Functional Networks as a Novel Approach for Prediction of Permeability in a Carbonate Reservoir

Information and Computer Science Department
Center of Petroleum and Minerals
King Fahd University of Petroleum and Minerals
Dhahran, Saudi Arabia, 31261
{sebakhy,aazeez,mujahed,aalmajed,mrawal,fazzedin,raharja}@kfupm.edu.sa

Abstract

Permeability is one of the most difficult properties to predict, especially in carbonate reservoirs. Permeability prediction is a challenge to reservoir engineers due to the lack of tools that measure them directly. The most reliable data of permeability, obtained from laboratory measurements on cores, do not provide a continuous profile along the depth of the formation. This paper presents functional networks as a novel approach for forecasting permeability using Well Logs in a Middle Eastern Carbonate reservoir. Unlike the standard artificial neural network, functional network is a problem driven, in these networks there are no weights associated with the links connecting neurons, and it uses unknown neuron functions, that are learned from given families of linearly independent functions during the training process. Appropriate families can be chosen for each specific problem, such as, polynomials, Fourier, exponential, and trigonometric functions. This new computational intelligence scheme will overcome the weakness of the common softcomputing techniques, such as, neural networks and decision trees limitations. Two types of functional network models, separable and associativity functional networks are used to predict the permeability. Functional networks permeability model is chosen based on the minimum description length criterion, which takes care of both overfitting and complexity problems. A comparative study is carried out to compare their performance with those of statistical techniques and conventional neural network models. Preliminary results show that the new framework is flexible, more accurate, and comparable in performance to those of artificial neural networks and statistical techniques. Results were obtained with only the simplest structures of functional networks. It is possible to use more complex non-linear forms that lead to better accuracy, efficient results, and outperform the most common statistics, data mining techniques.

1. Introduction

Permeability is defined as the ability of porous rock to transmit fluid, the reservoir management and development requires accurate knowledge of permeability. Permeability is one of the most crucial reservoir properties and it is very hard to accurately obtain for reservoir description. Generally, permeability is directly measured in either core analysis or measured based on the use of wireline logs. Last two decades, numerous efforts have been made to forecast the permeability using well log data with the aid of available core-measured permeability. There are many empirical correlations for forecasting permeability based on wireline logs and the corresponding core permeability data, most of them were developed using equations of state (EOS) or statistical regression [1, 2] or feedforward neural networks (FFN) [1, 2, 6] or fuzzy logic (FL) [31,32,33]. However, they often do not perform very accurate and suffer from a number of drawbacks, such as, both FFN and FL are black boxes modeling schemes that are based on the trial-and-error approach. Furthermore, the parameters of neural networks architecture have to be optionally guessed in advance and the training algorithm parameters were determined based on guessing initial random weights, learning rate, and momentum.

The most recent work by the authors in [9] presented a state-of-the-art review of the use of neural networks for predicting permeability from well logs. The main contribution of this paper is to propose functional networks as a new framework for estimating the permeability in a carbonate reservoir and then we overcome the above weakness of computational intelligence schemes. Therefore, the built calibration model based on functional networks with known a wireline log data and core-measured permeability will be utilized to predict permeability for unseen wireline logs within the same wireline logs or different wireline logs. In this paper, we used the same real-world industry wireline logs used in [31] and follow the same quality control criterion to get
ready of the un-useful observations in the provided wireline logs. Thus, functional networks can be used to forecast the permeability values within multiple wireline logs and then the geologist will be able to build decision for well drilling with high confidence and less risk. The rest of this paper is organized as follows. Section 2 provides the related work and literature review. The proposed intelligent system modeling paradigm is described in detail in Section 3. Functional networks utilization in predicting permeability and the most common statistical quality measures are briefly described in section 4. The implementations and comparative studies with the most published empirical correlations are shown in Section 5. Both performance results and details discussions are drawn in Section 6. The conclusion and recommendations for future works are drawn in Section 7.

