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A Scaled Model Study of Waterjet Drilling

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Abstract: Laboratory experimental results of waterjet drilling have rarely been scaled up to the field scale. This article presents the scaling criteria for designing waterjet drilling laboratory experiments for simulating a given oilfield operation. Dimensional analysis is used to derive scaling groups for the waterjet drilling technique. The proposed scaling approach meets all important requirements of this drilling process. Experiments were conducted to determine the strength and the relation between the rate of penetration (ROP) and depth of penetration (DOP) with drilling time. Experimental results are scaled up for field application. Laboratory measurements with such models accurately duplicate the behavior of the drilling performance of a reservoir. Such modeling is the most effective tool for the study of drilling behavior, performance, and management in the reservoir field.

Keywords: depth of penetration, dimensional analysis, rate of penetration, rock properties, scaling approaches, scaling group

INTRODUCTION

Laboratory experiments are extensively used to investigate various issues in the petroleum industry. Such experiments are representative of the reservoirs as a whole if they are carried out with models that are properly scaled. The scaling concept is very effective and reliable in science and engineering applications. Scaled physical models have the unique advantage of capturing all physical phenomena occurring in a particular process. A scaled model is designed on the basis of the principle of similarity. Such a model is characterized by the same ratios of dimensions, forces, velocities, and temperatures. The performance of reservoirs and drilling is governed by the related variables. These variables can be combined by dimensionless groups. There are two basic available methods in the literature by which the dimensionless groups can be obtained (Geertsma et al., 1956; Loomis and Crowell, 1964; Rojas, 1985; Islam, 1987). The methods used are dimensional analysis and

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inspectional analysis. These methods and their applications in the petroleum industry have been have discussed extensively. Researchers have mainly focused their work on oil displacement and recovery processes (Pujol and Boberg, 1972; Farouq Ali and Redford, 1977; Lozada and Farouq Ali, 1987, 1988; Kimber et al., 1988; Islam and Farouq Ali, 1990, 1992; Bansal and Islam, 1994; Islam et al., 1994; Sundaram and Islam, 1994). However, there is no available literature that deals with the scaling criteria and its applications on petroleum drilling. The objectives of this article are to study the relevant variables in drilling technology for dimensionless groups and to analyze the important groups by introducing a new approach for scaling criteria for proper scaling from model to prototype.

It should be noted that the complete set of scaling criteria is very difficult, if not impossible, to satisfy. Therefore some of the similarity groups must be relaxed in order to satisfy the most important parameters of the specific activity. Scaling of the phenomena considered to be least important to a particular process might be relaxed without significantly affecting the major features of the process. The choice of an approach depends on the importance of the phenomena that are not scaled by that approach. As an example, if one considers such an approach, for which model and prototype have the same morphology, the same fluids, and are operated under the same conditions of pressure and temperature, the scaling groups, such as geometric factors, morphology factors, and ratio of gravitational to viscous forces, are completely satisfied. The criteria used most widely for high-pressure models are outlined by Pujol and Boberg (1972). The high-pressure models typically employ the same fluids in the model as found in the prototype field (Kimber et al., 1988).

RESERVOIR ROCK PROPERTIES

For the last several decades, considerable research has been conducted on the theoretical aspects of rock properties and drilling techniques. The rock properties that influence the coal and rock cutting process have been reported extensively in recent literature. Bilgin et al. (2006) reported a comprehensive study of various rock properties. The rock properties in joints were investigated at a field site on Vancouver Island, British Columbia (Bessingera et al., 2003). They found that different physical properties of rocks have dominant effects on a drilling program. They mainly addressed the effects of joints and fractures on rock properties. Hakala et al. (2006) studied the theory of anisotropy concerning the intact rock moduli via the strain compliance matrix, a description of the core sample testing methods, and interpretation of results for migmatic mica gneiss from two site investigation boreholes. They provided a detailed test procedure and rock property measurements. Kahraman et al. (2003) studied the penetration rates of percussive drills in the field and tested rock both in the field and in the laboratory for different physical

Table 1. Rock properties for different rock types (Kahraman et al., 2003)

Rock type	Density (g/cm^3)	Compressive strength (MPa)	Brazilian tensile strength (MPa)	Point load strength (MPa)
Limestone	2.73	123.8	6.6	5.3
Altered sandstone	2.55	20.1	1.2	1.1
Metasandstone	2.73	25.7	5.8	4.2
Sandstone	3.00	149.2	16.1	11.2
Dolomite	2.92	68.0	6.0	3.5
Diabase	2.96	110.9	10.1	10.3
Marl	2.20	39.5	5.2	2.7

properties of rocks. Following this, the penetration rates were correlated with the rock properties for the development of reliable equations. They concluded that the uniaxial compressive strength, the Brazilian tensile strength, the point load strength, and the Schmidt hammer value are the dominant rock properties affecting the penetration rate of percussive drills. The physical properties of reservoir rock are presented in Table 1.

