Jour. of Engineering
Vol.9, No.2 (1999) 31-39

A CURVILINEAR GRID SYSTEM GENERATOR FOR PATTERN NUMERICAL SIMULATOR

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ABSTRACT

Pattern geometries have been used extensively in Reservoir Engineering, to evaluate sweep efficiency and various improved hydrocarbon recovery processes.

A curvilinear grid is generally preferred to avoid the grid orientation problems associated with rectangular systems as well as to handle anisotropy.

The method presented here calculates a curvilinear grid for a one-eight (1/8) of a five-spot pattern. The symmetry of pattern geometry is used to determine the remaining coordinates.

The conversion of dimensionless coordinates to actual ones is based on L, the half length of the five-spot pattern.

The curvilinear gird points discussed in this paper are located at the intersection of corresponding streamlines and iso-potentials.

The point of intersection is determined numerically. The Newton-Raphson technique is used to solve the system of equations describing the streamline and iso-potential functions. The spacing between iso-potential lines may be equal or logarithmic spacing.

Introduction

The problem of gridding is widely discussed in the open literature. Aziz [1] gave an overview which may be considered as the state of the art at the time of its publication. A more recent statement on girding; albeit a brief one, may be found in Watts paper [2]. Sharpe [3] in a recent publication dealt with the general gridding systems by analogy with radial elliptical and other grids. Other authors [4,5] have recently presented gridding methods based on streamlines that can be used for upscaling or basic reservoir simulation. The present work discusses specifically a special purpose curvilinear grid system generations for pattern injection.

The program presented in this paper calculates (x,y) coordinates of grid points in a curvilinear system. These grid points are the intersection of (M+1) streamlines with (N+1) iso-potential lines.

Morel-Seytoux's [6] equations describing the streamline functions and the equipotential functions at any point are solved using Newton-Raphson technique to determine their point of intersection. The detailed formulations to calculate these points for 1/8 of a 5-spot pattern are presented in the following paragraphs.

Formulation Of The Problem

A Fortran program has been developed to determine curvilinear grid systems for a pattern geometry. More specifically for a one-eight (1/8) of a five-spot pattern.

Given a set of streamlines (ψ 's) and iso-potentials (ϕ 's), grid blocks are determined by the intersection of the streamlines and iso-potentials as shown in Figure 8 of Reference [6].

The problem here is to find the x and y coordinates of each intersection away from discontinuities.

For a repeated five-spot pattern, the real iso-potential function ϕ (x,y), and the streamlines function ψ (x,y) are reported by Morel-Seytoux [6] as :

$$\phi(x,y) = \frac{1}{4\pi} \ln\left(\frac{1 - Cn^2x \; Cn^2y}{Cn^2x + Cn^2y}\right) \tag{1}$$

$$\psi(x,y) = \frac{1}{2\pi} \tan^{-1}\left(\frac{Sny \; Dny \; Cnx}{Snx \; Dnx \; Cny}\right) \tag{2}$$

Figure 1 is based on Figure 8 from Reference [6]. The distance AB indicated in Figure 1 is taken to represent r_w or wellbore radius. The grid system is started at point B to avoid the discontinuity at A. The iso-potential along arc BC is given by Equation (1). The streamline function of each streamtube cutting arc BC can be determined from Equation (2).