2. Related Work and Literature Review

The most popular empirical correlations to estimate the permeability from wireline logs in literature are the statistical regression approaches [1, 2], state of equations (physical model) [3, 5, 32], and the standard feedforward neural networks [1, 2, 6, 10]. Unfortunately, the developed empirical correlations are often limited and global correlations are usually less accurate compared to local correlations. The most existing empirical correlations are often determined from log-derived parameters, such as, porosity…etc. However, the estimated permeability values are not accurate and these empirical correlation formulas are not universally applicable. Alternatively, the standard neural networks have been increasingly applied to predict reservoir properties using well log data [6]. Recently, comparative studies were carried out, their results show that the neural networks have reliable and stable performance with great potential for generating more-accurate predictions [7, 8].

The most recent work by the authors in [9] presented a state-of-the-art review of the use of neural networks for predicting permeability from well logs. In their work, the network was used as a nonlinear regression tool to develop transformation between well logs and core permeability. Such a transformation can be used for estimating permeability in uncorrelated intervals and wells. The neural network was trained by a training set with four well logs (GR, NPHI, RHOB, and RT) with the core permeability. Results show accurate and reliable performance. However, the artificial neural networks often do not perform very accurate in all situations and suffer from a number of drawbacks, such as, feedforward neural network (FFN) is a black box modeling scheme that is based on the trial-and-error approach. FFN architectural parameters have to be guessed in advance, such as, number and size of hidden layers and the type of transfer function(s) for neurons in the various layers. Moreover, the training algorithm parameters were determined based on guessing initial random weights, learning rate, and momentum. Although acceptable results may be obtained with effort, it is obvious that potentially superior models can be overlooked. The considerable amount of user intervention not only slows down model development, but also works against the principle of 'letting the data speak'. Furthermore, each correlation for predicting permeability were developed for a certain range of reservoir fluid characteristics and geographical area with similar input wireline logs. Thus, the accuracy of such correlations is critical and not often known in advance.

Recently, fuzzy logic has been introduced as well in the field of petroleum engineering, it has been applied to solve distinct oil and gas applications, such as, petro-physical problems, lithofacies and permeability prediction from well logs, see [30, 31, 32, 33, 35, 36] for more details. The predictions of both Lithofacies and permeability were used the Gaussian membership and fuzzy clustering algorithm. The authors in [32], predict permeability by dividing the core permeability values into ten equal bin sizes on a log scale, then forecasting problem was converted into a classification problem, and they obtained better petrophysical estimates compared to the conventional techniques. The authors in [37] also applied fuzzy rules to predict porosity and permeability from five compositional and textural characteristics of sandstone in the Yacheng Field (South China Sea). The five input attributes were the relative amounts of rigid grains, ductile grains and detrital matrix, grain size and the Trask sorting coefficient. All the porosity and permeability data were firstly divided into certain number of clusters by fuzzy c-means clustering algorithm. The corresponding sandstone characteristics for each cluster were used to general the fuzzy linguistic rules. Each fuzzy cluster produced one fuzzy if-then rule with five input statements. The formulated rules were then used to make linguistic prediction by combining individual conclusion from each rule. If a numerical output was desired, a defuzzification algorithm [38] could be used to extract a crisp output from a fuzzy set. The results showed that the fuzzy modeling gave better results compared to those presented in [34].

The authors in [35] presented a simple but practical fuzzy interpolator for predicting permeability from well logs in the North West Shelf (offshore Australia). The basic idea was to simulate local fuzzy reasoning. When a new input vector (well logs) was given, the system would select two training vectors which were nearest to the new input vector and build a set of piece-wise linear inference rules with the training values, in which the membership value of the training values was one. The study used well log and core data from two wells and the performance was tested at a third well, where actual core data were available for comparison. The accuracy of the permeability predictions at the test well was although similar to that obtained from the authors’ previous neural fuzzy
technique; the fuzzy approach was greater than 7,000 times faster in terms of CPU time.

