A Comprehensive Material Balance Equation with the Inclusion of Memory During Rock-Fluid Deformation

M.E. Hossain *Dalhousie University, Canada*

M.R. Islam *Dalhousie University, Canada*

Abstract

With the advent of fast computational techniques, it is time to include all salient features of the material balance equation (MBE). The inclusion of time dependent porosity and permeability can enhance the quality of oil recovery predictions to a great extent. This alteration occurs due to change of pressure and temperature of the reservoir which causes a continuous change of rock-fluid properties with time. However, few studies report such alterations and their consequences. This study investigates the effects of permeability, pore volume, and porosity with time during the production of oil. Moreover, a comprehensive MBE is presented. The equation contains a stress-strain formulation that is applicable to both rock and fluid. In addition, this formulation includes memory effect of fluids in terms of a continuous time function. Similar time dependence is also invoked to the rock strain-stress relationship. This formulation results in a highly non-linear MBE, with a number of coefficients that are inherently non-linear (mainly due to continuous dependence on time). This enhances the applicability of the model to fractured formations with dynamic features. Such formulation is different from previous approach that added a transient term to a steady state equation. For selected cases, the MBE is solved numerically with a newly developed non-linear solver. A comparative study is presented using field data. Results are compared with conventional MBE approach. This comparison highlights the improvement in the recovery factor (RF) as much as 5%. This new version of MBE is applicable in the cases where rock/fluid compressibilities are available as a function of pressure from laboratory measurements or correlations. It is also applicable where non-pay zones are active as connate water and solid rock expansions are strong enough with pressure depletion. Finally, suitability of the new formulation is shown for a wide range of applications in petroleum reservoir engineering.

1 Introduction

The MBE is the most fundamental equation that is used predicting petroleum reservoir performance. However, it is well known that the material balance equation that is actually solved is not a comprehensive one. The commonly used form of MBE has a number of assumptions that are not always justified. In the past, such assumptions were necessary due to limitations in computational techniques. Now a day, a high power modern computing technique is minimising the challenges of a very high accuracy and efficiency in complex calculations in reservoir simulation. Therefore, it is not necessary to have approximate solutions in reservoir engineering formulations. The MBE, one of the most widely used techniques in reservoir engineering, is an excellent examples of this. MBE is used for estimating the original hydrocarbon-in-place. It is also used for calculating the decline in average reservoir pressure with depletion. In the majority of cases, the conventional formulation of the material balance is satisfactory. However, certain circumstances, which are sometimes unpredictable, demand formulations with greater accuracy. Proper understanding of reservoir behaviour and predicting future performance are necessary to have knowledge of the driving mechanisms that control the behaviour of fluids within reservoirs. The overall performance of oil reservoir is mainly determined by the nature of energy (i.e. driving mechanism) available for moving oil toward the wellbore. There are basically six driving mechanisms that provide the natural energy necessary for oil recovery. These mechanisms are rock and liquid expansion drive, depletion drive, gas cap drive, water drive, gravity drainage drive and combination drive [Dake, 1978; Ahmed, 2002].

A lot of research works have been conducted for the last 50 years (Havlena and Odeh, 1963; Havlena and Odeh, 1964; Ramagost and Farshad, 1981; Fetkovich et al., 1991; Fetkovich et al., 1998; Rahman et al., 2006a). All these former researchers use the expansion drive mechanism in developing MBE for gas reservoir. In this study, time dependent rock/fluid properties (i.e. expressed in terms of memory function), expansion of oil, water, rock, and dissolved gas in oil and water are incorporated. In addition, total water associated with the oil reservoir volume is taken care. "Associated" water comprises connate water, water within interbedded shales and nonpay reservoir rock, and any limited aquifer volume [Fetkovich et al., 1998]. This study is an attempt to investigate the effects of expansion drive mechanism in a typical oil field using the PVT data available in the literature. Two new, rigorous MBE for oil flow in a compressible formation with residual fluid saturations are presented. One MBE is numerically solved. A new dimensionless parameter, C_{epm} in MBE is identified to represent the whole expansion drive mechanism. It is also explained how this parameter can predict the behaviour of MBE. All water and rock volumes associated with the reservoir and available for expansion, including a limited aquifer volume with formation fluid and rock volume expansion are added in this dimensionless parameter.

2 New comprehensive MBE model developments

2.1 New MBE for a compressible undersaturated oil reservoir

To develop a MBE for an undersaturated reservoir with no gascap gas, the reservoir pore is considered as an idealised container. MBE is derived by considering the whole reservoir as a homogeneous tank of uniform rock and fluid properties. We are considering oil and water as the only mobile phases in the compressible rock and the residual fluid saturation ($c_s \neq 0$, $c_o \neq 0$, $c_w \neq 0$, $S_{oi} \neq 0$, $S_{gi} \neq 0$, $S_{wi} \neq 0$). The derivation includes pressure-dependent rock and water compressibilities (with gas evolving solution). The inclusions of a limited aquifer volume, all water and rock volumes associated with the reservoir available for expansion are recognised. The volumetric balance expressions can be derived to account for all volumetric changes which occur during the natural productive life of the reservoir. Therefore, MBE can be presented in terms of volume changes in reservoir barrels (rb) where all the probable fluids and media changes are taken care as

Pore volume occupied by the oil initially in place + originally dissolved gas at time, t = 0 and p_i, rb

=

Pore volume occupied by the remaining oil at a given time, t and at p, rb

+

Change in oil volume due to oil expansion at a given time t and at p, $(-\Delta V_o)$, rb

+

Change in water volume due to connate water expansion at a given time t and at p, $(-\Delta V_w)$, rb

+

Change in dissolved gas volume due to gas expansion at a given time, t and at p, $(-\Delta V_g)$, rb

+

Change in pore volume due to reduction a given time, t and at p, (ΔV_s), rb

+

Change in associated volume due to expansion and reduction of water and pore volume at a given time, t and at p, (ΔV_A), rb

+

Change in water volume due to water influx and water production at a given time, t and at p , rb

(1)

Where:

ΔV_A = change in associated volume due to expansion and reduction of water and pore volume at a given time, t and at p, (ΔV_A), rb

ΔV_g = change in gas volume due to oil expansion at a given time t and at p, $V_{gi} - V_g$, rb

ΔV_o = change in oil volume due to oil expansion at a given time t and at p, $V_{oi} - V_o$, rb

ΔV_s = change in pore volume due to rock contraction at a given time t and at p, $V_{si} - V_s$, rb

ΔV_w = change in water volume due to oil expansion at a given time t and at p, $V_{wi} - V_w$, rb

V_g = gas volume at a reduced pressure p, rb

V_o = oil volume at a reduced pressure p, rb

V_s = solid rock pore volume at a reduced pressure p, rb

V_w = water volume at a reduced pressure p, rb

V_{gi} = gas volume at initial pressure p_i, rb

V_{oi} = oil volume at initial pressure p_i, rb

V_{si} = solid rock pore volume at initial pressure p_i, rb

V_{wi} = water volume at initial pressure p_i, rb

Now

a) Pore volume occupied by the initial oil in place and originally dissolved gas = NB_{oi}
b) Pore volume occupied by the remaining oil at p = $(N - N_p)B_o$