FACTORS AFFECTING THE DRILLING OPERATION

In drilling operations of a reservoir, the rate of penetration (ROP) is the most important criterion among the variables related to this technology. The factors that affect ROP are numerous and are not completely understood. Some of the more recognizable variables that affect the ROP are well explained by Gatlin (1960). In conventional rotary drilling systems, there are some both human and mechanical factors that can affect mud properties. In waterjet drilling, these factors are not normally so important compared to formation characteristics and drilling fluid (water) properties. Therefore formation characteristics such as compressive strength, hardness, abrasiveness, state of underground stress (i.e., overburden pressure), elasticity (i.e., brittle or plastic), fluid content and interstitial pressure, porosity, permeability, and temperature are considered in developing scaling groups.

METHODS AND DERIVATION OF DIMENSIONLESS GROUPS

Dimensionless Analysis

The theory of dimensional analysis has been described in many books and articles (Bridgman, 1931; Langhaar, 1951; Geertsma et al., 1956; Loomis

Table 2. Relevant variables related with the waterjet drilling technique

No.	Variable	No.	Variable	No.	Variable
1	A	9	ROP	17	q_{w}
2	d	10	t	18	t_{T}
3	D	11	T	19	ϕ_{b}
4	g	12	u	20	ρ_{b}
5	h	13	c_{p}	21	ρ_{w}
6	K	14	c_{pw}	22	μ_{w}
7	k	15	h_{c}	23	σ
8	p	16	k_{w}	24	τ

and Crowell, 1964; Isaacson and Isaacson, 1975; Rojas, 1985; Islam, 1987). Dimensional analysis requires knowledge of the complete set of relevant variables influencing the process. The first step is to identify the relevant variables of the problem and make an arrangement in a set of dimensionless groups using the Buckingham pi theorem. Table 2 shows the relevant variables for waterjet drilling activities. Based on these relevant variables, and using Buckingham pi theorem, the dimensionless groups have been derived and are presented in Table 3.

DESIGN OF A SCALED PROTOTYPE FOR WATERJET DRILLING

Scaled models are important in order to be able to interpret laboratory data for field applications. This statement is also true for prototypes. However,

Table 3. Complete set of dimensionless groups using dimensional analysis for laboratory and field

$\pi_1 = \left[\dfrac{A}{d^2}\right]$	$\pi_6 = \left[\dfrac{t^2Tc_p}{d^2}\right]$	$\pi_{11} = \left[\dfrac{t^3Tk_w}{d^4\rho}\right]$	$\pi_{16} = \left[\dfrac{\rho_w}{\rho}\right]$
$\pi_2 = \left[\dfrac{gt^2}{d}\right]$	$\pi_7 = \left[\dfrac{t^2Tc_{pw}}{d^2}\right]$	$\pi_{12} = \left[\dfrac{q_wt}{d^3}\right]$	$\pi_{17} = \left[\dfrac{\mu_wt}{\rho_wd^2}\right]$
$\pi_3 = \left[\dfrac{t^3Th_c}{d^3\rho}\right]$	$\pi_8 = \left[\dfrac{D}{d}\right]$	$\pi_{13} = \left[\dfrac{ROPt}{d}\right]$	$\pi_{18} = \left[\dfrac{t^2\sigma}{d^2\rho}\right]$
$\pi_4 = \left[\dfrac{t^2p}{\rho d^2}\right]$	$\pi_9 = \left[\dfrac{K}{d^2}\right]$	$\pi_{14} = \left[\dfrac{t_T}{t}\right]$	$\pi_{19} = \left[\dfrac{t^2\tau}{d^2\rho}\right]$
$\pi_5 = \left[\dfrac{ut}{d}\right]$	$\pi_{10} = \left[\dfrac{t^3Tk}{d^4\rho}\right]$	$\pi_{15} = [\phi]$	$\pi_{20} = \left[\dfrac{h}{d}\right]$

to design a prototype based on experimental analysis in the laboratory, the following steps must be followed once a model drilling condition is selected for study. These steps include selection of the model data, model operating conditions, scaling approach, reference quantities, and design of the scaled prototype.