The authors in [36] applied a neural-fuzzy approach to develop an optimum set of rules for nonlinear mapping between porosity, grain size, clay content, P-wave velocity, P-wave attenuation and permeability. The rules developed from a training set were used to predict permeability in another data set. The prediction performance was very good. The study also showed that the integrated technique discovered clear relationships between P-wave velocity and porosity, and P-wave attenuation and clay content, which were useful to geophysicists.

In (2007), the authors in [31] developed friendly graphical user interface standalone software to estimation the permeability from wireline logs in a Middle Eastern carbonate reservoir based on the adaptive neuro-fuzzy logic inference systems (ANFIS), which is a hybrid intelligent systems modeling approach. This software will be able to handle training of the model and then predict the permeability for the new layers or wells. In this software, the user has the option to model the given input data using either Grid Partition (GP) or Subtractive Clustering (SC) schemes [38] and then select the most suitable membership function though the training and testing process. In addition, based on the selected input sets of parameters, a user can choose to predict permeability for either the same well (validation that assess the strength or the weakness of the calibration model) or to predict the permeability for different well logs where core date is not available. Furthermore, they trained ANFIS using multiple well logs and determine the permeability for different well logs with unknown core permeability. They compared the performance between predicting the core permeability (k) versus log10(k) using different combinations of the input data, namely, CT, DRHO, DT, DR, MSFL or log10(MSFL), NPHI, PHIT, RHOB, RT, and SWT. They train the software using the stratified sampling scheme to have a fair separation for both training and testing subsets, say 70% for training and 30% for testing and validation. The entire process implementations were carried out based on on different six wireline log input parameters based on the highest significant value with the individual input with the desired output permeability. The results show that the best combinations were chosen from DT, log10(MSFL), NPHI, PHIT, RHOB, and SWT. From the published results, it can be concluded that the ANFIS model predict permeability with a very high accuracy, but still less accuracy in the both validation and testing. The built ANFIS model give an acceptable permeability prediction for a new well in a formation, the calibration ANFIS model that is trained using combination of well logs from surrounding wells.

3. Functional Networks

Recently, functional networks have been introduced by [11, 12, 17, 18, 19, and 20] as a generalization of the standard neural networks. It dealt with general functional models instead of sigmoid-like ones. In these networks the functions associated with the neurons are not fixed but are learnt from the available data. There is no need, hence, to include weights associated with links, since the neuron functions subsume the effect of weights. Functional networks allow neurons to be multi-argument, multivariate, and different learnable functions, instead of fixed functions. Functional networks allow converging neuron outputs, forcing them to be coincident. This leads to functional equations or systems of functional equations, which require some compatibility conditions on the neuron functions. Functional networks have the possibility of dealing with functional constraints that are determined by the functional properties of the network model.

Functional networks as a new modeling scheme has been used in solving both prediction and classification problems. It is a general framework useful for solving a wide range of problems in probability, statistics, signal processing, pattern recognition, functions approximations, real-time flood forecasting, science, bioinformatics, medicine, structure engineering, and other business and engineering applications, see [11, 12, 13, 14, 15, 16] and [22-28], and the references therein for more details. The performance of functional networks has shown bright outputs for future applications in both industry and academic research of science and engineering based on its reliable and efficient results. Several comparative studies have been carried to compare its performance with the performance of the most popular prediction/classification modeling data mining, machine learning schemes in literature [12, 13]. The results show that functional networks performance outperforms most of these popular modeling schemes in machine learning, data mining, and statistics communities [12, 13]. Dealing with functional networks in prediction and classification required some concepts and definitions, which can be briefly summarized as follows:

