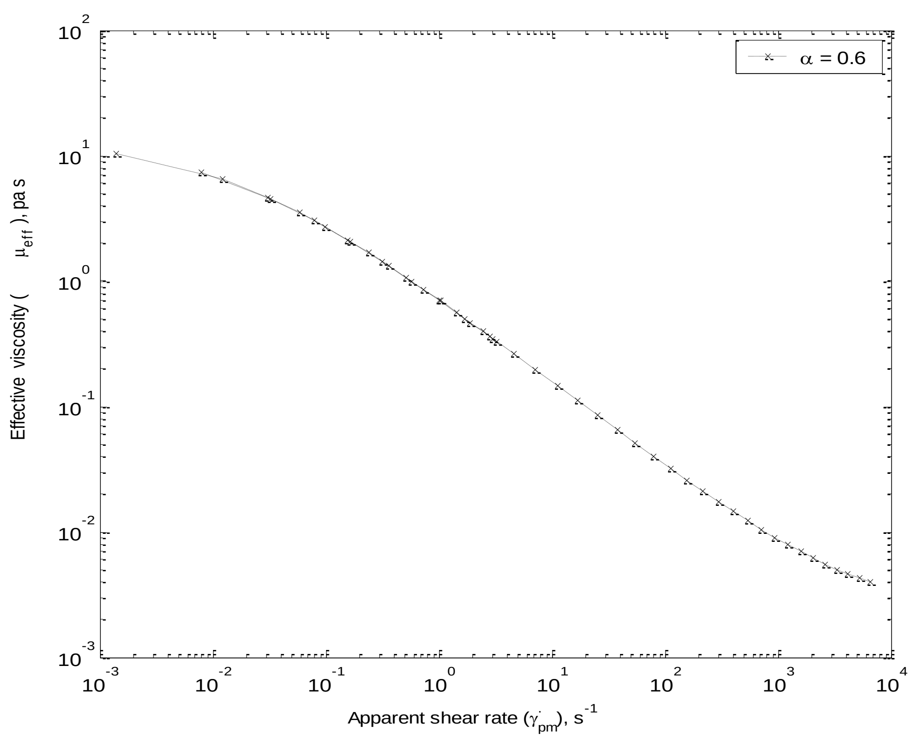

FIGURE 12: Variation of Effective viscosity with apparent shear rate for $\alpha = 0.6$

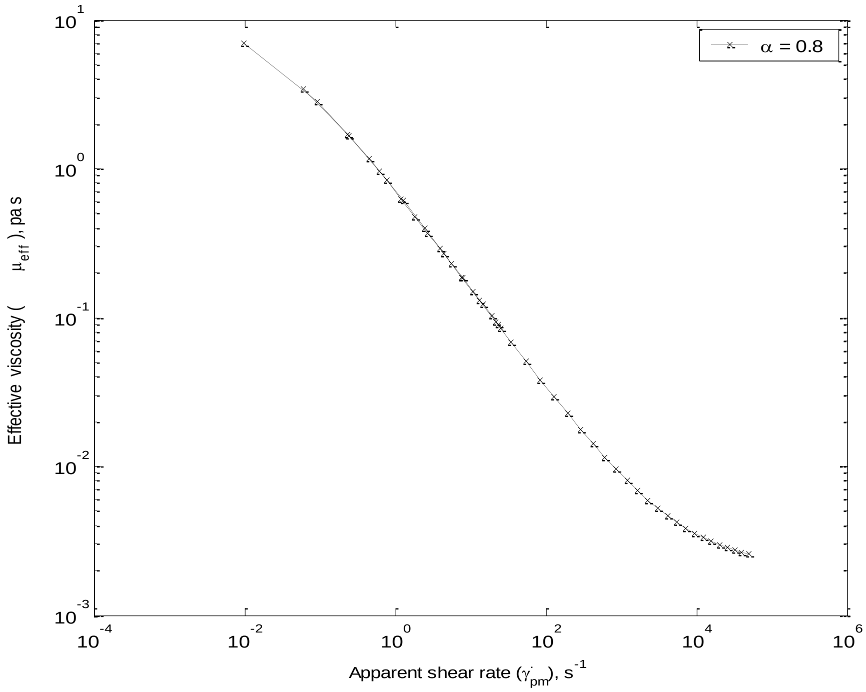

FIGURE 13: Variation of Effective viscosity with apparent shear rate for $\alpha = 0.8$

Figure 14 shows the variation of effective viscosity verses shear rate for different α values in a log-log plotting. In this figure, it is clearer how the fluid memory plays a role to viscosity-shear rate relation. All data generated for different α values are overlapped. However the ranges of data vary with the increase of α value which is already explained earlier.