Where

B_o = oil formation volume factor at a given time t and a reduced pressure p, rb/stb

B_{oi} = oil formation volume factor at initial pressure p_i, rb/stb

N = initial oil in place (e.g. initial volume of oil in reservoir), =$[V\phi(1 - S_{wi})/B_{oi}]$, stb

N_P = cumulative oil production at a given time t and a reduced pressure, p, stb

V = total reservoir volume, ft^3

ϕ = variable rock porosity with space and time, volume fraction

S_{wi} = water saturation at initial pressure p_i

The isothermal fluid and formation compressibilities are defined according to the above discussion as [Dake, 1978; Ahmed, 2000; Rahman et al., 2006a]:

Oil:

$$c_o = -\frac{1}{V_o}\frac{\partial V_o}{\partial p}\bigg]_T \tag{2}$$

Water:

$$c_w = -\frac{1}{V_w}\frac{\partial V_w}{\partial p}\bigg]_T \tag{3}$$

Gas:

$$c_g = -\frac{1}{V_g}\frac{\partial V_g}{\partial p}\Big]_T \tag{4}$$

Solid rock formation:

$$c_s = \frac{1}{V_s}\frac{\partial V_s}{\partial p}\Big]_T \tag{5}$$

If p_i is the initial reservoir pressure and p is the average reservoir pressure at current time t, one can write down the expressions for ΔV_o, ΔV_w, ΔV_g, and ΔV_s by integrating Equation (2) through Equation (5), assuming compressibilities to be pressure dependent, and by subsequent algebraic manipulations as (Rahman et al., 2006a):

$$\Delta V_o = -V_{oi}\left(1 - e^{\int_p^{p_i} c_o\, dp}\right) \tag{6}$$

$$\Delta V_w = -V_{wi}\left(1 - e^{\int_p^{p_i} c_w\, dp}\right) \tag{7}$$

$$\Delta V_g = -V_{gi}\left(1 - e^{\int_p^{p_i} c_g\, dp}\right) \tag{8}$$

$$\Delta V_s = V_{si}\left(1 - e^{-\int_p^{p_i} c_s\, dp}\right) \tag{9}$$

Where,

c_g = reservoir gas compressibility at a reduced pressure p, psi^{-1}

c_o = reservoir oil compressibility at a reduced pressure p, psi^{-1}

c_s = reservoir rock formation compressibility at a reduced pressure p, psi^{-1}

c_w = reservoir water compressibility at a reduced pressure p, psi^{-1}

p = current reservoir pressure (at time t), $psia$

p_i = initial reservoir pressure, $psia$

In addition to above, associated volume is also included in developing a MBE with expansion drive mechanism. The "associated" volume is an additional reservoir part which is not active in oil/gas production. However, this part of the reservoir may accelerate the oil recovery by its water and rock expansion-contraction. Fetkovich et al. (Fetkovich et al., 1991; Fetkovich et al., 1998) defined this term as "the nonnet pay part of reservoir where interbedded shales and poor quality rock is assumed to be 100% water-saturated". The interbedded nonnet pay volume and limited aquifer volumes are referred to as "associated" water volumes and both contribute to water influx during depletion. They used a volume fraction, M to represent the associated volume effects in the conventional MBE (Dake, 1978; Ahmed, 2000; Craft and Hawkins, 1959; Havlena and Odeh, 1963; Havlena and Odeh, 1964) for gas reservoir. M is defined as the ratio of associated pore volume to reservoir pore volume. If we consider this associated volume change, this part will be contributing as an additional expansion term in MBE. Therefore, the volume change would be:

$$\Delta V_A = -(-\Delta V_{Aw}) + \Delta V_{As} = V_{Awi}\left(e^{\int_p^{p_i} c_w\, dp} - 1\right) + V_{Asi}\left(1 - e^{-\int_p^{p_i} c_s\, dp}\right) \tag{10}$$

Where

ΔV_{As} = change in associated volume due to rock expansion at a given time t and at p, $V_{Asi} - V_{As}$, rb

ΔV_{Aw} = change in associated volume due to water expansion at a given time t and at p, $V_{Awi} - V_{Aw}$, rb

V_{Asi} = associated solid rock pore volume at initial pressure p_i, rb

V_{Awi} = associated water volume at initial pressure p_i, rb

Note that ΔV_o, ΔV_w, and ΔV_g have negative values due to expansion, and ΔV_s has a positive value due to contraction. The initial fluid and pore volumes can be expressed as

$$V_{oi} = \frac{NB_{oi}}{1-S_{wi}} S_{oi} \tag{11}$$

$$V_{wi} = \frac{NB_{oi}}{1-S_{wi}} S_{wi} \tag{12}$$

$$V_{gi} = \frac{(NB_{oi})\times(R_{soi}/B_{oi})\times B_{gi} + (NB_{oi})\times(R_{swi}/B_{wi})\times B_{gi}}{1-S_{wi}} S_{gi} \tag{13}$$

$$V_{si} = \frac{NB_{oi}}{1-S_{wi}} \tag{14}$$

$$V_{Awi} = M\frac{NB_{oi}}{1-S_{wi}} \text{ (as 100\% water saturated e.g., } S_w = 1.0) \tag{15}$$

$$V_{Asi} = M\frac{NB_{oi}}{1-S_{wi}} \tag{16}$$

Where

B_{wi} = water formation volume factor at initial pressure p_i, rb/stb

B_{gi} = gas formation volume factor at initial pressure p_i, rb/scf

S_{wi} = water saturation at initial pressure p_i

S_{gi} = gas saturation at initial pressure p_i

R_{si} or R_{soi} = gas solubility at initial reservoir pressure e.g. initial solution gas-oil ratio, scf/stb

R_{swi} = initial solution gas-water ratio, scf/stb

Substituting Equations (11) through (16) in Equations (6) to (10) respectively, one can write down the equations as:

$$\Delta V_o = -V_{oi}\left(1 - e^{\int_p^{p_i} c_o\,dp}\right) = -\frac{NB_{oi}}{1-S_{wi}} S_{oi}\left(1 - e^{\int_p^{p_i} c_o\,dp}\right) \tag{17}$$

$$\Delta V_w = -V_{wi}\left(1 - e^{\int_p^{p_i} c_w\,dp}\right) = -\frac{NB_{oi}}{1-S_{wi}} S_{wi}\left(1 - e^{\int_p^{p_i} c_w\,dp}\right) \tag{18}$$

$$\Delta V_g = -V_{gi}\left(1 - e^{\int_p^{p_i} c_g\,dp}\right) = -\frac{(NB_{oi})\times(R_{soi}/B_{oi})\times B_{gi} + (NB_{oi})\times(R_{swi}/B_{wi})\times B_{gi}}{1-S_{wi}} S_{gi}\left(1 - e^{\int_p^{p_i} c_g\,dp}\right) \tag{19}$$

$$\Delta V_s = V_{si}\left(1 - e^{-\int_p^{p_i} c_s\,dp}\right) = \frac{NB_{oi}}{1-S_{wi}}\left(1 - e^{-\int_p^{p_i} c_s\,dp}\right) \tag{20}$$

$$\Delta V_A = \Delta V_{Aw} + \Delta V_{As} = M\frac{NB_{oi}}{1-S_{wi}}\left(e^{\int_p^{p_i} c_w\,dp} - 1\right) + M\frac{NB_{oi}}{1-S_{wi}}\left(1 - e^{-\int_p^{p_i} c_s\,dp}\right) \tag{21}$$