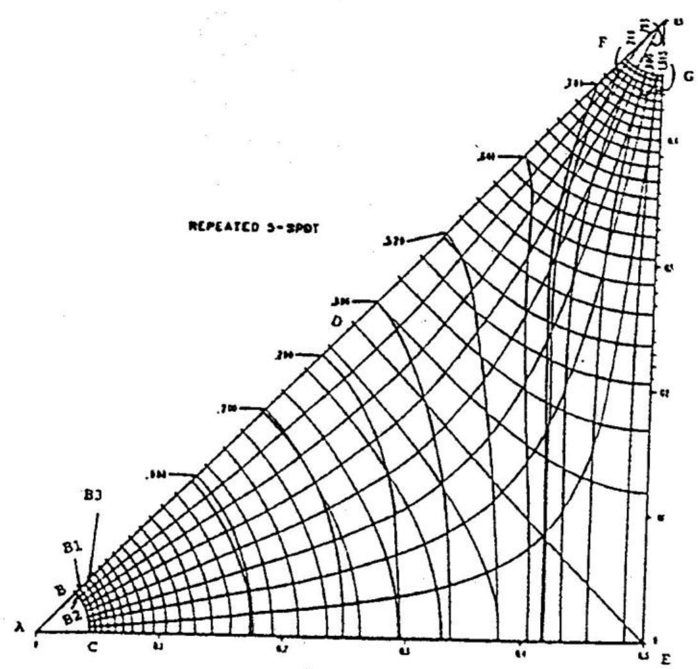

Figure 1. Pattern Geometry Showing Streamlines and Isopotentials

If the x, y coordinates of points B1 and B2, from figure 1, are known then ϕ (x,y) and ψ (x,y) can be calculated from Equations (1) and (2), respectively. The problem here is to find the x, y coordinates of the intersection point B3, that is given B1(x1, y1) and B2(x2, y2) find B3(x3, y3), knowing that ϕ (x1, y1) = ϕ (x3, y3) and ψ (x2, y2) = ψ (x3, y3).

In general, if i is the i'th streamline or streamtube and j the j'th iso-potential line, the coordinates x and y of the intersection between i and j can be determined by solving the following system of equations :

$$\psi(i-1,\ j) = \psi(i,j)$$

$$\phi(i,\ j-1) = \psi(i,j)$$

This system of equations may be solved using the Newton-Raphson technique.

Mathematical Model

Equations (1) and (2) are rewritten such that :

$$F_1(x,\ y) = e^{4\pi\phi(x,\ y)}\ [Cn^2x + Cn^2y] + Cn^2x\ Cn^2y-1 \qquad (3)$$

$$F_2(x,y) = \tan\ [2\pi\ \psi(x,y)]\ Snx\ Dnx\ Cny - Sny\ Dny\ Cnx \qquad (4)$$

Let $K_1 = e^{4\pi\phi(x,\ y)}$ and $K_2 = \tan\ [2\pi\ \psi(x,\ y)]$ for convenience.

Let R(x) be the residual vector where

$X = (x,y)$ and $R(X) = (F_1(x,y),\ F_2\ (x,y))$

The solution to R(X) such that :

$$R(X) = 0 \qquad (5)$$

Is obtained in two steps :

1) $$X^{k+1} = X^k - \frac{R(X^k)}{R'(X^k)}$$

Here X^k is the value of X at the k'th iteration level and R'(X) is the Jacobian Matrix that is :

$$R'(X^k) = \frac{\partial R(X^k)}{\partial X^k} \qquad (6)$$

$$\text{Let } \delta X = X^{k+1} - X^k = -\frac{R(X^k)}{R'(X^k)} \qquad (7)$$

or

$$R(X^k) = \delta X R'(X^k) \qquad (8)$$

2) The second step of this procedure is to solve the linear system of equations depicted by Equation (8) :

$$\begin{bmatrix} \frac{\partial F_1(x,y)}{\partial x} & \frac{\partial F_1(x,y)}{\partial y} \\ \\ \frac{\partial F_2(x,y)}{\partial x} & \frac{\partial F_2(x,y)}{\partial y} \end{bmatrix} \times \begin{bmatrix} \Delta x \\ \\ \Delta y \end{bmatrix} = \begin{bmatrix} -F_1(x,y) \\ \\ -F_2(x,y) \end{bmatrix} \tag{9}$$

The analytical derivatives of $F_1(x, y)$ and $F_2(x, y)$ with respect to x and y are given below :

$$\frac{\partial F_1(x,y)}{\partial x} = -2\left(K_1 + Cn^2 y\right) \text{ Cnx Snx Dnx} \tag{10}$$

$$\frac{\partial F_1(x,y)}{\partial y} = -2\left(K_1 + Cn^2 x\right) \text{ Cny Sny Dny} \tag{11}$$

$$\frac{\partial F_2(x,y)}{\partial x} = K_2 \text{ Cny Cnx } \left(Dn^2 x - K_2 \; Sn^2 x\right) \tag{12}$$

$$\frac{\partial F_2(x,y)}{\partial y} = K_2 \text{ Snx Dnx Sny Dny} + \text{Cny Cnx } \left(K_2 \; Sn^2 y - Dn^2 y\right) \tag{13}$$

From Equation (9), we have :

$$\frac{\partial F_1(x,y)}{\partial x}\Delta x + \frac{\partial F_1(x,y)}{\partial y}\Delta y = -F_1(x,y) \tag{14}$$

$$\frac{\partial F_2(x,y)}{\partial x}\Delta x + \frac{\partial F_2(x,y)}{\partial y}\Delta y = -F_2(x,y) \tag{15}$$