Suppose that, we use the set \( X = \{x_1, \ldots, x_p\} \) to be the set of nodes, such that each node \( x_i \) is associated with a variable \( X_i \). The neuron (neural) function over a set of nodes \( X \) is a tuple \( \langle x, f, z \rangle \), where \( x \) is a set of the input nodes, \( f \) is a processing function and \( z \) is the output nodes, such that \( z = f(x) \), where \( x \) and \( Z \) are two non-empty subsets of \( X \). The best way to illustrate the functional networks and its way of representation is to use an example [12]. As it can be seen in Fig.1, a functional network consists of: a) several layers of storing units, one layer for containing the input data \( x_i; i = 1,2,3, \)
4), another for containing the output data (x7) and none, one or several layers to store intermediate information (x5 and x6); b) one or several layers of processing units that evaluate a set of input values and deliver a set of output values (fj); and c) a set of directed links. Generally, functional networks extend the standard neural networks by allowing neuron functions fj to be not only true multia rgument and multivariate functions, but to be different and learnable, instead of fixed functions. In addition, the neuron functions in functional networks are unknown functions from a given family, such as, polynomial, exponential, Fourier...etc, to be estimated during the learning process. Furthermore, functional networks allow connecting neuron outputs, forcing them to be coincident [12, 18, and 19].

![Functional network architecture: An example](image)

The functional network uses two types of learning: a) structural learning b) parametric learning. In structural learning, the initial topology of the network, based on some properties available to the designer is arrived at and finally a simplification is made using functional equation to a simpler architecture. In parametric learning, usually activation functions by considering the combination of "basis" functions are estimated by using the least square, steepest descent and mini-max methods [29]. In this paper we use the least square method for estimating activation functions in both classification and prediction.

Generally, functional network is a problem driven, which means that the initial architecture is designed based on a problem in hand. In addition, to the data domain, information about the other properties of the function, such as associativity, commutativity, and invariance, are used in selecting the final network. In the functional network, neuron functions are arbitrary known or unknown to be learned from the provided data, but in neural networks they are a sigmoid, linear or radial basis and other functions. In functional networks, neuron functions in which weights are incorporated are learned, and in neural networks, weights are learned. Neural networks work well if the input and output data are normalized in a specific range, such as, in between 0 and 1, but in functional networks there is no such restriction. Furthermore, It can be pointed out that neural networks are special cases of functional networks [12]. Dealing with functional networks required the following steps through the networks implementations and learning process:

**Statement of the problem:** Specify the initial topology based on the domain of the problem in hand.

- Simplify the initial architecture using functional equations and the equivalence concept and check the uniqueness condition of the desired architecture, see [18, 19] for more details.
- Gathering the required data and handle multicollinearity problem with the implementation of the required quality control check before the functional networks implementation.

The learning procedures and training algorithm based on either Structure or Parametric learning by considering the combinations of linear independent functions, $\Psi = \{\psi_{s1}, \psi_{s2}, \ldots, \psi_{sm}\}$, for all $s$ to approximate the neuron functions, that is,

$$ g_s(x) = \sum_{i=1}^{m_s} a_{si} \psi_{si}(x) \quad \text{for all } s, \quad (1) $$

where the coefficients $a_{si}$ are the parameters of functional networks. The most popular linearly independent functions in literature are:

$$ \Psi = \{1, X, \ldots, X^m\}, \text{ or} $$

$$ \Psi = \{1, \cos(X), \ldots, \cos(X), \sin(X)\}, m = 2l $$

$$ \Psi = \{1, e^x, e^{-x}, \ldots, e^{mx}, e^{-mx}\}, $$

where $m$ is the number of elements in the combination of sets of linearly independent function. The parameters in (1) can be learned using one of the known optimization (loss criterion) techniques, such as, least squares, conjugate gradient, iterative least squares, minimax, or maximum likelihood estimation.

**Model selection and validation.** The best functional network model is chosen based on the minimum description length and some other quality measurements, such as, the correlation coefficients and the root-mean-squared errors. The selection is achieved using one of the well known selection schemes, such as, exhaustive selection, forward selection, backward elimination, backward forward selection, and forward backward elimination.

Finally, if the fifth step is satisfactory, then the functional networks model is ready to be used in predicting unseen new unseen data sets from real-world industry applications.

The learning method of a functional network consists of obtaining the neuron functions based on a set of data

$$ D = \{Y, X\}, \text{where } X \in \mathbb{R}^p \text{ represents the matrix of}$$
size \((n \times p)\) of \(p\) input feature variables and the column \(Y \in R\) (prediction) or \(Y \subseteq R\) (classification) is an output.