Substituting Equation (17) through (21) in Equation (1), one can write down the MBE as:

$$N_p B_o - \left(W_e - W_p B_w\right) = N\left(B_o - B_{oi} + B_{oi}\, C_{epm}\right) \tag{22}$$

where

$$C_{epm} = \left\{ \frac{S_{oi}\left(e^{\int_p^{p_i} c_o\, dp} - 1\right) + S_{wi}\left(e^{\int_p^{p_i} c_w\, dp} - 1\right) + S_{gi}\left(e^{\int_p^{p_i} c_g\, dp} - 1\right)\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right) B_{gi} + \left(1 - e^{-\int_p^{p_i} c_s\, dp}\right) + M\left[\left(e^{\int_p^{p_i} c_w\, dp} - 1\right) + \left(1 - e^{-\int_p^{p_i} c_s\, dp}\right)\right]}{1 - S_{wi}} \right\} \tag{23}$$

Here

B_w = water formation volume factor at a given time t and a reduced pressure $p,\ rb/stb$

W_p = cumulative water production at $p,\ stb$

W_e = cumulative water influx into reservoir at $p,\ rb$

C_{epm} = parameter of effective compressibility due to residual fluid, dissolved gas and formation for the proposed MBE, dimensionless

Equation (22) is the new, rigorous and comprehensive MBE for an undersaturated reservoir with no gascap gas and above the bubble-point pressure. The above MBE is very rigorous because it considers the fluid and formation compressibilities as any functions of pressure. It is deeming oil and water as the only mobile phases in the compressible rock. This rigorous MBE is applicable for a water-drive system with a history of water production in an undersaturated reservoir. Here, associated volume ratio, the residual and dissolved phase saturations are also considered. The Equation (23) is an expression of the proposed dimensionless parameter, C_{epm} where all the probable and available expansions are illustrated. When the water influx term is not significant [*i.e.*, $W_e = 0$], Equation (22) may be modified to:

$$N_p B_o + W_p B_w = N\left(B_o - B_{oi} + B_{oi}\, C_{epm}\right) \tag{24}$$

Equation (24) can be written in the form of straight line's MBE as (Havlena and Odeh, 1963; Havlena and Odeh, 1964):

$$N = \frac{F}{E_0 + E_{cepm}} \tag{25}$$

where

$$F = N_p B_o + W_p B_w$$

$$E_0 = B_o - B_{oi}$$

$$E_{cepm} = B_{oi}\, C_{epm}$$

Now, if we consider a constant data of oil, water, gas and formation compressibilities (*e.g.*, compressibilities are not functions of pressure), Equation (22) remains unchanged. However, Equation (23) can be modified by integrating the power of exponents as:

$$C_{epm} = \left\{ \frac{\begin{array}{c} S_{oi}\left(e^{c_o(p_i-p)}-1\right)+S_{wi}\left(e^{c_w(p_i-p)}-1\right)+S_{gi}\left(e^{c_g(p_i-p)}-1\right)\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+\left(1-e^{-c_s(p_i-p)}\right) \\ +M\left[\left(e^{c_w(p_i-p)}-1\right)+\left(1-e^{-c_s(p_i-p)}\right)\right] \end{array}}{1-S_{wi}} \right\} \tag{26}$$

Equation (26) is still a rigorous expression for use in the case of constant compressibilities. This equation can be further approximated by the exponential terms for small values of the exponents as:

$$e^{c_o(p_i-p)} \approx 1 + c_o(p_i - p) \tag{27}$$

$$e^{c_w(p_i-p)} \approx 1 + c_w(p_i - p) \tag{28}$$

$$e^{c_g(p_i-p)} \approx 1 + c_g(p_i - p) \tag{29}$$

$$e^{-c_s(p_i-p)} \approx 1 - c_s(p_i - p) \tag{30}$$

Substituting Equations (27) through (30) in Equation (26) and considering the average reservoir pressure, it becomes:

$$C_{epm} = \frac{S_{oi}\,c_o + S_{wi}\,c_w + S_{gi}\,c_g\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + c_s + M\,(c_w + c_s)}{1-S_{wi}}\Delta p \tag{31}$$

Where:

$\bar{p}$ = volume average reservoir pressure at p, psi

Δp = average pressure drop in the reservoir at p, = $p_i - \bar{p}$, psi

The dimensionless parameter, C_{epm} expressed in terms of fluids (oil, water and gas) and formation compressibilities, and saturation is same to that of conventional MBE if we neglect the expansion of dissolved gas in oil and water, and the associated volume expansion. This issue will be discussed later.

2.2 Conventional MBE

The general form of conventional MBE can be written as [Dake, 1978; Ahmed, 2000; Craft and Hawkins, 1959; Havlena and Odeh, 1963; Havlena and Odeh, 1964]:

$$N_pB_0 + W_pB_w = N(B_o - B_{oi} + B_{oi}C_{eHO}) \tag{32}$$

where

$$C_{eHO} = \frac{S_{oi}\,c_o + S_{wi}\,c_w + c_s}{1-S_{wi}}\Delta p \tag{33}$$

C_{eHO} = parameter of effective compressibility due to residual water and formation for the Havlena and Odeh [1963] MBE, dimensionless

Equation (24) resembles the MBE (Equation (32)) of Havlena and Odeh (1963, 1964) for a volumetric and undersaturated reservoir with no gascap except the pattern of dimensionless parameter, C_{epm} and Δp. Equation (32) can further be written as in the straight line form of Havlena and Odeh MBE as:

$$N = \frac{F}{E_0 + E_{ceHO}} \tag{34}$$

where

$F = N_p B_o + W_p B_w$

$E_0 = B_o - B_{oi}$

$E_{ceHO} = B_{oi}\, C_{eHO}$

Now, if we neglect dissolved gas saturation and associated volume expansion in Equation (31) and define the pressure at time t as the average pressure, the equation becomes as:

$$C_{epm} = \frac{S_{oi}\, c_o + S_{wi}\, c_w\ + c_s}{1 - S_{wi}} \Delta p \tag{35}$$

The right hand side of above equation is same as stated in Equation (33).

