Efficiency, size, benchmarks and targets for bank branches: an application of data envelopment analysis

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This paper describes an application of data envelopment analysis (DEA) to the performance assessment of Portuguese bank branches. The analysis shows how DEA can complement the profitability measure currently used at the bank. The use of an efficiency-profitability matrix enabled the characterisation of the branches’ performance profile. Consistent with the bank’s development objectives, the analysis focused on the relation between branch size and performance. Two alternative target setting strategies were explored. One eliminates pure technical inefficiencies by focusing on the selection of appropriate benchmarks. The other attains the branches’ most productive scale size through the elimination of scale inefficiencies, with minimal changes to branches’ scale size.

Keywords: data envelopment analysis; banking; efficiency; benchmarks; size; target setting

Introduction

The purpose of this paper is to describe an application of data envelopment analysis (DEA) to the performance assessment of branches of a Portuguese bank. This bank currently uses two different methods to analyse the performance of its branches. The first, called system of incentives and motivation (SIM), focuses on the branches’ volume of business. Every four months, each branch is assigned business volume objectives. These objectives are specified for each of the products and services considered of strategic importance by senior management, such as current and savings deposits, investment funds, credits or insurances. The SIM proved to be a valuable tool for establishing a clear incentive scheme for branch staff. The staff of the best performing branches receive a monetary bonus. The second more recent method is called earning analysis system (EAS) and determines the profitability at the branch level. EAS information provides the bank with a clearer picture of the key costs and revenues for branch activities. However, both the SIM and EAS have limitations as methods for performance assessment. SIM is particularly valuable as a method to stimulate staff, but does not take into account the costs of delivering services. Also, setting targets for sales motivation or bonus is different to planning a branch network for efficient operation. In contrast, profitability measures alone, such as EAS, ignore the branches’ efficiency and potential for improved performance. Also, they are difficult to explain and be accepted by branch staff.

The primary aim of the study was to develop an enhanced performance measurement method to assess and improve branches’ performance. The use of an efficiency-profitability matrix enabled the characterisation of the branches’ performance profile. Profitability was not included directly in the DEA model as an output because is should be seen as a different dimension from efficiency issues. The DEA results can be used alongside the EAS profitability measure to achieve an overall picture of network performance. This information can also enhance the SIM method for setting business volume objectives, as the DEA measure has the advantage of simultaneously taking into account both the volume of business and the corresponding resource consumption.

The bank’s recent strategy has focused on growth of business levels, through the acquisition of other financial institutions. This has required rationalisation of existing branches and redeployment of surplus staff to new ones. The general policy has been to open small branches with four members of staff. The relation between branch size and performance was explored to identify the optimal scale size for the existing and new branches as well as to obtain some insights on whether merging with branches from the recently acquired networks could be beneficial. The study also focused on the development of target setting strategies to increase overall efficiency levels. The selection of appropriate benchmarks was given particular attention, as well as the identification of the scale size that maximises the branches’ productivity for a given mix of products.

This paper is structured as follows: The following section outlines the DEA models and the alternative methods for testing a branch’s returns to scale. This is followed by a review of previous studies on the efficiency of bank branches. Then a case study of the branch network is
described, focusing on the choice of inputs and outputs, the integration of efficiency and profitability measures, and the effect of branch size on efficiency and target setting. Conclusions and future implementations of the results are discussed in the final section.

Background on DEA

DEA is a linear programming based technique for measuring the relative efficiency of a fairly homogeneous set of decision making units (DMUs) in their use of multiple inputs to produce multiple outputs. It identifies a subset of efficient ‘best practice’ DMUs and for the remaining DMUs, the magnitude of their inefficiency can be determined by comparison to a frontier constructed from the ‘best practices’. DEA derives a single summary measure of efficiency for each DMU. For the inefficient DMUs, DEA derives efficient input and output targets and a reference set (or peer group), corresponding to the subset of efficient DMUs to which they were directly compared. The genesis of the DEA approach lies in the work by Farrell.1 Based on Farrell’s work, the DEA model was operationalised and popularised by Charnes et al.2 The input oriented linear programming version of the model introduced by Charnes et al.2 can be formulated as follows:

\[
\min \theta_0 = e \sum_{j=1}^{n} s_j^{-} - e \sum_{i=1}^{m} s_i^{-} \tag{1}
\]

subject to

\[
x_{ij} \theta_0 - \sum_{j=1}^{n} x_{ij} \lambda_j - s_i^{-} = 0, \quad i = 1, \ldots, m,
\]