In the case of pattern classification, we assumed that the observation vector is of fixed length, say \(p\), and that \(Y\) is a set of class labels \(A_k\), for all \(k = 1, 2, \ldots, c\). Therefore, we assume that we have given a training set \(D = \{y_i, x_{i1}, \ldots, x_{ip}\}\) for all \(i = 1, 2, \ldots, n\) of feature variables \(x_{i1}, \ldots, x_{ip}\), where \(x_j = [x_{ij} \ldots x_{uj}]\), for \(j = 1, \ldots, p\) drawn from \(c\) classes, and the categorical response variable \(Y\), for all \(i = 1, 2, \ldots, n\). We use the lower case letters \(x_{i1}, \ldots, x_{ip}\) for all \(i = 1, \ldots, n\) to refer to each observation of the predictor variables, and \(y_i = k\) to refer to class \(A_k\) for all \(k = 0, 1, \ldots, c\), where \(c \geq 2\). We use a symbol \(\pi_{ik}\) for the probability that observation \(i\) falls in the class \(A_k\), that is,

\[
\pi_{ik} = P(x_i \in A_k | x_{i1}, \ldots, x_{ip}) = P(y_i = k | x_{i1}, \ldots, x_{ip}),
\]

where \(\pi_{ik} \geq 0\); \(k = 0(1)c\); \(i = 1(1)n\); and \(\sum_{k=0}^{c} \pi_{ik} = 1\), \(i = 1,2,\ldots,n\).

In this paper we propose two distinct functional networks called the Generalized Associativity Model and the Separable Functional Networks Model to approximate \(f(x_{i1}, \ldots, x_{ip})\). We briefly expressed these functional networks models as [12, 18, 19]:

The **Generalized Associativity** model which leads to the additive model:

\[
f(x_{i1}, \ldots, x_{ip}) = \sum_{j=1}^{k} h_j(x_j), \tag{5}
\]

The corresponding architecture is shown in Fig.2.

**Fig. 2. Additive Functional Networks model topology**

1. The **Separable functional networks model** which considers a more general form for the unknown function, \(f(x_{i1}, \ldots, x_{ip})\):

\[
f(x_{i1}, \ldots, x_{ip}) = \sum_{s=1}^{w_0} \sum_{s=1}^{w_k} C_{n-r} h_i(x_s) h_j(x_r), \tag{6}
\]

where \(C_{n-r}\) are unknown parameters and the sets of functions \(\Phi_s = \{\phi_s : r = 1, \ldots, q_s\}\), \(s = 1, 2, \ldots, k\), are linearly independent. An example of this functional network for \(k = 2\) and \(q_1 = q_2 = q\) is shown in Figure 3. Equations (5) and (6) are functional equations since their unknowns are functions. Their corresponding functional networks are the graphical representations of these functional equations.

**Fig. 3. The functional network for the separable model with \(p = 2\) and \(q_1 = q_2 = q\).**

We note that the graphical structure is very similar to a neural network, but the neuron functions are unknown. Our problem consists of learning \(h_1, h_2, \ldots, h_k\) in (5) and \(C_{n-r}\) in (6). In order to obtain \(h_1, h_2, \ldots, h_k\) in (5), we approximate each \(h_j(x_j)\), for \(j = 1, 2, \ldots, k\), by a linear combination of sets of linearly independent functions \(\psi_{js}\) defined above, that is,

\[
h_j(x_j) = \sum_{s=1}^{q_j} a_{js} \psi_{js}(x_j); j = 1, 2, \ldots, p, \tag{7}
\]
and the problem is reduced to estimate the parameters $a_{js}$, for all $j$ and $s$, see [12, 18, and 19] for more details.