2.3 A comprehensive MBE with memory for cumulative oil recovery

Equation (31) can be written using variable pressure expression as:

$$C'_{epm} = \frac{\left[S_{oi}c_o + S_{wi}c_w + S_{gi}c_g\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + c_s + M(c_w + c_s)\right] \times [p_i - p_{(t)}]}{1 - S_{wi}} \tag{36}$$

In Equation (36), the average pressure decline for a particular time t from the start of production may be calculated using the time-dependent rock/fluid properties with stress-strain model. The mathematical explanation and the derivation of the stress-strain formulation are described in Hossain et al. (2007a). They gave the stress-strain relationship as follows:

$$\tau_T = (-1)^{0.5} \times \left(\frac{\partial\sigma}{\partial T}\frac{\Delta T}{\alpha_D M_a}\right) \times \left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial\xi\,\partial x}\right)d\xi}{\Gamma(1-\alpha)}\right]^{0.5} \times \left(\frac{6\, K\, \mu_0\, \eta}{\frac{\partial p}{\partial x}}\right)^{0.5} \times e^{\left(\frac{E}{R\, T_T}\right)} \frac{du_x}{dy} \tag{37}$$

Where:

E	= activation energy for viscous flow, Btu/mol
K	= operational parameter
M_a	= Marangoni number
R	= universal gas constant, $Btu/mole - °F$
t	= time, s
u	= filtration velocity in x direction, ft/s

u_x	= fluid velocity in porous media in the direction of *x* axis, *m/s*
T_T	= temperature of the reservoir at time, t, °F
T_o	= initial reservoir temperature at time, $t = 0$, °F
$\frac{du_x}{dy}$	= velocity gradient along y-direction, $1/s$
$\left\lvert\frac{d\sigma}{dT}\right\rvert$	= the derivative of surface tension σ with temperature and can be positive or negative depending on the substance, $lb_f/ft -$ °F
ΔT	= $T_T - T_0 =$ temperature difference, °F
β	= coefficient of thermal expansion, $1/$°F
σ	= surface tension, lb_f/ft
α	= fractional order of differentiation, dimensionless
α_D	= thermal diffusivity, ft^2/s
ρ_f	= density of fluid, lb_m/ft^3
ρ_r	= a density of fluid, lb_m/ft^3
μ	= dynamic viscosity of reservoir fluid at temperature T, cp
μ_0	= fluid dynamic viscosity at initial reservoir temperature T_0, *pa-s*
τ_T	= shear stress at temperature *T*, *Pa*
ξ	= a dummy variable for time i.e. real part in the plane of the integral, *s*
η	= ratio of the pseudopermeability of the medium with memory to fluid viscosity, $ft^3 s^{1+\alpha}/lb_m$

The above equation reduces to (Hossain, 2008):

$$p_i - p_{(t)} = -\frac{6\,K\,\mu_0\,\eta\left(\frac{\Delta T}{\alpha_D M_a}\frac{\partial\sigma}{\partial T}\right)^2 \times \left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial\xi\,\partial x}\right)d\xi}{\Gamma(1-\alpha)}\right] \times e^{2\left(\frac{E}{R\,T_T}\right)} \times \left(\frac{du_x}{dy}\right)^2}{\tau_T^2}\,u_x\Delta t \tag{38}$$

The change of pressure with time and space can be calculated using the stress-strain Equation (38). This change of pressure is directly related to oil production performance of a well. Therefore, substituting Equation (38) into Equation (36):

$$C'_{epm} = \frac{\left[S_{oi}c_o + S_{wi}c_w + S_{gi}c_g\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + c_s + M(c_w + c_s)\right] \times \left[-\frac{6\,K\,\mu_0\,\eta\,(L-x)\left(\frac{\partial\sigma}{\partial T}\frac{\Delta T}{\alpha_D M_a}\right)^2\left[\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial\xi\,\partial x}\right)d\xi\right] e^{2\left(\frac{E}{R\,T_T}\right)}\left(\frac{du_x}{dy}\right)^2}{\tau_T^2\,\{\Gamma(1-\alpha)\}}u_x\,\Delta t\right]}{1 - S_{wi}} \tag{39}$$

where C'_{epm} is the modified dimensionless parameter which depends on rock/fluid memory and other related fluid and rock properties and Δt is the time difference between the start of production and a particular time which is actually time, t.

The C'_{epm} of Equation (39) can be used in Equation (22) to represent the time dependent rock/fluid properties and other properties which are related to the formation fluid and formation itself. Therefore, Equation (22) can be written as (Hossain, 2008):

$$N_p = \phi A_3 + \frac{1}{B_o}\left(W_e - W_p B_w\right) \tag{40}$$

where

$$N = \frac{V \phi (1 - S_{wi})}{B_{oi}}$$

$$A_3 = \frac{V (1 - S_{wi})}{B_{oi} B_o}\left(B_o - B_{oi} + B_{oi} C'_{epm}\right)$$

The Equation (40) represents a new rigorous MBA with memory where the every possibility of time dependent rock/fluid properties is considered.

3 Significance of C_{epm}

The dimensionless parameter, C_{epm}, in the below Equation (41) can be considered as the effective strength of the energy source for oil production in expansion drive oil recovery. This is only due to the compressible residual fluids (oil, water and dissolved gas) and rock expansions of the reservoir. This value does not account for the oil compressibility as stated by other researchers [Dake, 1978; Fetkovich et al., 1991; Fetkovich et al., 1998; Ahmed, 2000; Rahman et al., 2006b]. If we see the expression of C_{epm} as presented in Equation (23), it is a function of the current reservoir pressure, fluid compressibilities, initial saturations, dissolved gas properties engaged in water and oil, and associated volume fraction. C_{epm} in Equation (26) is still a function all those parameters except a set of constant compressibilities instead of variable compressibilities. The final simplified form of C_{epm} presented in Equation (31) is dependent on current average reservoir pressure drop and other related parameters as stated above. The dimensionless parameter, C_{epm} is an important parameter in the proposed MBE (Equation (22)) because it can be used as an analytic tool to predict how the MBE will behave for the relevant input data. An elaborate discussion is stated in results and discussion.

Some specific significant properties may be well explained to elucidate the new version of C_{epm}, presented in Equations (23) and (26). To explain these significance of C_{epm}, Equation (22) can be rearranged as:

$$\frac{1}{B_o}\left(1 - C_{epm} - \frac{W_e - W_p B_w}{N B_{oi}}\right) = \frac{1}{B_{oi}}\left(1 - \frac{N_p}{N}\right) \tag{41}$$

The Equation (41) is used to explain the effects of C_{epm}, where two cases have been considered:

3.1 Water drive mechanism with water production

The Equation (41) gives a limit of expansion plus water drives mechanism for initial fluid situations, water influx and water production. This limit may be expressed as:

$$0 \le \left(C_{epm} - \frac{W_e - W_p B_w}{N B_{oi}}\right) < 1.0 \tag{42}$$

The Equation (42) is true for any given average reservoir pressure. The lower limit in Equation (42) is due to the fact of C_{epm}, W_e and W_p where all these parameters zero at the initial reservoir pressure. The upper limit is characterised by the fact that the right-hand side term in Equation (41) is zero, when all the original oil-in-place has been produced. However, practically it is not possible to reach the production level up to that marks. Therefore, the upper limit should be less than 1. Hence, it may be concluded that if the numerical values beyond these range comes out, there might be some problem in input data or there might be a problem in calculating or assigning the average reservoir pressure. So, it is a tool to diagnose or predict the reservoir behaviour in the early stage of production.