Selection of the Model Data

One of the most important conditions of scaling is that dimensionless properties must be the same functions of dimensionless variables in the model and prototype. Therefore fluid properties as well as other data must be scaled properly. The first step in designing a scaled prototype is selection of the parameters representing the model environment. Table 4 presents the model data for waterjet drilling on a laboratory scale. Some of these data are based on experimental results.

Selection of Model Operating Conditions

Different operating conditions are required depending on the process to be scaled. These include operating pressure, water injection rate (i.e., velocity at the tip of nozzle), temperature of the drilling fluid (water), density of the drilling fluid, and diameter and cross-sectional area of the nozzle. Table 4 shows the operating condition data.

Selection of a Scaling Approach

Generally it is impossible to satisfy all of the dimensionless groups simultaneously. Therefore compromises are necessary to make the selection of

Table 4. Laboratory data for waterjet drilling

No.	Variables	Laboratory value
1	Nozzle diameter of the drill bit (d)	1 mm (0.0394 in)
2	Cross-sectional area of drill bit tip (A)	0.785 mm^2 (0.00123 in^2)
3	Drilling time for beeswax (t)	10 sec
4	Pressure of the system in the laboratory (p)	75.0 psi
5	Waterjet velocity at the tip of the drill bit (u)	2546.48 cm/sec (83.55 ft/sec)
6	Depth of penetration for beeswax (D)	10 mm (0.3937 in)
7	Flow rate of drilling fluid (water) (q_w)	20 cm^3/sec (0.0076 ft^3/sec)
8	Rate of penetration for beeswax, (ROP)	50.0 mm/h
9	Compressive strength of beeswax (σ)	526.72 kPa

a scaling approach. There are six scaling approaches in oil displacement and oil recovery processes (Lozada and Farouq Ali, 1987, 1988; Kimber et al., 1988; Islam and Farouq Ali, 1990, 1992). These researchers discuss the advantages and disadvantages of different approaches and their effects on different related variables, mechanisms, and applications. However, for better scaling and understanding of the waterjet drilling technique, a new approach—same drilling fluid, same gravity force, different porous media, different pressure drop, geometric similarity and different temperature—is proposed. This approach is similar to Approach 5 of Lozada and Farouq Ali (1987), Approach 1 of Kimber et al. (1988), and Approach 6 of Islam and Farouq Ali (1992). The basic difference with the proposed approach is the temperature and gravity force. In order to scale gravitational forces properly, a different pressure drop is used in this approach. In waterjet drilling, geometric similarity, temperature and pressure of the drilling fluid, and the reservoir porous media are the main parameters to be considered in scaling. There is no capillary force action in drilling activities, which leads to different pressure drops in the system. In this approach, geometric similarity, viscous forces, and gravitational forces are satisfied. Therefore different pressure drops and different porous media are necessary in order to have a feasible model in the laboratory.

Selection of Reference Quantities

The purpose of selecting reference quantities is to represent each parameter using an appropriate approach. The system will be invariant as the scale is changed from the model to the field or prototype. The proper value will make the dimensionless parameter the same in the model as in the field. For example, the reference quantities for the fluid properties are selected to make them the same dimensionless functions of dimensionless pressure, temperature, and composition in the model as in the field. If the pressure varies within the same limits in the field as in the model, any value of a fluid property will be suitable as a reference quantity.

Design of the Scaled Prototype

To design a scaled prototype based on model data for the waterjet drilling technique, the following steps should be followed. Consider a scaling factor a that is defined as the diameter of the nozzle of the prototype (i.e., the model is reduced in diameter by a scaling factor a and employing the same fluids as the prototype):

$$a = \frac{d_{field}}{d_{lab}},\ b = \frac{g_{field}}{g_{lab}},\ \text{and } c = \frac{\rho_{field}}{\rho_{lab}}.$$

The similarity groups presented in Table 3 are the complete set of scaling criteria. These groups must be satisfied in order to scale completely the waterjet drilling activity. These requirements are very difficult to satisfy; therefore, some of the similarity groups cannot be satisfied and are ignored or relaxed in order to satisfy the most important parameters. The resulting subset is called the relaxed set of similarity groups.

