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A Variable Cell Model for Simulating Gas Condensate Reservoir Performance

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ABSTRACT

A variable cell model for simulating gas condensate reservoir performance has been developed. The model closely approximates predictions of conventional compositional models but would only require a fraction of the computation time. The characteristic trend of oil saturation profiles, which is exhibited when gas condensate wells produce below dew point pressure, was utilized to divide the reservoir into three concentric regions. Each region is simulated by one cell, and the volume of each cell is varied as production continues. Dew point pressure and phase behavior of the reservoir fluid control expansion of the inner first and second cells and contraction of the exterior third cell.

The model was used to study the effect of liquid flow on well-stream fluid composition. Also, the influence of reservoir fluid phase equilibria data on the extent of two phase flow dominated region was investigated.

INTRODUCTION

It is unfortunate that compositional modeling is computationally expensive primarily because flash calculations must be made over and over for each cell. Particularly for PC computers it is desirable to reduce computing time. A significant saving in computing time can be achieved by minimizing the number of cells required to adequately model the behavior of a gas condensate reservoir. On the other hand, single block - tank type models - are a faster alternative to multicell compositional models, but the simpler models do not provide the accuracy desired for prudent reservoir engineering decisions.

Studies of depletion performance of gas condensate reservoirs report the existence of a relatively high, near-constant, oil saturation in the vicinity of the producing wells shortly after the well flowing pressure falls below the dew point pressure of original hydrocarbon fluid. This saturation buildup occurs even when the average reservoir pressure is still above the dew point pressure. Between this region near the wellbore and the exterior region of the reservoir in which the fluid exists as a single gaseous phase, lies a transition region in which the oil saturation decreases as the distance from the wellbore increases. The size of the transition region, which is concentric in shape, varies as depletion increases.

In this study, a 1-D, single well, radial model specifically designed to deal effectively with this observed saturation profile using only three cells was constructed. A unique feature of this model is that the dimensions of all three cells vary as reservoir depletion proceeds. Here, the inner cell represents the high oil saturation region, the second cell represents the transition region, and the third cell represents the exterior region. The volume of these cylindrical cells varies as a function of the dew point pressure and the phase behavior of the reservoir fluid. Using the techniques devised, this model accurately models the behavior of gas condensate reservoirs.

References and illustrations at end of paper.

MATHEMATICAL FORMULATION

Statement of the Problem

Studies dealing with depletion performance of gas condensate reservoirs indicate that a characteristic trend is exhibited by oil saturation profiles around producing wells [1,2,3]. Figure 1 depicts a typical one. This trend is a manifestation of interplay among factors controlling two phase flow in porous media, such as relative permeability, viscosity, pressure and fluid phase behavior.

A producing formation can be divided into three concentric regions. The interior region in the vicinity of the wellbore is dominated by a relatively high, near-constant oil saturation. The intermediate region is a transition region where oil saturation decreases as the distance from the wellbore increases. Depending on the pressure in the exterior region, the oil saturation may be small or zero. As recovery continues, both the interior and intermediate regions expand, while the exterior region shrinks. The diagram of the variable cell model is presented in Fig. 2.

Compositional modeling of gas condensate reservoirs is a must if one has to obtain reliable performance predictions. Unfortunately, it is computationally expensive due to repeated flash calculations for each cell which consume a great portion of computing time. A significant saving in computing time can be achieved through minimizing the number of cells required to adequately model the behavior of gas condensate reservoirs.

Mathematical Formulation

The IMPES formulation of Neghiem, Fong, and Aziz [4] appears to be more advantageous than the other available formulations in the literature. It requires less computer memory than the Young and Stephenson, and Coats formulations, plus it consumes less computation time. Moreover, it is the simplest to program and implement [5]. In radial models fully implicit formulations are usually used, however, computational cost on a per time step basis is more expensive than that of the IMPES formulations [6]. A study appearing lately in the literature used an IMPES formulation in a one-dimensional radial model. Thus it was decided to use Neghiem formulation.

As subsequently will be shown, the variable cell formulation can achieve a great saving in computation time. The mathematical description of this model is based upon the following assumptions:

1. The well produces at a constant flow rate in an isotropic, homogeneous, cylindrical, closed boundary reservoir of constant thickness.
2. Porosity and permeability are independent of pressure and temperature and are constant.
3. Water phase is immobile.
4. Gravitational forces, and capillary forces between oil, gas and water are neglected.
5. Fluid flow between cell one and the wellbore, and between cell two and cell one can be approximated by steady state flow equations. Fluid flow between cell three and cell two can be approximated by a pseudo steady state flow equation.
6. Interphase thermodynamic exchange in the reservoir is rapid compared with fluid flow so that the gas and oil in each cell are in phase equilibrium.

Mathematical Equations and Solution Method

Compositional material balance equations, thermodynamic equilibrium equations, and constrains equations on mole fractions and hydrocarbon saturations are solved using the iterative sequential method of Neghiem, Fong and Aziz. The unknowns solved for in each cell are: P, Z, X, Y, L, V, S_o and S_g. After which the following calculational procedure is performed.