3.2 Depletion drive mechanism with no water production

When depletion drive mechanism (with no water influx) with no water production is considered, Equation (41) gives a limit for this drive mechanism as:

$$0 \leq C_{epm} < 1.0 \tag{43}$$

The limits are identified using the same argument as the water drive mechanism, presented in the previous section. If the cited limits are violated at a given time, there is no chance of calculating any reasonable values of the average reservoir pressure. Therefore, the limits are the indications of decision tool about the reservoir and fluid properties and decline criteria.

4 Numerical simulation

The numerical results of the dimensionless parameters, C_{epm} and C_{eHO} based on the models presented by Equations (23), (26), (31), and (33) can be obtained by solving these equations. A volumetric undersaturated reservoir with no gascap gas is considered for the simulation. The reservoir initial pressure is $p_i = 4000\ psi$.

Table 1 presents the rock and fluid properties that have been used in solving the above mentioned equations. Trapezoidal method is used to solve the exponential integral. All computation is carried out by Matlab 6.5. To calculate IOIP using Havlena and Odeh (1963, 1964) straight line method, the Virginia Hills Beaverhill Lake field [Ahmed, 2002] data and an additional data of Table 1 are considered. The initial reservoir pressure is 3685 psi. The bubble-point pressure was calculated as 1500 psi. Table 2 shows the field production and PVT data.

Table 1 Reservoir rock and fluid properties for simulation

Rock and fluid properties [Hall, 1953; Dake, 1978; Ahmed, 2000]	
B_{gi} = 0.00087 rb/scf	c_w = 3.62 x 10^{-6} psi^{-1}
B_{oi} = 1.2417 rb/stb	R_{soi} = 510.0 scf/stb
B_{wi} = 1.0 rb/stb	R_{swi} = 67.5 scf/stb
c_g = 500.0 x 10^{-6} psi^{-1}	S_{gi} = 20%
c_o = 15.0 x 10^{-6} psi^{-1}	S_{oi} = 60%
c_s = 4.95 x 10^{-6} psi^{-1}	S_{wi} = 20%

5 Results and discussion

5.1 Effects of compressibilities on dimensionless parameters

Figures 1(a)–(d) present the variation of dimensionless parameters with average reservoir pressures when associated volume fraction does not take care. The figures give a general idea of how the fluid and formation compressibilities play a role on MBE when pressure varies. The plotting of Figure 1(a) is based on Equation (23) where variable compressibilities of the fluids and formation are taken care. The trend of the curve is a non-linear exponential type. When reservoir pressure starts to deplete, C_{epm} increases and it reaches its highest value at $p = 0$. Figure 1(b) is plotted based on Equation (26) where constant compressibilities of the fluids and formation are considered. The trend of the curve is still in the form of a non-linear exponential type. The numerical data and shape of the curve is almost same as Figure 1(a). Figure 1(c) is plotted using the Equation (31) where constant compressibilities and an approximation of the exponential terms are considered. A straight line curve produces where the numerical values are less than that of Figures 1(a) and 1(b). The dimensionless parameter from the use of conventional MBE (Equation 33) shows a straight line curve where the numerical values are much less than that of the previous presentation (Figure 1(d)).

Table 2 The field production and PVT data (Example 11-3: of Ahmed, 2002)

Volumetric average pressure psi	No. of producing wells	B_o rb/stb	N_p mstb	W_p mstb
3685	1	1.3102	0	0
3680	2	1.3104	20.481	0
3676	2	1.3104	34.750	0
3667	3	1.3105	78.557	0
3664	4	1.3105	101.846	0
3640	19	1.3109	215.681	0
3605	25	1.3116	364.613	0
3567	36	1.3122	542.985	0.159
3515	48	1.3128	841.591	0.805
3448	59	1.3130	1273.530	2.579
3360	59	1.3150	1691.887	5.008
3275	61	1.3160	2127.077	6.500
3188	61	1.3170	2575.330	8.000

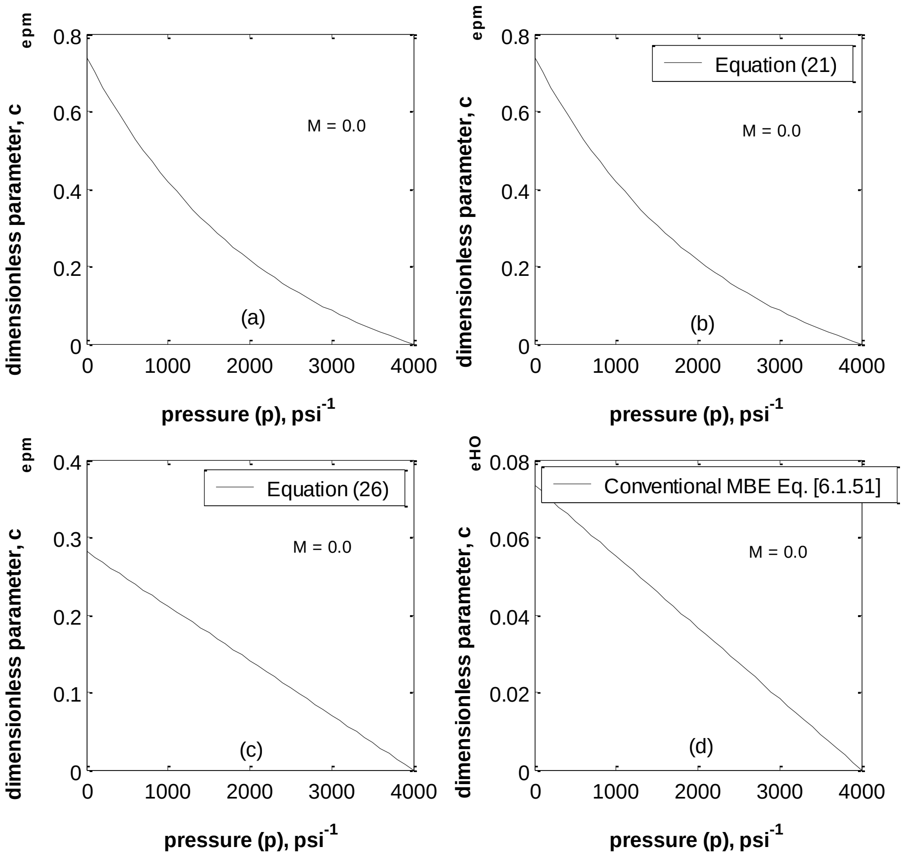

Figure 1 Dimensionless parameter variation with pressure for different equations

5.2 Comparison of dimensionless parameters based on compressibility factor

Figure 2 explains how the values of the dimensionless parameter can vary with reservoir pressure for a given set of compressibility and saturation. It compares between the dimensionless parameter, C_{epm} of the proposed model and the conventional dimensional parameter, C_{eHO}. The curves of the figure have been generated using Equations (23), (26), (31) and (33). The depleted reservoir pressure of $p = 0$ gives the maximum value of C_{epm} or C_{eHO}. The constant or variable compressibility (Equations (23), and (26)) does not make any significant difference in computation up to a certain level of accuracy, which is approximately 10^{-3} %. At low pressures, the magnitude of C_{epm} increases very fast comparing with other two Equations (26) and (28). The pattern and nature of the curves are already explained in Figure 1. The change of dimensionless parameter is low for conventional MBE. However, when the expression of the proposed MBE's parameter is simplified, it turns to reduce the magnitude of the dimensionless parameter. This cleanly means that the simplified version of the proposed model will be the same if it will be further simplified. Therefore, it may be concluded that the use of conventional MBE over estimate the IOIP. The proposed model is closer to reality. This issue will be discussed in later section.