After multiplying Equation (14) by $\frac{\partial F_2(x,y)}{\partial x}$ and Equation (15) by $\frac{\partial F_1(x,y)}{\partial x}$ and simplifying one can solve for Δy first as follows :

$$\Delta y = \frac{\left[\frac{\partial F_1(x,y)}{\partial x} . F_2(x,y) - \frac{\partial F_2(x,y)}{\partial x} . F_1(x,y)\right]}{\left[\frac{\partial F_2(x,y)}{\partial x} . \frac{\partial F_1(x,y)}{\partial y} - \frac{\partial F_2(x,y)}{\partial y} . \frac{\partial F_1(x,y)}{\partial x}\right]}$$

Δx is then computed from either Equation (14) or (15). For example :

$$\Delta x = \frac{\left[-F_2(x,y) - \frac{\partial F_2(x,y)}{\partial y} \Delta y \right]}{\frac{\partial F_2(x,y)}{\partial x}}$$

The program checks whether $\frac{\partial F_1(x,y)}{\partial x}$ is zero before using Equation (14).

The overall solution procedure is now outlined :

(i) An initial guess is made for the unknowns x and y.

(ii) A check is made to see if R(X) the residual vector is smaller than a specified tolerance.

(iii) R'(X) and R(X) are computed as a function of the primary unknowns given in the first step above. Then, Equation (8) is solved for δx.

(iv) Given δx update X such that $X^{k+1} = X^k + \delta X$ and record tha maximum difference that is :

$$|X^{k+1} - X^k|_{max} \tag{16}$$

Check whether Equation (16) is smaller than a specified tolerance. If so, convergence has been achieved and the computation is terminated. If not, one goes to the next step.

(v) X^{k+1} is now the new guess and the procedure is restarted from the first step.

Discussions And Results

The present program has been written in single precision. The accuracy under these conditions is considered sufficient.

The convergence criteria utilized are :

(1) Both $F_1(x, y)$ and $F_2(x,y)$ are less than or equal to 10^{-6} and

(2) $|X^{k+1} - X^k|_{max} \leq 10^{-5}$

Convergence is generally achieved in five to six iterations as long as one stays away from the discontinuities at A and DE as shown in Figure 1.

There are two choices for delineating the iso-potential lines along BD (Fig. 1)

IOPT = 1 is for an equal spacing while,

IOPT = 2 is for a logarithmic spacing allowing more grid cells near the wellbore as shown in Figure 1.

The program computes the intersections coordinates in the area BCDE.

The symmetry of pattern geometry is utilized to determine the coordinates of the intersections in area DEFG shown in Figure 1.

The only data required to run this program are :

M = number of grid cells between streamtubes.

N = number of grid cells between iso-potential lines.

IOPT = 2 for a logarithmic spacing between iso-potential lines.

IOPT ≠ 2 for equal spacing between iso-potential lines.

L = actual length of AE in Figure 1. Note that $(2L)^2$ is the area of the five-spot pattern.

The results of the Fortran Program are given in Table 1. Figure 2 is a plot of these data.

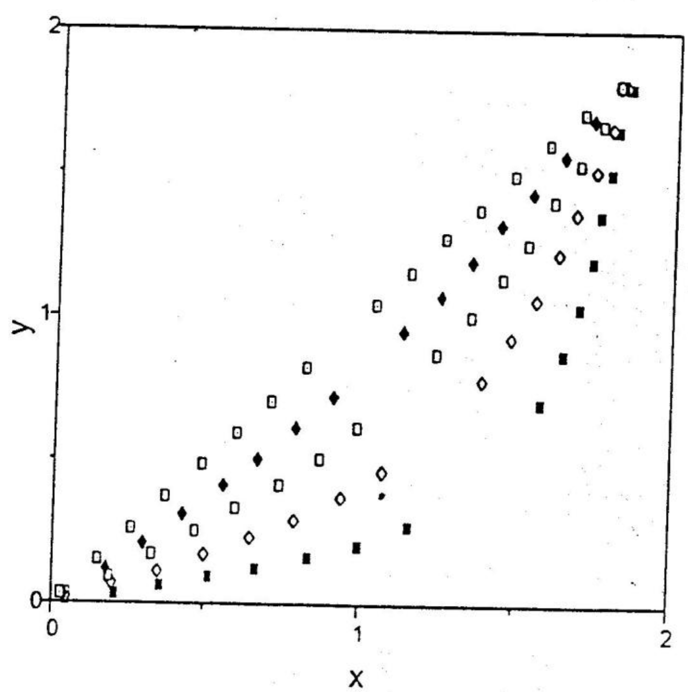

Figure 2. Intersections of Streamlines with Isoptential Lines.