However, for better scaling of waterjet drilling, the geometric similarity scale is the first scaling factor to be calculated in designing a scaled model. The scaling factor a is chosen such that the model is of a size suitable for a laboratory. In the scaled model, the time scale is one of the important issues because it determines the length of experimental time and total duration. The effects of pressure can be studied by a comparison of the drilling of two different pressure runs at the same gravitational:viscous force ratio. Different pressure forces in the model and prototype are the most influential parameter in waterjet drilling. The ROP and depth of penetration (DOP) are dependent on this force. The volumetric flow rate needs to be scaled and satisfied for reservoir drilling conditions because the gravitational: viscous force ratio demands the same average linear velocity in the model and prototype. It does not care what the pressures of the model and prototype are. Permeability must also be taken into account when simulating the rock sample in a laboratory. Therefore, a new dimensionless group is derived using the similarity groups in Table 3.

Development of New Dimensionless Groups

To find the Reynold's number, a new π term is formed that does not include t:

$$\pi_{21} = \frac{\pi_5}{\pi_{17}} = \left[\frac{ut}{d}\right] \Big/ \left[\frac{\mu_w t}{\rho_w d^2}\right] = \left[\frac{\rho_w u d}{\mu_w}\right] \Rightarrow \text{Reynold's number.}$$

Now, $[\pi_{21\,lab}]$ must be equal to $[\pi_{21\,field}]$ which means $[\rho_w ud/\mu_{w\,lab}] = [\rho_w ud/\mu_{w\,field}]$. As the same drilling fluid (water) is used in laboratory and field, $[\rho_{w\,lab}] = [\rho_{w\,field}]$ and $[\mu_{w\,lab}] = [\mu_{w\,field}]$, which gives $[\mu d_{lab}] = [\mu d_{field}]$. Therefore

$$\left[\frac{u_{field}}{u_{lab}}\right] = \frac{d_{lab}}{d_{field}} \Rightarrow \left[\frac{u_{field}}{u_{lab}}\right] = \frac{1}{a}$$

In order to match the Reynold's numbers in the laboratory and the field, laboratory experiments should be performed at a velocity much higher than that in the field. If this is not achievable in the laboratory, it would indicate that the Reynold's number cannot be satisfied as a scaling group.

If the same fluid is used, the gravity and capillary group cannot be satisfied. In waterjet drilling, there is no capillary action. Another new π

term is formed using π_2, π_5, and π_{17}:

$$\pi_{22} = \frac{\pi_2}{\pi_{17}} = \left[\frac{gt^2}{d}\right] \Big/ \left[\frac{\mu_w t}{\rho_w d^2}\right] = \left[\frac{g\rho_w dt}{\mu_w}\right] \Rightarrow$$

$$\pi_{23} = \frac{\pi_{22}}{\pi_5} = \left[\frac{g\rho_w dt}{\mu_w}\right] \Big/ \left[\frac{ut}{d}\right] = \left[\frac{g\rho_w d^2}{u\mu_w}\right]$$

In π_{23}, d^2 can be replaced by using the scaling group π_9, which gives the new scaling group $\pi_{24} = [g\rho_w K/u\mu_w] = \frac{\textit{gravitational force}}{\textit{viscous force}}$. This is a measure of the natural tendency of the fluid of greater density to seek the lower levels of the formation. Now, with a model of the same fluid (water) and considering the same gravitational forces in the field and laboratory (using the proposed approach), π_{24} becomes

$$\left[\frac{g\rho_w K}{u\mu_w}\right]_{lab} = \left[\frac{g\rho_w K}{u\mu_w}\right]_{field} \Rightarrow \left[\frac{K}{u}\right]_{lab} = \left[\frac{K}{u}\right]_{field} \Rightarrow \left[\frac{K_{field}}{K_{lab}}\right] = \frac{u_{field}}{u_{lab}}$$

$\left[\frac{K_{field}}{K_{lab}}\right] = \frac{1}{a}$. Therefore $K_{field} = \frac{1}{a} \times K_{lab}$.

Thus the permeability in the laboratory needs to be a times greater in the laboratory as in the field. Such an approach calls for the use of a different porous medium in the laboratory than the in field (Geertsma et al., 1956; Lozada and Farouq Ali, 1987). Table 5 was developed for different important scaling parameters based on the above procedure and applying the proposed approach.