Generally, by setting appropriate input and applying system identification to study the defect prone classes identification, one can customize the characteristics of input value according to wishful output. In this paper, we follow the same procedures in both [12, 19] and choose the least squares criterion to learn the parameters, but the additive model requires add some constrains to guarantee uniqueness. Alternatively, one can choose different optimization criterion based on his interest. The main advantage in choosing the least squares method is that the least squares criterion leads to solve a linear system of equations in both prediction and classification problems.

4. Functional Networks Utilization in Permeability Prediction

To implement both functional networks and other existing empirical correlation models, the entire available data set is usually randomly divided into a training set and a testing or validating set. The training set is used for neural network model building and the testing set is used to evaluate the predictive capability of the network model. The Cross-Validation is used to assess the predictive performance of neural network.

4.1. Initialization and Setting up Parameters

We use either associativity or separable functional networks model with only family of linearly independent families (basis) with polynomial degree at most 2. In addition, we implement the multilayer perceptron feedforward networks (MLPFFN) with both pure linear and sigmoid activation neuron functions based on two/three hidden layers. As it is commonly done in literature [39]. For the sake of both statistical regression and neural networks, the input data sets were normalized based on equation (8) to make sure that the utilized input variables were independent of measurement units. Thus, the predictors are normalized to interval of $[0,1]$.

$$x_i^{(\text{new})} = \frac{(x_i^{(\text{old})} - \min(x_i))}{(\max(x_i) - \min(x_i))}; \quad i = l(1)n; \quad (8)$$

Through the implementation, we use stratified sampling technique to make sure that we get the same proportion as in the original data [40]. We randomly hold a total of ($k=\text{round}(n/5)$ or $k=\text{round}(n/10)$ observations) from the provided data set. Therefore, to evaluate the performance of each modeling scheme on a real-application, we use either 5-fold or 10-fold cross validation. Thus, we fold the given data into 5 or 10 parts, and we use 0.80 or 0.70 of the provided data as a training set for building the calibration model and 0.20 or 0.30 of the provided data for testing set to test (external validate) the model, respectively numerous of runs. We compute the suitable statistical quality measures (correlation coefficient and root mean squared errors) for the utilized schemes, such as, multiple linear/nonlinear regression and feedforward neural networks. The input parameters for each technique were chosen in such away to give the best performance. The results are summarized by computing both root mean-squared errors (RMSE) and correlation coefficient (R or $R^2$) over all runs.

4.2. The Utilized Statistical Quality Measures

The most common statistical quality measures can be written in mathematical formulae as follows:

- **Root Mean Squares Error**: Measures the data dispersion around zero deviation:

$$RMSE = \left[ \frac{1}{n} \sum_{i=1}^{n} E_i^2 \right]^{1/2}, \quad (9)$$

where $E_i$ is a relative deviation of an estimated value from an experimental input data sets.

$$E_i = \left( \frac{y_{\text{exp}} - y_{\text{est}}}{y_{\text{exp}}} \right) \times 100; \quad i=1(1)n; \quad (10)$$

- **Correlation Coefficient**: It represents the degree of success in reducing the standard deviation by regression analysis, defined as:

$$r = \sqrt{\frac{\sum_{i=1}^{n} (y_{\text{exp}} - \bar{y})(y_{\text{est}} - \bar{y})}{\sum_{i=1}^{n} (y_{\text{exp}} - \bar{y})^2 \sum_{i=1}^{n} (y_{\text{est}} - \bar{y})^2}}, \quad (11)$$

where $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_{\text{exp}}$.

- **Average Percent Relative Error**: It is the measure of the relative deviation from the experimental data, defined as:

$$E_r = \frac{1}{n} \sum_{i=1}^{n} |E_i|, \quad (12)$$

where $E_i$ is a relative deviation of an estimated value from an experimental value, $E_i = \left| \frac{y_i^{\text{est}} - y_i^{\text{exp}}}{y_i^{\text{exp}}} \right|\times 100; \quad i=1, \ldots, n$.