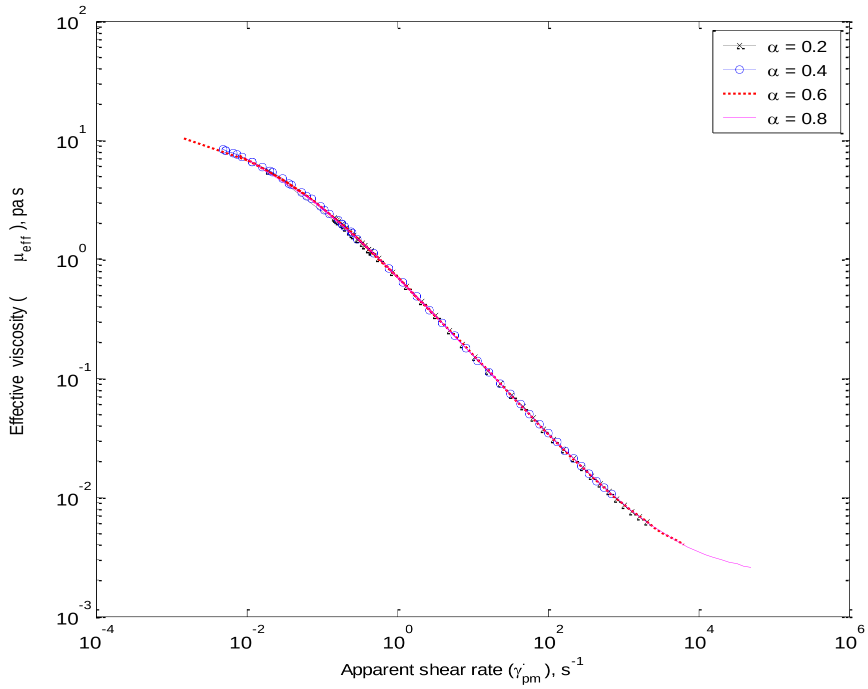

FIGURE 14: Variation of Effective viscosity with apparent shear rate for $\alpha = 0.2\ to\ 0.8$

5.4 Comparison of proposed viscosity model with Carreau-Yasuda model

Figure 15 shows the variation of viscosity verses shear rate of the proposed model (Eq. (8)) for different α values to compare the Carreau-Yasuda model (Eq. (2)) in a log-log plotting. All the data generated by solving these two models are overlapped with each other except the range of data variation. For the same conditions and input data, the proposed model gives the more information than Carreau-Yasuda model. The proposed model provides a wider range of data in both zero shear and infinite shear region. The existence of Carreau-Yasuda model is only in power-law region if we compare it with proposed model. It is also noted that all the α values data lie in the transition and power-law region which are very difficult to capture and explain. If α increases, the data range extend to reach the other two regions, zero shear and infinite shear. Therefore it may be concluded that the proposed model is more appealing and represent able in defining the rheological properties of the shear-thinning fluid flow in porous media.