Table 5. Scale-up of different important scaling parameters based on proposed approach

No.	Number of scaling group	Description of the group	Relation between laboratory and field
1	π_1	Geometric similarity scale	$A_{field} = a^2 \times A_{lab}$
2	π_2	Time scale	$t_{field} = \sqrt{a} \times t_{lab}$
3	π_4	Pressure scale	$p_{field} = ac \times p_{lab}$
4	π_5	Velocity scale	$u_{field} = \sqrt{a} \times u_{lab}$
5	π_8	Depth of penetration scale	$D_{field} = a \times D_{lab}$
6	π_{12}	Flow rate scale	$q_{w-field} = a^{5/2} \times q_{w-lab}$
7	π_{13}	Penetration (ROP) scale	$ROP_{field} = \sqrt{a} \times ROP_{lab}$
8	π_{18}	Compressive strength scale	$\sigma_{field} = ac \times \sigma_{lab}$
9	π_{19}	Compressive stress scale	$\tau_{field} = ac \times \tau_{lab}$
10	π_{24}	Permeability scale	$K_{field} = 1/a \times K_{lab}$

RESULTS AND DISCUSSION

Scaled Prototype Data

Based on the proposed approach and applying the derived relationship between different relevant variables and dimensionless groups in the previous section, Table 6 was developed. The table gives the scaling ratios that would apply to an example case where the model:field diameter ratio is 1:102. The case assumes that the field and model fluids are the same; therefore no fluid property groups are considered. Also, initial saturation pressure and temperature conditions are assumed to be equal. To develop these data, the solid rock matrix density (ρ_s) and beeswax matrix density (ρ) are considered as 2.2 gm/cm^3 (167) and 0.970 g/cm^3 (60.475), respectively.

Empirical Relations for the Prototype

Figures 1a and 1b show the variation of DOP with drilling time for field and laboratory data. The laboratory data are based on experimental results (Hossain et al., 2007). The field data are developed by the scaled-up process described earlier in this article. Initially DOP increases with a slight increase in drilling time. However, after 3.38 hr (Figure 1a), DOP starts to decrease. D_{field} reaches approximately 1131.0 mm at this time. Hossain et al.

Table 6. Comparison of laboratory and field data

No.	Variables	Laboratory value	Field value
1	Nozzle diameter of the drill bit (d)	1 mm (0.0394 in)	102.0 mm (4.0 in)
2	Cross-sectional area of the drill bit tip (A)	0.785 mm^2 (0.00123 in^2)	8167.14 mm^2 (12.80 in^2)
3	Drilling time for beeswax (t)	10 sec	101 sec
4	Pressure of the system in the laboratory (p)	75.0 psi	2,1125.26 psi
5	Waterjet velocity at the tip of the drill bit (u)	2546.48 cm/sec (83.55 ft/sec)	25,718.19 cm/sec (843.81 ft/sec)
6	Depth of penetration for beeswax (DOP)	10 mm (0.3937 in)	1020.0 mm (40.16 in)
7	Flow rate of the drilling fluid (water) (q_w)	20 cm^3/sec (0.0076 ft^3/sec)	2259452.64 cm^3/sec (798.57 ft^3/sec)
8	Rate of penetration for beeswax (ROP)	50.0 mm/h	504.98 mm/h
9	Compressive strength of beeswax (σ)	526.72 kpa	1,48361.28 kpa

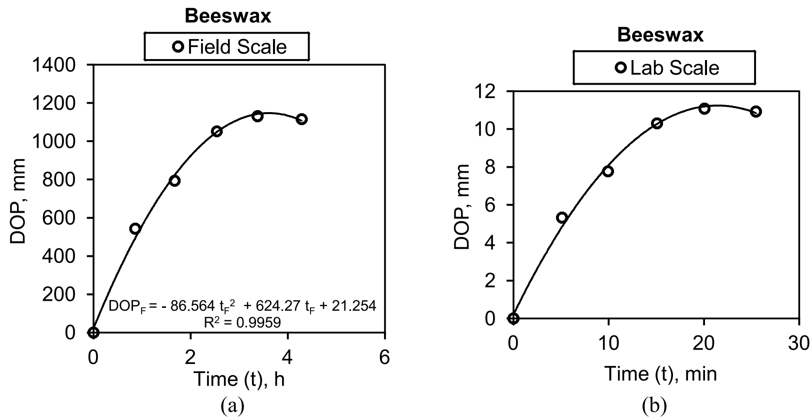

Figure 1. Variation of DOP with drilling time for (a) field and (b) lab scale.