1. Expansion of Second Cell

After producing the well for a period of time the pressure in the outer cell remains above the dewpoint pressure while the pressure in the first and second cell falls below the dewpoint pressure of the fluid system. The radius of the outer boundary of this cell is increased as the dewpoint pressure location moves away from the wellbore. We seek to make the outer boundary of the second cell coincide with the radius of the dewpoint pressure. This can be achieved by first locating the radius of the dewpoint pressure and then expanding the volume of the second cell to that point. A step by step procedure is shown here:

a. The molar flow rate from the third cell to the second cell is calculated as follows:

$$q_3 = T_3^n \left(\frac{P_3^{n+1} - P_2^{n+1}}{\left(\frac{r_e^2}{r_e^2 - r_2^2}\right) \ln \frac{r_e}{r_2} - \frac{r_2^2}{4r_e^2} - 0.75} \right) \quad \text{........(1)}$$

b. Because pseudo steady state conditions exist in the third cell the molar flow rate q_{3n} at a radius r_{sn} where $r_{sn} > r_s$ (r_s is the outer boundary radius of second cell) can be related to q_3 by:

$$q_{3n} = q_3 - q_3 \left(\frac{\pi\phi h \, (r_{sn}^2 - r_s^2)}{\pi\phi h \, (r_e^2 - r_s^2)} \right) \quad \text{.........(2)}$$

$$q_{3n} = q_3 \left(\frac{r_e^2 - r_{sn}^2}{r_e^2 - r_s^2} \right) \quad \text{................(3)}$$

c. The molar flow rate at r_{sn} and P_{dew} is:

$$q_{3n} = T_3^n \left(\frac{P_3^{n+1} - P_{dew}}{\left(\frac{r_e^2}{r_e^2 - r_{sn}^2}\right) \ln \frac{r_e}{r_{sn}} - \frac{r_{sn}^2}{4r_e^2} - 0.75} \right) \quad \text{.....(4)}$$

Now Eq. (3) is substituted in Eq. (4):

$$q_3 \left(\frac{r_e^2 - r_{sn}^2}{r_e^2 \quad r_s^2} \right) = T_3^n \left(\frac{p_3^{n+1} - p_{dew}}{\left(\frac{r_e^2}{r_e^2 - r_{sn}^2} \right) \ln \frac{r_e}{r_{sn}} - \frac{r_{sn}^2}{4r_e^2} - 0.75} \right) \quad ..(5)$$

d. Eq. (5) is one equation in one unknown and may be solved by an iterative method such as Newton-Raphson's.

2. Expansion of First Cell

The volume expansion in the first cell is proportional to condensate volume in the second cell. A reasonable assumption is made by letting the volume increment to the first cell equal to one half the condensate volume in the second cell.

The new radius of the first cell (r_{tn}) can be calculated as follows:

Volume of condensate in second cell

$$= \pi\phi h(1 - S_w)(r_s^2 - r_t^2)S_{o2} \quad ....(6)$$

Volume increment to first cell

$$= \pi\phi h(1 - S_w)(r_{tn}^2 - r_t^2) \quad ....(7)$$

Multiplying Eq. (6) by 1/2, equating it to Eq. (7) and solving for r_{tn} gives,

$$r_{tn} = \sqrt{r_t^2 + 1/2(r_s^2 - r_t^2)\ S_{o2}} \quad ....(8)$$

3. Preservation of Molar Balance

As the second cell expands the single phase third cell shrinks, thus moles lost from the third cell are gained by second cell. The composition of these moles is the composition in the third cell. Let $N_{3,2}$ denote the number of moles transferred from third cell to the second cell. $N_{3,2}$ can be calculated as follows:

$$N_{3,2} = V_{h3}(\rho_o S_o + \rho_g S_g)_3 \left(\frac{r_{sn}^2 - r_s^2}{r_e^2 - r_s^2} \right) \quad ....(9)$$

Let N_3 be the number of moles remaining in the third cell; N_3 can be calculated as follows:

$$N_3 = V_{h3}\ (\rho_o S_o - \rho_g S_g)_3 - N_{3,2} \quad ....(10)$$

In a similar manner as the first cell expands moles in the second cell are transferred to the first cell. Let $N_{2,1}$ denote the number of moles transferred from the second cell to the first cell. $N_{2,1}$ can be calculated as follows:

$$N_{2,1} = V_{h2}(\rho_o S_o + \rho_g S_g)_2 \left(\frac{r_{tn}^2 - r_t^2}{r_s^2 - r_t^2} \right) \quad .... (11)$$

If N_1 is the new number of moles in the first cell, it can be calculated as follows:

$$N_1 = V_{h1}\ (\rho_o S_o + \rho_g S_g)_1 + N_{2,1} \quad ....(12)$$

Similarly, N_2, the new number of moles in the second cell, can be calculated as follows:

$$N_2 = V_{h2}\ (\rho_o S_o + \rho_g S_g)_2 + N_{3,2} - N_{2,1} \quad ....(13)$$

Since the second cell represents the transition region in the reservoir where the oil saturation is a maximum at the inner boundary of the cell, close in magnitude to the oil saturation in the first cell, and decreases to zero at the outer boundary of the cell, it follows that the composition of the fluid in the vicinity of the inner boundary of the cell is similar to the composition in the first cell and may be approximated by it without loss of accuracy.