- **Average Absolute Percent Relative Error**: It is the measure of the relative absolute deviation from the experimental data, defined as:

$$E_{\text{min}} = \min_{i=1}^{n} |E_i| \quad (14)$$
Relative Error: To define the range of error for each correlation, the calculated absolute percent relative error values are scanned to determine the minimum and maximum values. They are defined as:

\[ E_{\min} = \max_{i=1}^{n}|E_i| \]

\[ E_{\max} = \max_{i=1}^{n}|E_i| \]

5. Implementations and Comparative Studies

Based on the above explanation and the setting up parameters within Section 4, and the same procedures achieved in [31], the implementation of the calibration functional networks model is built on 70% of the provided data (learning model) and it was validated and tested using the remaining 30% of the provided data for testing set to test (external validate) the built model. The implantations are repeated for both prediction and validation processes based on 100/500 times using MATLAB V7 under Pentium M personal computer, then compute the above discussed statistical quality measures, such as, RMSE and \( R^2 \) over the entire number of runs.

We draw a graph for the average of the predicted output versus the actual response variable over all runs. Both RMSE and \( R^2 \) values are shown on this graph as well. This graph helps us to decide which forecasting model is better in its performance. In the graph, each modeling scheme is represented by a symbol; a good forecasting scheme should have a higher \( R^2 \) and lowest RMSE. In addition, corresponding to this graph, we summarize the results in Tables. In these Tables, the highest RMSE and \( R^2 \)'s are given in boldface.

6. Results and Discussions

Based on the above theoretical discussion, we observed that after building the calibration functional networks model to predict permeability using eight different well log data with the aid of available core-measured permeability, the calibration model becomes ready for testing and evaluation based on the stratified sampling and cross validation criteria. Comparative studies were carried out to compare the performance and accuracy of the new functional networks (FunNets) model versus statistical regression, neural networks, and the most common published empirical permeability correlations. The results of comparisons in both training and testing were summarized in different tables, but for the sake of both space and simplicity, we include only results for the first used wireline log in Table 1. The results show that FunNets modeling scheme outperforms most empirical correlations, statistical regression, and MLPFFN. In addition, functional networks modeling technique shown a high accuracy in predicting permeability in a carbonate reservoir values with a stable performance, lowest absolute percent relative error, lowest minimum error, lowest maximum error, lowest root mean square error, and highest correlation coefficient among other correlations for these documented two wireline logs. Furthermore, the built calibration model was used to estimate the permeability for new unseen well logs, which is a challenge achievement in the field of petroleum engineering.

We draw the required scatter plot of the root mean squared errors versus the correlation coefficients and versus depth used forecasting modeling techniques and the common published empirical permeability correlation schemes. Figures 1 through 8 show the results and its corresponding patterns for the both functional networks and statistical regression modeling schemes used in determining permeability or (log10(k)) based on the used two logs within the aid of core permeability, see [31] for more details. Theses graphs represent relationship between the core permeability and the estimated permeability values versus the depth. Therefore, by implementing the calibration functional networks model based on the provided input variables in the wireline logs, such as, CT, DRHO, DT, DR, MSFL or log10(MSFL), NPHI, PHIT, RHOB, RT, and SWT, we predict log10(k).

As it is shown in Figures 9 through, we observed that the histogram scatter plots for both “root mean squared errors and correlation coefficient (statistical quality measures) performance of functional networks and statistical regression in both training and testing processes. The performance of functional networks performance is reliable, stable and accurate compared to the regression performance. By using entire wireline log input parameters, we obtained functional networks with (\( R^2 = 96\% \), RMSE = 0.35 for training) and (\( R^2 = 93.2\% \), RMSE = 0.54 for testing). On the other hand, the permeability prediction based on the entire wireline log input parameters using nonlinear regression was (\( R^2 = 87\% \), RMSE = 0.65 for training) and (\( R^2 = 85.9\% \), RMSE = 0.66 for testing).