Table 1 : Results of Frotran Program

I	J	X	Y	I	J	X	Y	I	J	X	Y
1	1	0.0382	0.0382	3	1	0.0482	0.0245	5	1	0.0534	0.0085
1	2	0.1493	0.1493	3	2	01879	0.0963	5	3	0.2084	0.0333
1	3	0.2604	0.2604	3	3	0.3266	0.1696	5	3	0.3627	0.0592
1	4	0.3715	0.3715	3	4	0.4638	0.2456	5	4	0.5164	0.0872
1	5	0.4826	0.4826	3	5	0.5990	0.3257	5	5	0.6699	0.1187
1	6	0.5937	0.5937	3	6	0.7318	0.4116	5	6	0.8250	0.1561
1	7	0.4048	0.7048	3	7	0.8620	0.5050	5	7	0.9841	0.2038
1	8	0.8159	0.8159	3	8	0.9888	0.6076	5	8	1.1513	0.2710
1	9	1.0381	1.0381	3	9	1.2465	0.8653	5	9	1.5831	0.7027
1	10	1.1492	1.1492	3	10	1.3490	0.9921	5	10	1.6503	0.8699
1	11	1.2603	1.2603	3	11	1.4424	1.1223	5	11	1.6980	1.0291
1	12	1.3714	1.3714	3	12	1.5283	1.2551	5	12	1.7354	1.1841
1	13	1.4825	1.4825	3	13	1.6085	1.3903	5	13	1.7669	1.3377
1	14	1.5936	1.5936	3	14	1.6845	1.5274	5	14	1.7948	1.4914
1	15	1.7047	1.7047	3	15	1.7578	1.6662	5	15	1.8208	1.6457
1	16	1.8158	1.8158	3	16	1.8295	1.8059	5	16	1.8456	1.8007
2	1	0.0437	0.0318	4	1	0.0514	0.0167	6	1	0.0541	0.000
2	2	0.1707	0.1244	4	2	0.2006	0.0657	6	2	0.2110	0.000
2	3	0.2969	0.2178	4	3	0.3489	0.1163	6	3	0.3674	0.000
2	4	0.4220	0.3128	4	4	0.4959	0.1700	6	4	0.5235	0.000
2	5	0.5456	0.4098	4	5	0.6415	0.2287	6	5	0.6801	0.000
2	6	0.6673	0.5097	4	6	0.7861	0.2949	6	6	0.8396	0.000
2	7	0.7868	0.6128	4	7	0.9300	0.3724	6	7	1.0067	0.000
2	8	0.9040	0.7197	4	8	1.0728	0.4661	6	8	1.1914	0.000
2	9	1.1344	0.9501	4	9	1.3880	0.7813	6	9	1.8541	0.6626
2	10	1.2412	1.0672	4	10	1.4817	0.9241	6	10	1.8541	0.8474
2	11	1.3444	1.1868	4	11	1.5592	1.0680	6	11	1.8541	1.0145
2	12	1.4442	1.3085	4	12	1.6254	1.2125	6	12	1.8541	1.1740
2	13	1.5413	1.4321	4	13	1.6840	1.3582	6	13	1.8541	1.3306
2	14	1.6362	1.5572	4	14	1.7377	1.5052	6	14	1.8541	1.4866
2	15	1.7297	1.6834	4	15	1.7884	1.6535	6	15	1.8541	1.5431
2	16	1.8223	1.8103	4	16	1.8374	1.8026	6	16	1.8541	1.8000

Conclusions

This paper presented an algorithm to generate curvilinear grid systems for a pattern geometry.

The spacing between grid cells can be logarithmic as well as equal. The logarithmic option allows more grid blocks where it is needed that is around wellbores.

Test results show that fast convergence is achieved under minimum constraints that is with single precision and a fairly large tolerance.

The flexibility of this approach is reflected by the reduced amount of input data required, essentially the length of the pattern geometry and the number of grid cells required between streamtubes and iso-potentials.

Nomenclature

ϕ (x,y) = real potential function

ψ (x,y) = streamline function

Cn = elliptic cosine

Dn = elementary Jacobian elliptic function

Sn = elliptic sine

Acknowledgements

The author wishes to thank Kind Fahd University of Petroleum and Minerals for its support and Dr. A. S. Harouaka for his valuable comments.

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