4. Composition Calculation

The composition of third cell undergoes no change since it is still in the single phase state. The composition of the first cell is not going to be varied due to the aforementioned assumption that the composition of moles transferred from second cell to the first is similar to the first cell's composition. The second cell is where the change in composition occurs. Let Z_{2in} denote the new composition of second cell. Z_{2in} can be calculated as follows:

$$Z_{2in} = [Z_{2i} V_{h2} (\rho_o S_o + \rho_g S_g)_2 + Z_{3i}\ N_{3,2} - Z_{1i}\ N_{2,1}] N_2 \quad (14)$$

RESULTS AND DISCUSSIONS

In this study a three component system, methane-butane-decane, was used as the reservoir hydrocarbon fluid. At the pressures and temperatures considered the phase behavior of this system is very similar to that of typical gas condensate mixtures. The composition chosen in this study to represent a gas condensate fluid is given in Table 1. All the calculations in this study were made at a temperature of 160°F. The liquid mole fraction vs pressure for this system is shown in Fig. 3. The K-values for methane, butane and decane were obtained from the following equations respectively as a function of pressure:

$$K_1 = (1.128376E-07)P^2 - (1.100936E-03)P + 3.83606 \quad (15)$$

$$K_4 = (4.977797E-08)P^2 - (1.31997E-04)P + 0.3824369 \quad (16)$$

$$K_{10} = (6.103818E-08)P^2 - (2.813248E-04)P + 0.3494668 \quad (17)$$

Given the temperature, pressure and phase compositions, liquid and gas viscosities were calculated using the method of Lohrenz, et al. [7]. The relative permeability data curves used for the liquid and gaseous phases are similar to those given by Kiazeff and Naville [1]. Figure 4 shows these relative permeabilities as a function of liquid saturation.

Numerical Results

The primary purpose of this study was to develop a technique for varying cell sizes in a compositional model for simulating gas condensate reservoir depletion performance. It was required that results computed with the variable cell model closely approximate predictions made with a conventional compositional simulator in which cell sizes do not change. It was expected that the variable cell technique would require less computation time.

To gauge the accuracy of the variable cell model a conventional model containing six cells was constructed. Also, to examine whether the variable cell technique improves predictions, a conventional model containing the same number of cells as the variable cell model was constructed and used to predict depletion performance of the same gas condensate system. This examination reveals whether the predictions of the variable cell model are in closer agreement with those of the six cell or the three cell conventional model. If agreement is closer with the six cell model, then it can be argued that the variable cell model constitutes an improvement; on the other hand, if agreement is closer to the three cell model, it would mean that no substantial advantage is gained by using a variable cell model. Table 2 shows the data used in the variable cell model, while Table 3 shows the data used in the constant cell models.

The depletion performance of the lean gas condensate system was simulated with all three models. Figure 3 shows that this fluid has a dewpoint pressure of 3410 psi. Initial pressure in all runs was 4000 psi, and production rates was fixed at 5000 moles/day.

Convergence is measured using Eq. (18) as follows:

$$\varepsilon > \sqrt{\sum_{j=1}^{3}\left[\left(\frac{F_j}{V_{hj}({}_{o}S_{o} + {}_{g}S_{g})_j}\right)^{L+1}\right]^2} \quad ....(18)$$

in which the square root of the sum of the square of each cell's hydrocarbon molar balance error divided by the number of hydrocarbon moles in the cell is computed. This relative error measured was chosen to offset the large differences between the volumes of the cells. The solution is assumed to have converged when the error measure calculated by Eq. (18) is less than 10^{-4}. Normally, this would correspond to a change in pressure of less than .01 psi and a change in saturation of less than .001 between the final two iterations.

At the end of every time step the material balance error is calculated as follows:

= | (Initial Moles in place
- Cumulative production
- Current moles in plase)
÷ Cumulative production |

Using the convergence criteria given above, the models gave a material balance error of less than 10^{-3}.

Comparison of Pressure Profiles

The validity of the variable cell model ma be judged by the proximity of the variables it calculates to the "real" values calculated by the six constant cell model. Figure 5 compares the pressure profiles in the reservoir calculated by the three models after 190 days of production. The calculated wellbore flowing pressures are also plotted. The figure reveal that the variable cell model closely approximates the pressures calculated by the constant six cell model. In cells containing two phases a portion of the deviation can be attributed to the difference in the sizes of the cells in the two models. Considering that the variable cell model contains only one-half as many cells as the constant cell standard, the agreement is thought to be very good.

Comparison of Saturation Profiles

The unique feature of the variable cell compositional model is to change the sizes of the computing cells as the location of the dewpoint pressure and the associated high oil saturation region of the reservoir move outward from the producing well. Thus, throughout a simulation run each of the three cells deals with a region of the reservoir in which a certain type of condition dominates, i.e., the first cell represents the high oil saturation region, the second cell represents the transition region, and the third cell represents the outer single phase region. Figure 6 compares the saturation profiles calculated by the variable cell and the two constant-cell models. It can be seen that the predictions of the variable cell model more closely approximate those of the six cell model than do results from the constant three cell model.