We utilize the calibration functional networks model to predict the permeability of a new unknown well log without the aid of its core permeability. As it is shown in Figure 11, the obtained results are reliable and fall in the specific permeability domain, which is agreed with oil and gas industry expertise. This stable performance of functional networks with only second order degree basis polynomial show a bright direction of it to be used in different oil and gas industry applications, such as, such as, PVT properties and simulations, flow assurance, separation processes, multiphase flow regimes, history matching problems, uncertainty reasoning, risk analysis, and decision making.
The best selected calibration model after 500 runs can be written in the form:

$$k = 10^{1.645 - 0.1637 \log_{10}(MSFL) + 5.60 \text{NPHI} + 10.06 \text{PHIT}}$$

which depends only on three different inputs \(\log_{10}(MSFL),\) NPHI, and PHIT. In addition, PHIT has the highest significant on the permeability pattern, which is agreed with the physical properties of empirical state of equations.
Fig 5. Wireline log1: Prediction of $\log_{10}(k)$ based on FunNets with polynomial family of linearly independent of degree at most 2 and the entire input variables.

Fig 6. Wireline log1: The actual $\log_{10}(k)$ versus its estimated values based on FunNets using all input variables within the provided wireline data based on the second order polynomial linearly independent families.

Fig 7. Wireline log1: Prediction of $\log_{10}(k)$ based on statistical nonlinear regression.

Fig 8. Wireline log1: The actual $\log_{10}(k)$ versus its estimated values based on statistical nonlinear regression.

The same implementations process was repeated for the more wireline data used in [31] but for the sake of simplicity, we did not include it in this context. Figures 1 through 4 illustrate four scatter plots of the permeability.
data versus depth for both experimental and predicted permeability values using the wireline log data. These cross plots indicates the degree of agreement between the experimental and the predicted values based on the high quality performance of the functional networks modeling scheme. The reader can compare these patterns with the corresponding ones of the published neural networks modeling and common empirical permeability correlations.

Next, we investigate the significant of some of input parameters on the predicted permeability values to check and validate the history matching criterion. We conclude that developed FunNets modeling scheme has better and reliable performance compared to the most published modeling schemes and empirical correlations. The bottom line is that, the developed FunNets modeling scheme outperforms neural networks, statistical regression, and most common published empirical correlations in predicting permeability.

Table 1 The average performance of functional networks and statistical linear/nonlinear regression for the first wireline log

<table>
<thead>
<tr>
<th>Set of Input</th>
<th>Functional Networks</th>
<th>Nonlinear Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Training</td>
<td>Testing</td>
</tr>
<tr>
<td></td>
<td>$R^2$%</td>
<td>$R$</td>
</tr>
<tr>
<td>Five Variables</td>
<td>92.0</td>
<td>0.50</td>
</tr>
<tr>
<td>Entire wireline log</td>
<td>96.0</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Fig9. The correlation coefficient of both training and testing validation for two modeling schemes: Functional Networks and Statistical.

Fig10. The correlation coefficient of both training and testing validation for two modeling schemes: Functional Networks and Statistical.

Fig11. Use the best built calibration FunNets model based on the different wireline logs with its corresponding core permeability to estimate the permeability for a new unknown wireline log.
7. Conclusion and Recommendations

We conclude that the same functional networks mechanism can be utilized as well to estimate both porosity and fluid saturations. In addition, it can be used to identify the clustering of well logs for the recognition of lithofacies. We are confidence that functional networks will provide useful and reliable results. The performance of this new computational intelligence paradigm provide a bright direction for future utilization in the area of oil and gas industry and other real-world applications, such as, bioinformatics, security, and business.

We investigate the significant of some of input parameters on the predicted permeability values to check and validate the history matching criterion. We conclude that developed FunNets modeling scheme has better and reliable performance compared to the most published modeling schemes and empirical correlations. The bottom line is that, the developed FunNets modeling scheme outperforms neural networks, statistical regression, and most common published empirical correlations in predicting permeability.

This stable performance of functional networks with only second order degree basis polynomial show a bright direction of it to be used in different oil and gas industry applications, such as, such as, PVT properties and simulations, flow assurance, separation processes, multiphase flow regimes, history matching problems, uncertainty reasoning, risk analysis, and decision making.

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9. References