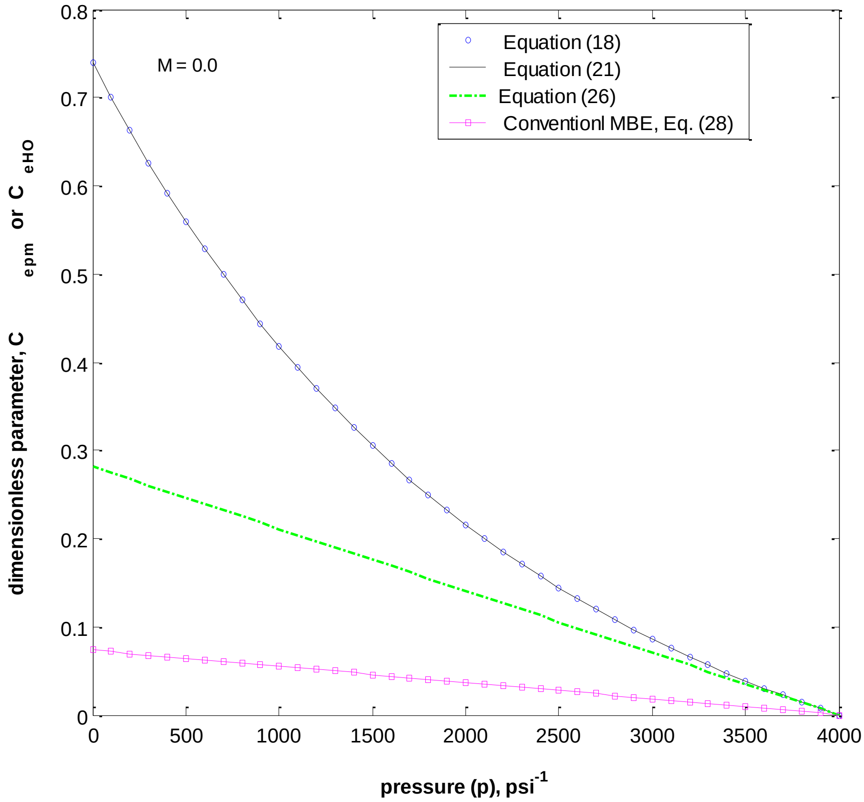

Figure 2 **Comparison of dimensionless parameters variation with pressure for different equations**

5.3 Effects of *M* on dimensionless parameter

If we consider the associated volume in the reservoir, all the available or probable pressure support from rock and water as well as from fluids are being accounted for the proposed MBE with dimensionless parameter. Figure 3 has been generated for a specific reservoir where several M values have been considered. The figure shows the variation of C_{epm} with average reservoir pressure for different M values. Figures 3(a)–(c) present the C_{epm} variation for the proposed Equations (23), (26) and (31) respectively. These curves have specific characteristics depending on the pressure dependence of rock and fluids (water, oil and dissolved gas) compressibilities. These curves have relatively less variant at high pressure, increase gradually as pressure decreases, and finally rise sharply at low pressure especially after 1,000 psi. All the curves in Figure 3 have the same characteristics except the numerical values of the dimensionless parameter, C_{epm}. For every equation, if M increases, the curve shifts upward in the positive direction of C_{epm}. The difference in C_{epm} due to M is more dominant at low pressure. This trend of the curve indicates that the matured reservoir feels more contributions from associated volume of the reservoir.

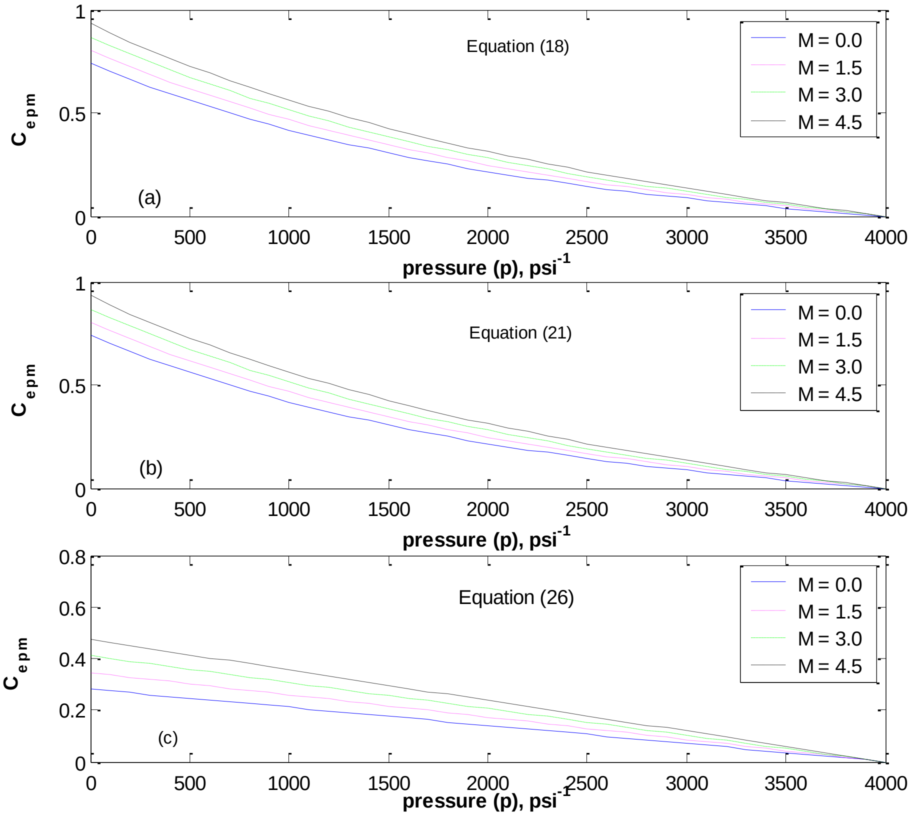

Figure 3 Dimensionless parameter variations with pressure at different M ratios

5.4 Effects of compressibility factor with M values

Figures 4(a)–(c) illustrate C_{epm} verses pressure for different proposed Equations (23), (26), and (31) at several M values of 0.0, 1.5, 3.0, 4.5 respectively. The shape and characteristics of all curves are same as Figure 3. When variable compressibilities are considered (Equation (23)) with pressure, there is a big difference at low pressure with constant compressibilities and exponential approximation Equation (Equation (31)) for all M values. However, there is no significant change in Equation (23) and (26) at different M values. It should be mentioned here that as M increases, C_{epm} increases. This is true for all the equations.