The condensation boundary, the locus of the dewpoint pressure, is plotted vs time in Fig. 7. In the variable model this boundary equals the outer radius of the second cell. This figure reveals that this boundary is nearly the same in the variable cell and the six cell model. Since the variable cell model is continually tracking the condensation boundary, it may be giving a more realistic representation of the movement of the dewpoint pressure in the reservoir. In an actual reservoir the dewpoint pressure moves gradually and continually away from the producing well. A constant cell model cannot reproduce this continuous transition, showing instead the condensation boundary only in a series of jumps. For example, as can be seen in Fig. 7, the condensation boundary appears at 20.428 ft after 45 days and remains there until 83 days, at which time it suddenly jumps to 58.395 ft.

Condensation begins as soon as the pressure falls below the dewpoint pressure in the inner cell, after which liquid saturation builds up rapidly. After a period of time the rate of liquid saturation falls off to a very low value, and liquid saturation becomes nearly constant in the inner cell at a value which is a function of the richness of the gas condensate system and the relative permeability of the formation.

Well-stream Composition

As depletion continues, the mole fraction of the methane - the light component - decreases, while the mole fraction of decane - the heavy component - increases in the vicinity of the wellbore. As shown in Fig. 8, a plot of the mole fractions in the first cell as a function of time reveals this fact. However, the changing composition in the first cell is not reflected by a change in the composition of wellstream fluid. Figure 9 shows the wellstream composition.

Effect of Fluid Phase Equilibria Data on Saturation

The simulation results discussed in the preceding section were performed for a lean gas condensate, with a maximum liquid mole percent of 4.75. In this section results are given for a rich gas condensate that has a maximum liquid mole percent of 21.43. The mole fractions of methane, butane and decane in this system are 0.77, 0.20, 0.03, respectively. The mixture has a dewpoint pressure of 2976 psi at a temperature of 160 F. Figure 10 shows the condensation of this rich fluid, as a function of pressure.

We examine the influence of richness on the magnitude of oil saturation in the vicinity of the producing well and on the two phase flow region. To allow comparison with the lean system performance the relative permeability data from Fig. 4 were used. In both cases initial pressure was approximately 600 psi above dewpoint pressure and production rate was maintained constant at 5000 moles per day. Figure 11 compares liquid saturation profiles for the two systems at equal times. These curves reveal that liquid saturation builds up more rapidly in the vicinity of the well and the extent of the two phase region is larger. Also, maximum liquid saturation near the wellbore for the lean system is smaller than for the rich system. Although the rich system has nearly four times the liquid content of the lean system, its stabilized oil saturation is only about 10% greater than that of the lean system.

Comments

Predictions of gas condensate reservoir performance with the variable cell model closely approximate predictions of a regular multicell model. The variable cell model requires less computation time to perform these predictions. Moreover, this significant reduction in computation time is gained with only a slight loss of accuracy. The variable cell model requires less time because a minimum number of cells are used, and because larger time steps are taken as production time increases. In one case time step size increased from a fraction of a day at start of production to more than seven days after 200 days of production. Thus, time step size increased by a factor of 35. For long time forecasting the savings in computation time can be substantial.

CONCLUSIONS

The following conclusions may be drawn from this study:

1. The Variable Cell Model adequately simulates gas condensate performance. Having three cells, the model's results are superior to those from tank type models, but would not, expectedly, reach the reliability of those from compositional models with many computing cells. The Variable Cell Model uses more computer time than a tank type model, but much less time than a larger multicell compositional model. Therefore, the variable cell model bridges the gap between simple tank type models and sophisticated multicell compositional models.

2. Liquid flow begins in the vicinity of the wellbore shortly after the wellbore flowing pressure falls below dewpoint pressure, even if the average reservoir pressure is above dewpoint pressure. For rich gas condensates this flow may begin within a day, but for lean gas condensates several days may pass before liquid flows in the formation.

3. As long as the average reservoir pressure is above the dewpoint pressure, liquid flow into the wellbore from the formation does not appreciably affect the well stream hydrocarbon fluid composition. The latter remains nearly constant and equal to the original gas condensate fluid composition.

4. A nearly constant liquid saturation is established around the wellbore shortly after the wellbore flowing pressure drops below dewpoint pressure. The magnitude of this liquid saturation is influenced by the richness of the gas condensate; however, the maximum liquid saturation formed by a lean system was only 10% lower than the maximum liquid saturation formed by a system four times richer in liquid fraction. The extent of the region of two phase flow is larger for a richer gas condensate, and thus causes greater productivity loss.