(2007) developed an empirical relation for laboratory experimental results for DOP. Equation (1) represents an empirical relation for DOP in an oilfield application:

$$DOP_F = -86.564t_F^2 + 624.27t_F + 21.254 \textit{ and } R^2 = 0.9959 \qquad (1)$$

Figures 2a and 2b show the variation of ROP with drilling time for laboratory and field data. The laboratory data are based on experimental results (Hossain et al., 2007). The field data are developed the by scaled-up process described earlier in this article. ROP decreases with a decrease in drilling time. Hossain et al. (2007) developed an empirical relation for laboratory experimental results for ROP. Equation (2) represents an empirical

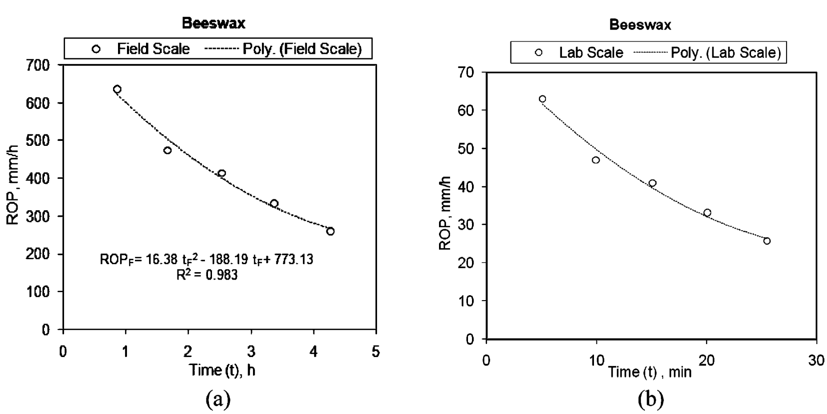

Figure 2. Variation of rate of penetration with drilling time for (a) field and (b) laboratory scale.

relation for ROP and drilling time after scaling up from laboratory to field scale in an oilfield drilling application:

$$ROP_F = 16.38t_F^2 - 188.19t_F + 773.13 \textit{ and } R^2 = 0.983 \quad (2)$$

CONCLUSIONS

This article introduces a new scaling approach for waterjet drilling. This study correctly scales up the formation properties and permeability using the proposed approach. So far, based on available literature reviews, there are no other existing approaches that can handle permeability except the proposed one. Moreover, the proposed approach creates the scaling option for compressive strength and stress during waterjet drilling which is not possible with other approaches. It is also noted that the approach considers different temperature scenarios in laboratory and field that leads to proper handling of the thermal conductivity of the rock matrix using similarity group 10. A scaled model is developed including a complete set of similarity groups for waterjet drilling. In addition, empirical models for drilling parameters such as DOP and ROP as a function of drilling time are established based on a scaled-up process for an oilfield drilling application. Various sample (i.e., beeswax or rock) properties such as density, permeability, and compressive strength are scaled up in the field scale.

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NOMENCLATURE

A	cross-sectional area of the drill bit tip, mm^2
d	nozzle diameter of the drill bit, mm
DOP_F	depth of penetration at field scale, mm
g	gravitational acceleration in x direction, m/sec^2
h	thickness of the beeswax sample, $[L]$, m
K	permeability of the beeswax sample, md
k	thermal conductivity of the beeswax sample, kJ/h-m-k
p	pressure of the system in the laboratory, pa
ROP_F	rate of penetration at field scale, mm/h
t	drilling time for beeswax at laboratory scale, min
t_F	drilling time at field scale, h
T	temperature of the drilling fluid (water), °C
u	waterjet velocity at the tip of the drill bit, m/sec
c_p	specific heat capacity of the beeswax sample, kJ/kg-k
c_{pw}	specific heat capacity of the drilling fluid (water), kJ/kg-k
h_c	convection heat transfer coefficient, kJ/h-m^2-k
k_w	thermal conductivity of the drilling fluid (water), kJ/h-m-k
$q_w = Au$	flow rate of the drilling fluid (water), ft^3/sec
t_T	thermal exposure time, min
ϕ	porosity of the beeswax, volume fraction
ρ	density of the beeswax sample, kg/m^3
ρ_w	density of the drilling fluid (water), kg/m^3
μ_w	viscosity of the drilling fluid (water), N-sec/m^3
σ	compressive strength of the beeswax, N/m^2
τ	compressive stress of the beeswax, N/m^2