5.5 Comparison of models based on RF

Figure 5 illustrates the underground withdrawal, F verses the expansion term $E_0 + E_{cepm}$ for the proposed MBE (Equation (25)) with Equations (26) and (31) where associated volume ratio is ignored. The conventional MBE (Equation (34)) with Equation (33) is also shown in the same graph. Using best fit curve fitting analysis, these plotting give a straight line passing through the origin with a slop of N. IOIP is identified as 73.41 mmstb, 68.54 mmstb and 175.75 mmstb for the MBE with Equations (26), (31), and (33) respectively. The corresponding recovery factors are calculated as 3.76%, 3.51%, and 1.46%. Therefore, the inclusions of the probable parameters increase the ultimate oil recovery. Here, linear plot indicates that the field is producing under volumetric performance ($W_e = 0$) which is strictly by pressure depletion and fluid and rock expansion.

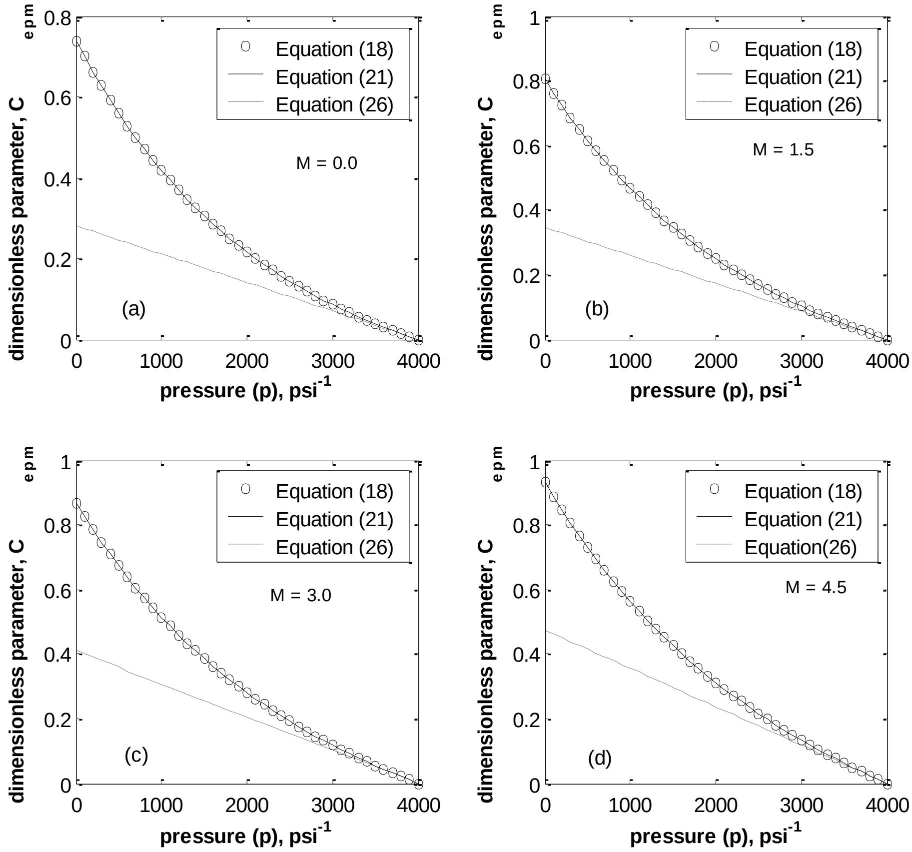

Figure 4 Dimensionless parameters variation with pressure for different equations

Figure 6 shows a plotting of $\left(F/E_0 + E_{cepm}\right)$ verses cumulative production, N_p for the proposed MBE (Equation (24)) with Equations (26) & (31), and $(F/E_0 + E_{ceHO})$ vs. N_p for the conventional MBE (Equation (32)) with Equation (33). In this figure, associated volume ratio is also ignored. The best fit plot for all the equations indicate that the reservoir has been engaged by water influx, abnormal pore compaction or a combination of these two [Dake, 1978; Ahmed, 2000]. In our situation we ignore the water influx ($W_e = 0$). Therefore, we may conclude that the reservoir behaviour is an indication of pore compactions and fluids and rocks expansion.