NOMENCLATURE

F = error term in material balance equation (moles)
h = thickness (ft)
K_i = equilibrium ratio of component i
K = permeability (md)
L = liquid phase mole fraction
N_j = number of moles in cell j (j=1,2,3)
$N_{3,2}$ = number of moles transferred from second cell to first cell
$N_{3,2}$ = number of moles transferred from third cell to second cell
P = pressure (psi)
q = flow rate (moles/day)
r = radius (ft)
r_e = external radius (ft)
r_s = outer radius of second cell (ft)
r_{sn} = new outer radius of second cell (ft)
r_t = outer radius of first cell (ft)
r_{tn} = new outer radius of first cell (ft)

r_w = wellbore radius (ft)
S = phase saturation (fraction)
V = vapor phase mole fraction
V = hydrocarbon pore volume of cell j (j=1,2,3)
X = mole fraction of component i in liquid phase
Y = mole fraction of component i in vapor phase
Z = overall mole fraction of component i

Greek Symbols

ϵ = error tolerance

ρ = density (moles/cu.ft)

$T = 2\pi kh \left(\frac{k_o}{\mu_o} \rho_o + \frac{k_g}{\mu_g} \rho_g \right)$ flow coefficient (moles/day-psi)

ϕ = porosity (fraction)

Superscripts

l = iteration number
n = old time level
n+1 = new time level

Subscripts

C = critical
g = gas
i = component
o = oil
w = water

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TABLE 1

Fluid Compsotion

Component	Composition
Methane	0.900
Butane	0.080
Decane	0.020

Dew Point Pressure 3410 Psia

TABLE 2

Variable Cell Model Data

Thickness (ft)	20
First cell radius (ft)	10
Second cell radius (ft)	20
Third cell radius (ft)	1364
Well radius (ft)	2.5
Permeability (md)	2.5
Porosity (fraction)	0.15
Water saturation (fraction)	0.20
Temperature (°F)	160
Initial Pressure (psi)	4000
Production rate (moles/day)	5000
Initial Composition (mole fraction)	
Methane	0.900
Butane	0.080
Decane	0.020

TABLE 3

Constant Cell Model Data

Thickness (ft)	20
Six Cell Model	
First cell radius (ft)	7.14
Second cell radius (ft)	20.43
Third cell radius (ft)	58.40
Fourth cell radius (ft)	166.93
Fifth cell radius (ft)	477.17
Sixth cell radius (ft)	1364.00
Three Cell Model	
First cell radius (ft)	20.43
Second cell radius (ft)	166.93
Third cell radius (ft)	1364.00
Well radius (ft)	2.5
Permeability (md)	2.5
Porosity (fraction)	0.15
Water saturation (fraction)	0.2
Temperature (°F)	160
Initial Pressure (psi)	4000
Production rate (moles/day)	5000
Initial Composition (mole fraction)	
Methane	0.900
Butane	0.080
Decane	0.020