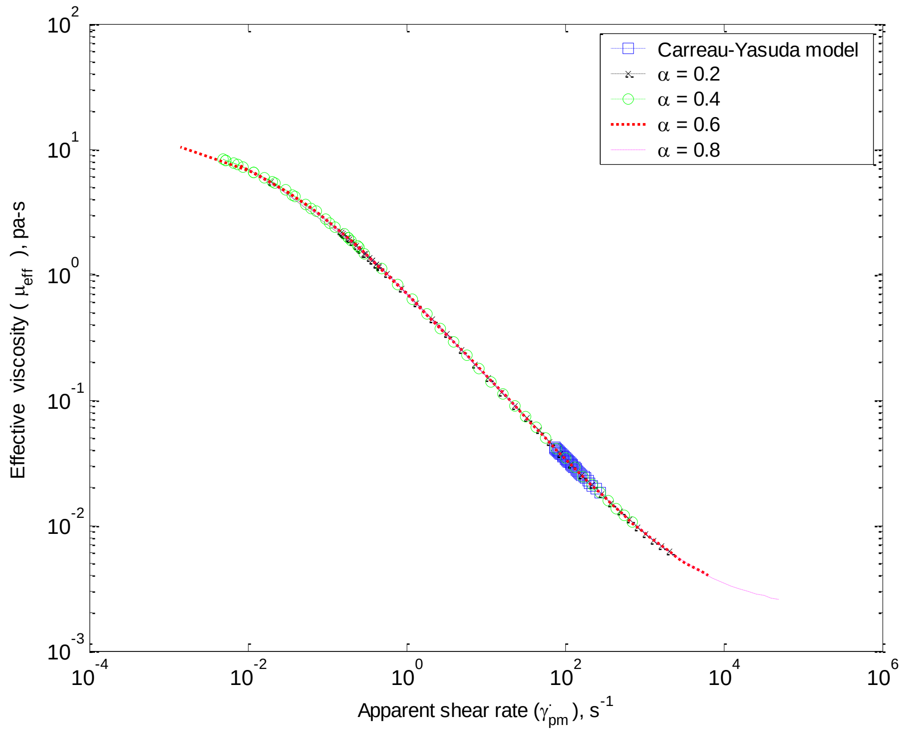

FIGURE 15: Comparison of proposed viscosity model with Carreau-Yasuda model.

6. CONSLUSION

In this study, two models are proposed to characterize the rheological behavior with memory for shear-thinning fluids. These models are validated with the available experimental data and also compared with currently used conventional models. The proposed models are effective in capturing physical phenomena. In addition, they are coherent with existing numerical models. However, the proposed models capture a wider range of information, covering fluids that would not be tractable with existing models. The shear rate-flow velocity has a non-linear variation, which poses the intriguing question as to why conventional linear relationships do not hold. The answer lies within considerations of memory effects. In this paper, we focused on the dependence of the shear rate on porosity, permeability, shape factor and flow velocity which is related to the effect of fluid memory. Also considered is the viscosity dependency on shear rate. This study concludes that fluid memory has a strong influence on shear-thinning fluid flow behavior during the propagation of a shear-thinning fluid in porous media (such as polymer flooding in the EOR process).

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8. NOMENCLATURE

a = parameter in Carreau–Yasuda model, dimensionless

A = cross sectional area of rock perpendicular to the flow of flowing fluid, m^2

k = initial reservoir permeability, m^2

L = length of a capillary or a core, m

n = power-law exponent for Carreau–Yasuda model, dimensionless
p = pressure of the system, N/m^2
Δp = differential pressure along a core or capillary of length L, N/m^2
Q = Au = initial volumetric flow rate, m^3/s
q_x = fluid mass flow rate per unit area in *x*-direction, $kg/m^2\ s$
t = time, s
u = Darcy velocity $(= Q/A)$, m/s
u_x = fluid velocity in porous media in the direction of x axis, m/s
α = fractional order of differentiation, dimensionless
α_{SF} = shape factor which is medium-dependent
$\dot{\gamma}_{pm}$ = apparent shear rate within the porous medium, $1/s$
ρ_0 = density at a reference pressure p_0, kg/m^3
φ = porosity of fluid media, m^3/m^3
μ = fluid dynamic viscosity, $pa\ s$
μ_0 = fluid dynamic viscosity at zero shear rate, $pa\ s$
μ_∞ = fluid dynamic viscosity at infinite shear rate, $pa\ s$
λ = time constant in Carreau–Yasuda model, s
η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3\ s^{1+\alpha}/kg$
ξ = a dummy variable for time i.e. real part in the plane of the integral, s

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