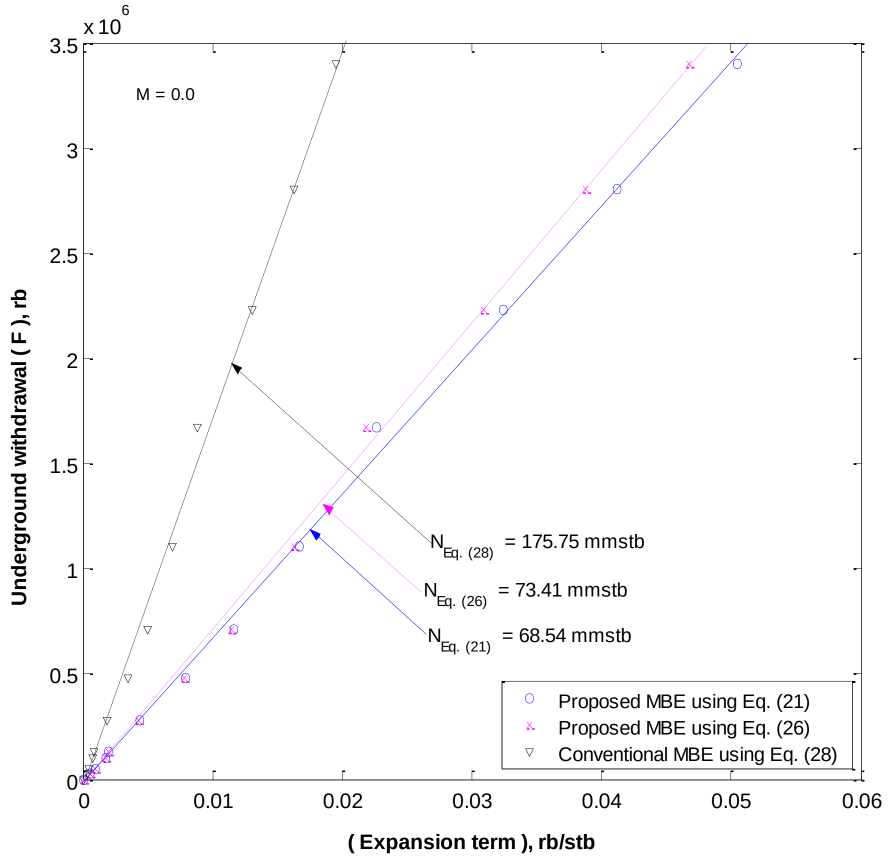

Figure 5 Underground withdrawal vs. Expansion term for *N* calculation

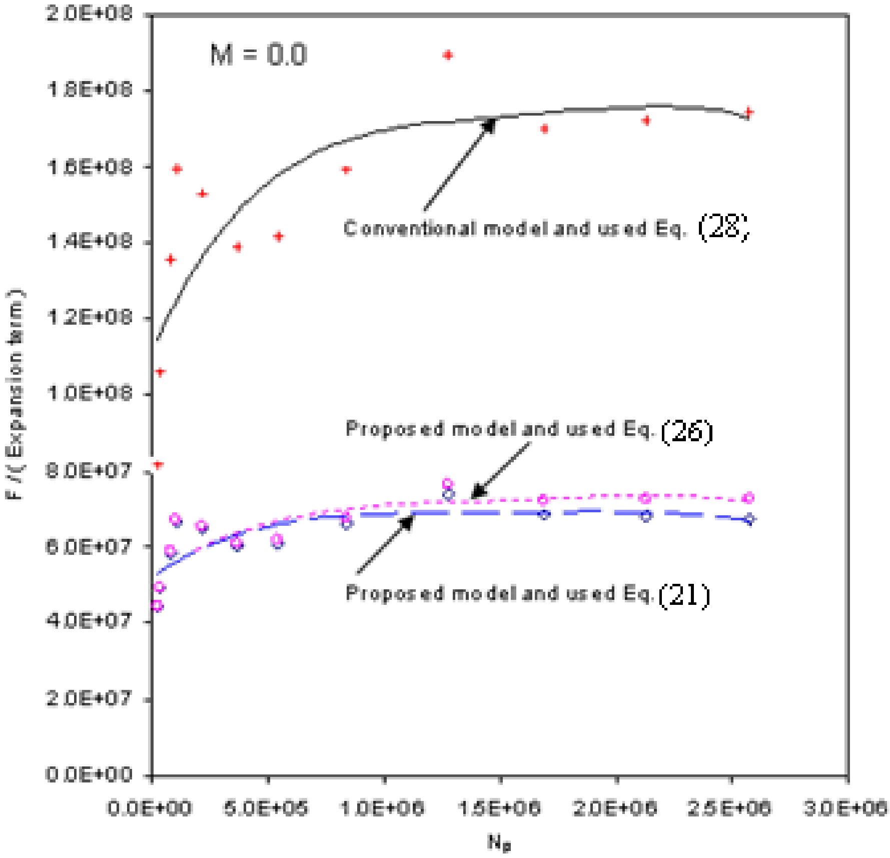

Figure 6 $F/(E_0 + E_{cepm}\ or\ E_{ceHO})$ vs. N_P

5.6 Effects of M on MBE

The recovery factor of the proposed model is higher than that of conventional MBE (Figure 5). Moreover, the comprehensive proposed MBE, Equation (25) with Equation (23) has higher RF than that of using Equation (26) with Equation (25). Therefore, to show the effects of M values, Figure 7 illustrates F vs. $(E_0 + E_{cepm})$ for only the proposed MBE with Equation (26). The straight line plotting passing through the origin of the figure gives 59.78 mmstb, 53.01 mmstb and 47.61 mmstb of IOIP for M = 1.0, 2.0, 3.0 respectively. The corresponding RF values are calculated as 4.31%, 4.86%, and 5.4%. So, RF increases with the increase of M values which correspond that if there is an associated volume of a reservoir, it should be considered in the MBE calculations otherwise there might be some error in getting the true production history of reservoir life.

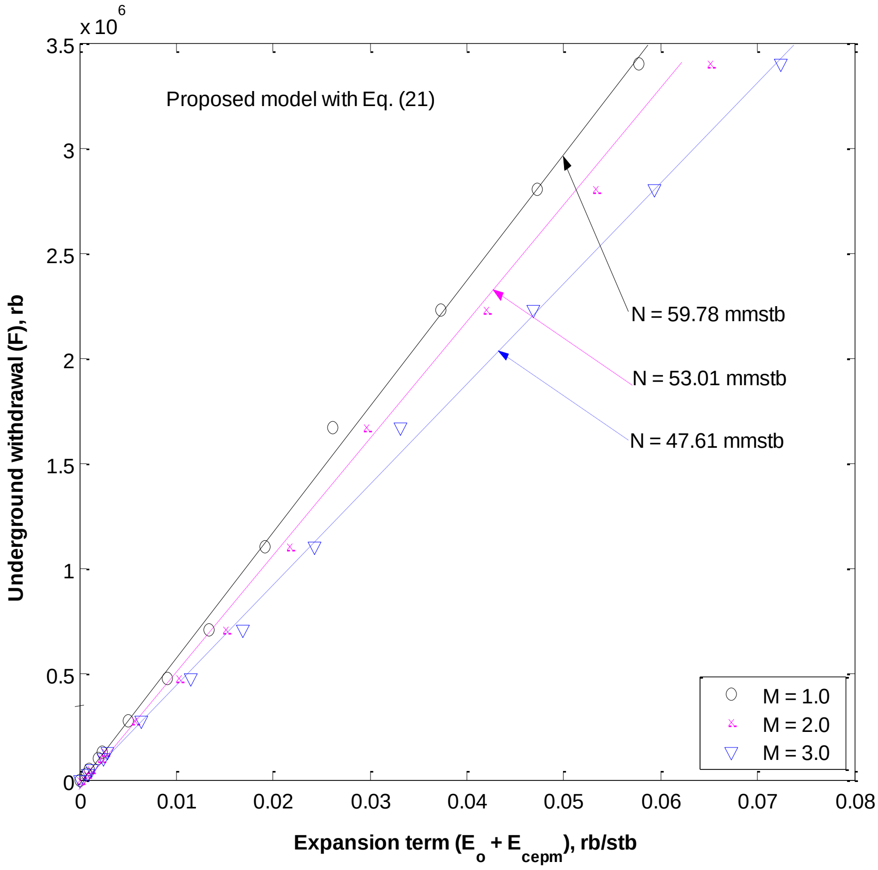

Figure 7 Underground withdrawal vs. Expansion term for different *M* values

6 Conclusions

A new comprehensive material balance equation has been established for an undersaturated oil reservoir with no gascap gas. The proposed MBEs have the core concepts of using variable compressibilities, residual fluid saturations and time dependent rock/fluid properties. The associated volume of the reservoir is also accounted to derive the generalised MBE. The MBE is greatly influenced by the compressibilities of fluids as well as rocks which help to increase the RF values in production history. If there exists an additional reservoir part which is not active in oil production (e.g. M values), RF is also affected by these M values. All these considerations offer the unique features of the proposed MBE with improved RF. Therefore, the inclusions of all probable parameters increase the ultimate oil recovery. A general idea of how the fluid and formation compressibilities play a role on MBE can be known by using this MBE. The available literature support that MBE has a linear relationship. However, the proposed MBE is a non-linear type which is obvious due to non-linear nature of pressure decline with time or distance. The input data can be scanned using the dimensionless parameter, C_{epm} expression.

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