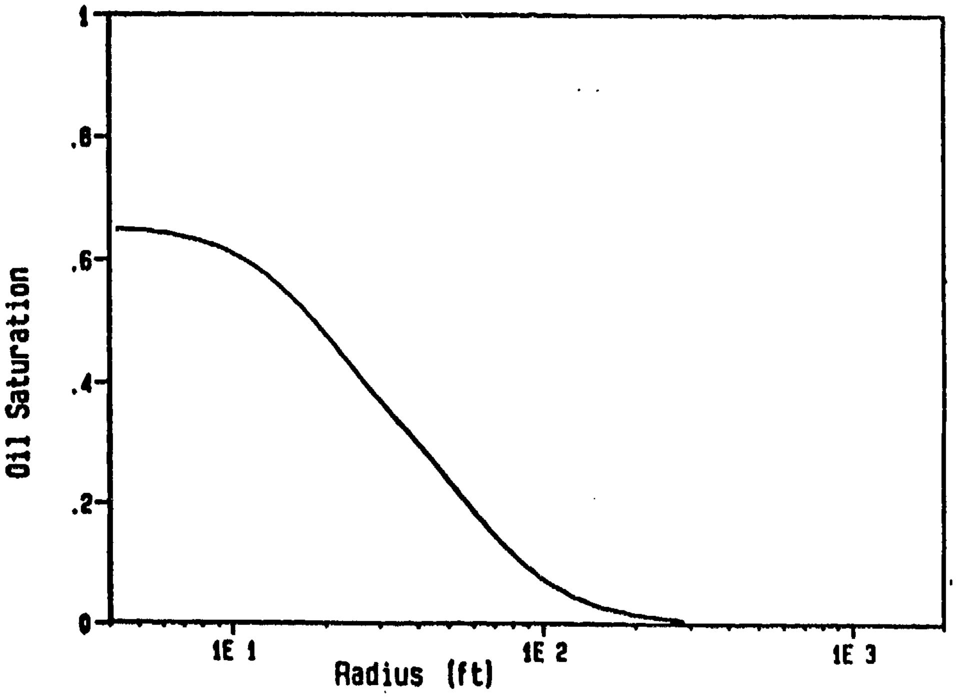

Figure 1
Oil Saturation Profile

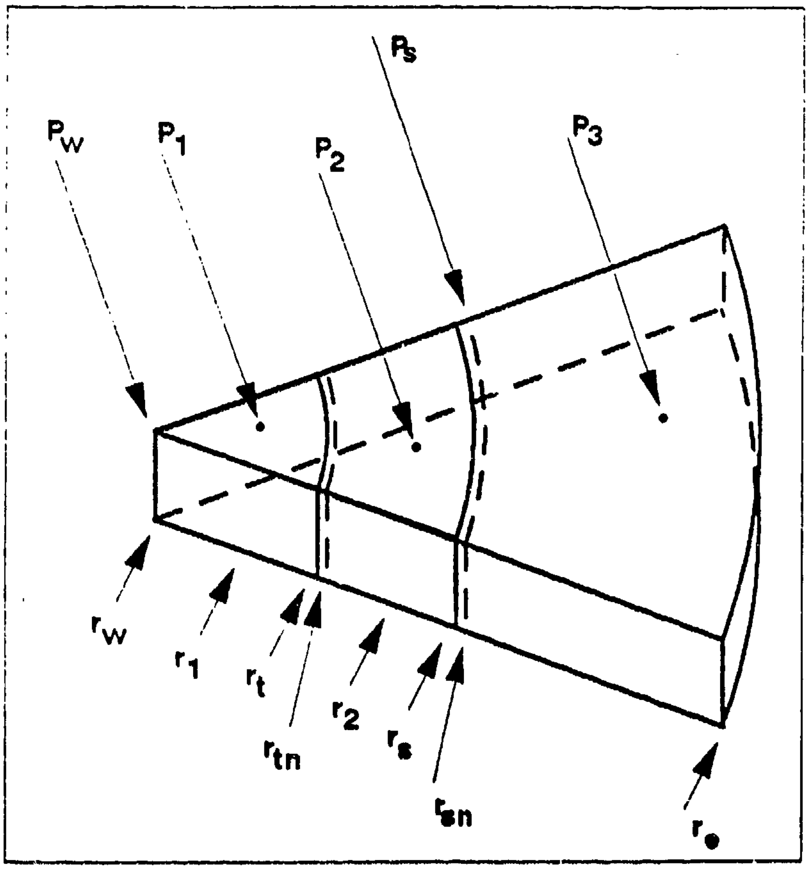

Figure 2
Variable Cell Model Diagram

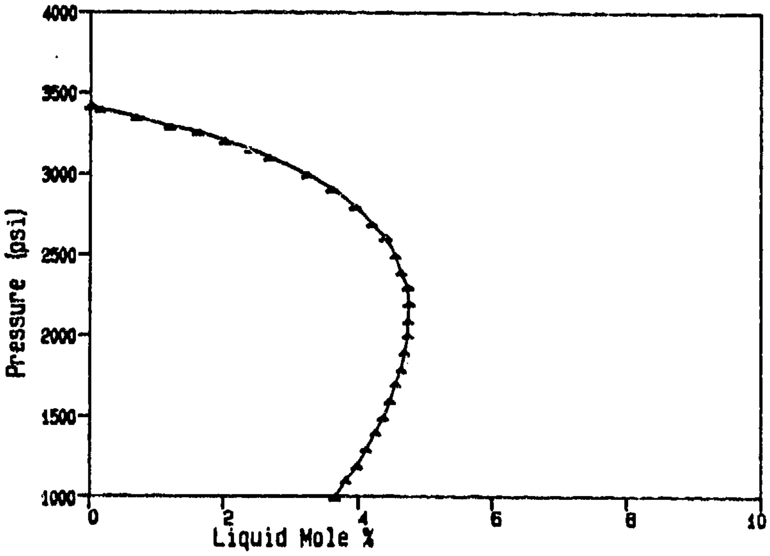

Figure 3
Liquid Mole Percent vs. Pressure

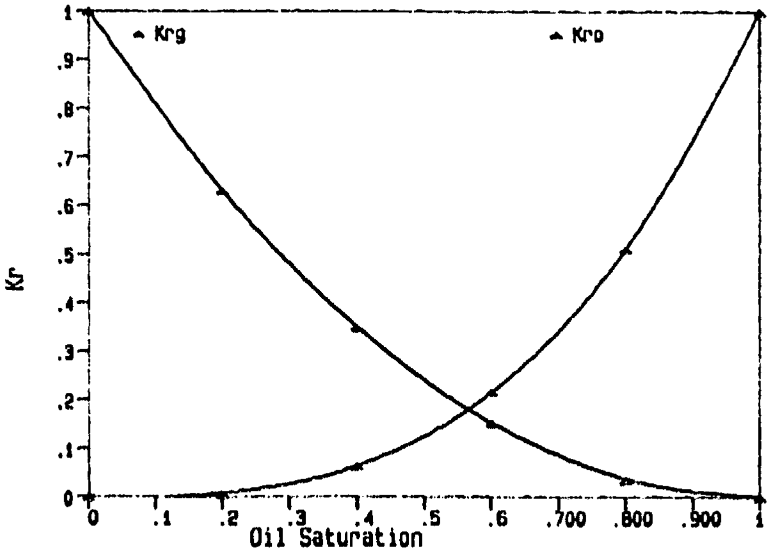

Figure 4
Relative Permeability Data

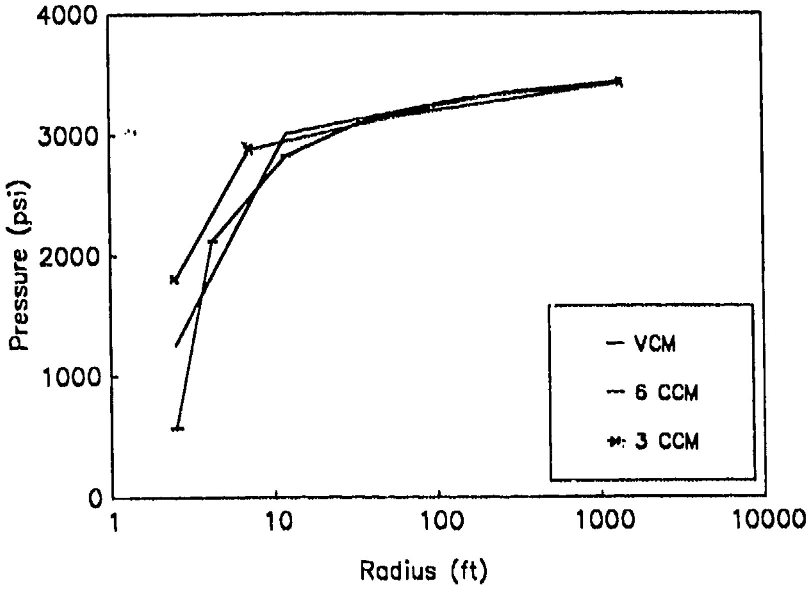

Figure 5

Pressure Profile (190 days)

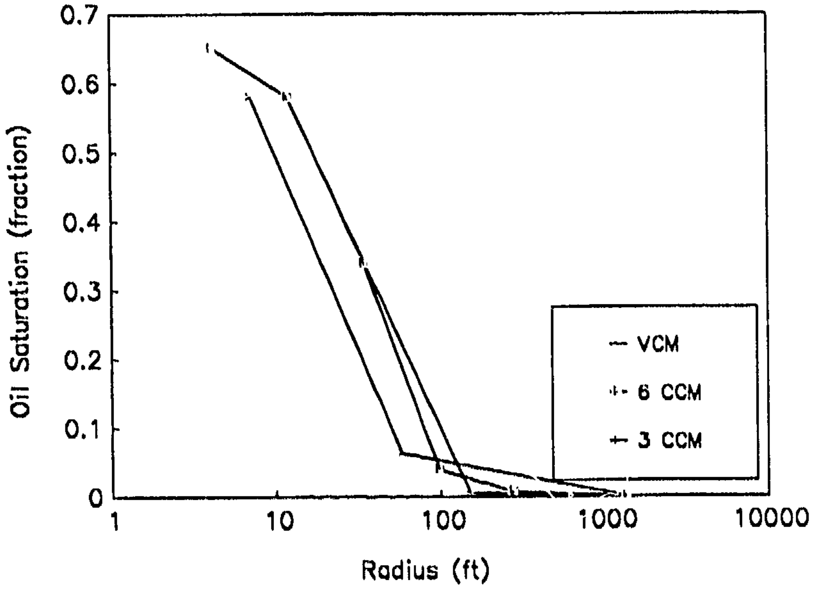

Figure 6

Saturation Profile (190 days)

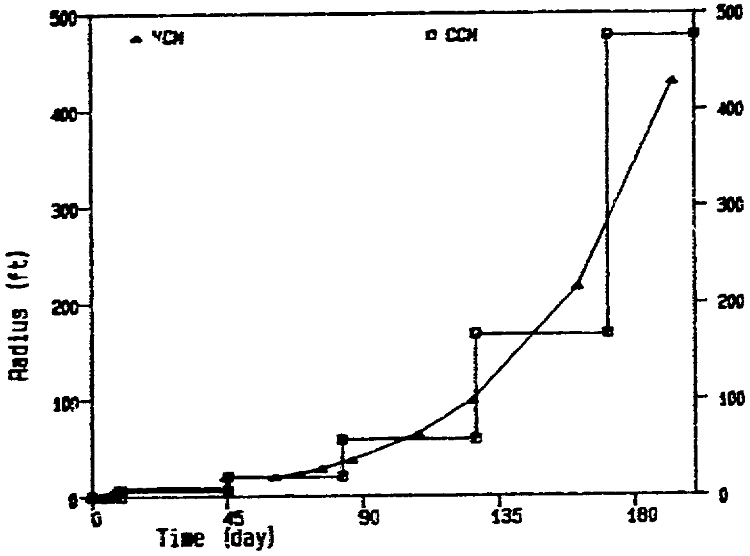

Figure 7

Condensation Radius vs. Time

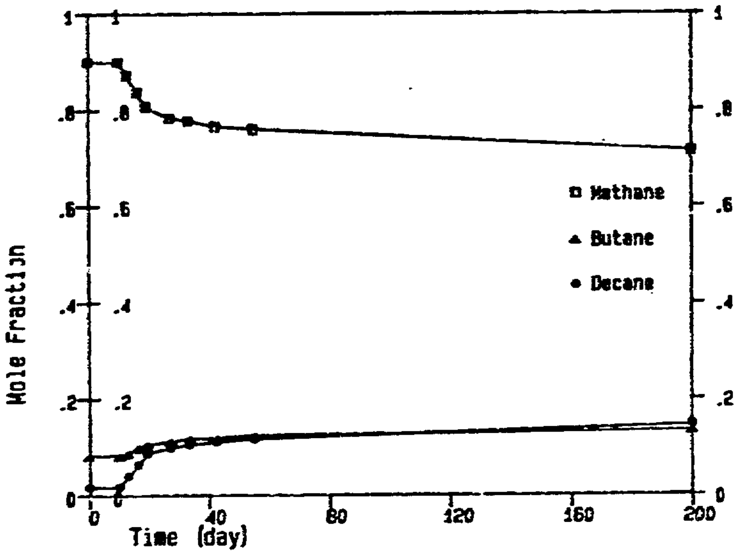

Figure 8

Composition in First Cell vs. Time

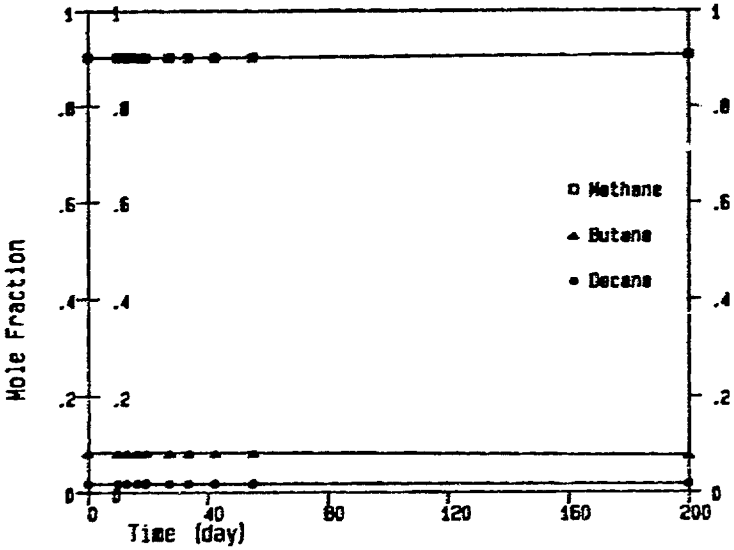

Figure 9

Composition of Well Stream Fluid vs. Time

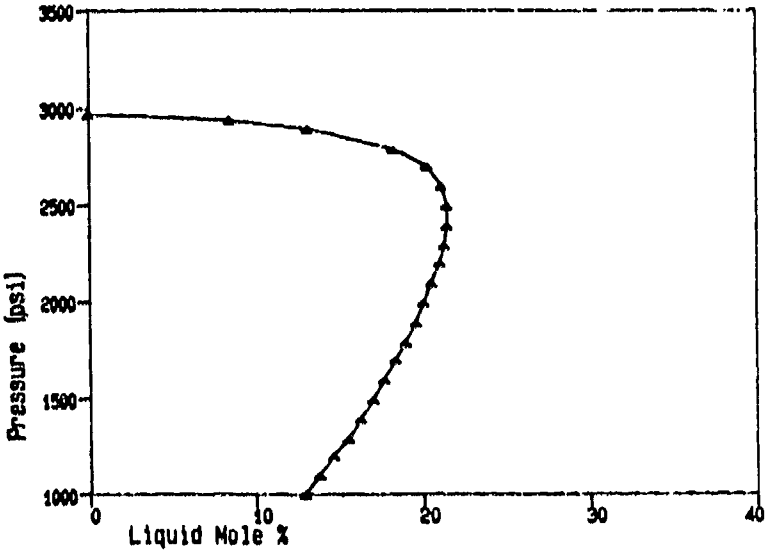

Figure 10
Rich System Liquid Mole Percent vs. Pressure

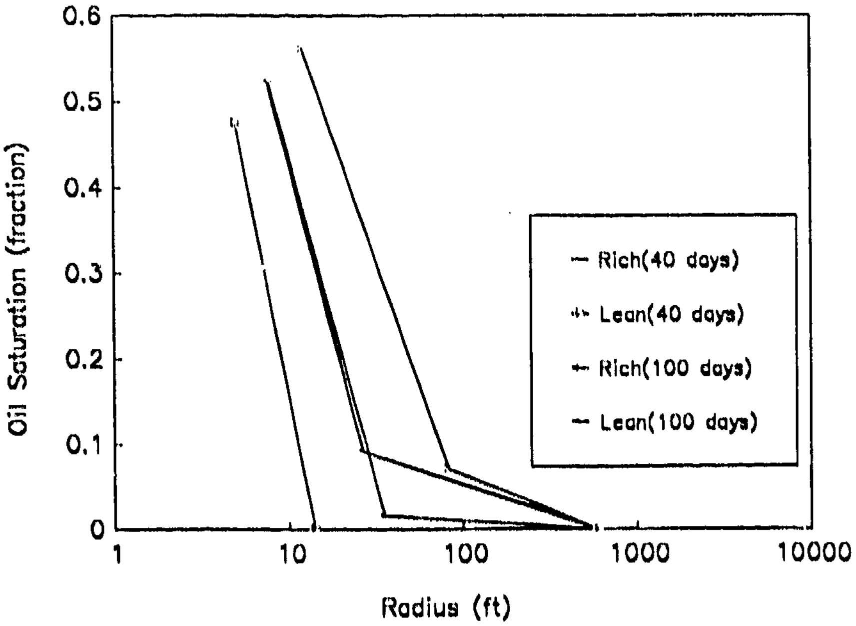

Figure 11
Saturation Profile