\[
\sum_{j=1}^{n} y_{ij} \lambda_j - s_i^{+} = y_{i0}, \quad r = 1, \ldots, t,
\]

\[
\lambda_j, s_i^{-}, s_i^{+} \geq 0, \quad \forall j, r \text{ and } i.
\]

where DMU0 is the DMU being evaluated in the set of \( j = 1, \ldots, n \) DMUs and \( x_{ij} \) and \( y_{ij} \) denote the observed level of the \( j \)th input and \( r \)th output at DMU \( j \). The value of \( \theta_0 \) is a measure of the technical efficiency (TE) of DMU0, which assumes the existence of constant returns to scale (CRS). However, some of the inefficiency detected using this model may be attributable to scale effects, which occur when operating at variable returns to scale (VRS). Banker et al.3 extended the original DEA model (1) to account for the existence of VRS. The VRS model can be obtained through the addition of a convexity constraint to model (1) requiring that the multipliers \( \lambda_j \) add up to 1. The scale efficiency (SE) of a DMU is obtained as the ratio of its technical efficiency (TE, assuming CRS) to its pure technical efficiency (PTE, assuming VRS).

The nature of returns to scale has been widely studied in DEA and three main methods for its estimation have been proposed. The first method, proposed by Banker et al.3 is based on the sign of the dual variable \( (w_0) \), or shadow price, of the convexity constraint on the VRS model. The second method, proposed by Banker4 is based on summing the optimal values of the multipliers \( \lambda_j \) on a CRS model. Both methods assume unique optimal solutions to the DEA models. Banker and Thrall5 generalised these methods to deal with multiple optimal solutions. These methods require either the estimation of all alternative values of \( w_0 \) (on the primal VRS model) or the sum of \( \lambda_j \) in all alternative optimal solutions (on the dual CRS model). Banker et al.6 and Banker et al.7 proposed simplified methods for testing the type of returns to scale, which avoid determining all alternative optimal solutions to the primal and dual models, respectively. The third method, proposed by Fare et al.8 requires solving three DEA models with CRS, VRS and non-increasing returns to scale (NIRS). For scale inefficient DMUs, the nature of returns to scale can be obtained by comparing the efficiency measure derived from a NIRS technology and a VRS technology. This method is not affected by the existence of multiple optimal solutions. Banker et al.9 prove the equivalence of all methods. This study is based on the second method for the analysis of all issues related to branches’ returns to scale.

Review of bank branch efficiency

During the late 1980s and particularly in the 1990s, the DEA method has been used extensively to evaluate banking institutions. However, only a few studies focused on the analysis of bank branches within the same financial institution. Published applications of DEA to bank branches are summarised in table format in the Appendix 1. This table contains information about the authors and year of publication, the inputs and outputs, the orientation and efficiency measure computed, the methods used and the country and sample size. For a comprehensive review of efficiency in financial institutions see Berger and Humphrey.9

This study extends the literature on bank branches’ efficiency by considering how DEA can complement conventional financial approaches to performance assessment, such as profitability. Also, it provides a detailed analysis of the impact of scale size on banking business. Despite being well documented at the bank level, there is only a limited number of studies focusing on branches’ optimal scale size.

The relation between efficiency and profits was first analysed by Oral and Yolalan10 and Oral et al.11 They defined two DEA models for analysing both efficiency and profitability. The profitability model consisted of a disaggregation of expenses and income, which were considered as inputs and outputs, respectively. However, the implications of using this model instead of a usual profitability measure such as profitability ratio were not discussed. Drake and Howcroft12 correlated the DEA technical efficiency score with cost-income ratios. The results indicated that more
efficient branches had lower cost-income ratios. The study by Schaffnit et al\textsuperscript{13} also concluded that the branch’s efficiency has a very clear positive effect on profit.

In relation to the association between branches’ size and efficiency, Giokas\textsuperscript{14} was the first study to use a VRS model. The average scale efficiency found was high, and the majority of branches had increasing returns to scale (IRS). However, the existence of multiple optimal solutions to the DEA models was not accounted for. The results of Drake and Howcroft\textsuperscript{12} were very similar, that is the scale efficiency detected was high and most branches had IRS. The optimal branch size, based on scale efficiency, appeared to be nine members of staff. Although Tulkens\textsuperscript{15} also used VRS models, the purpose of the study was a comparison of DEA and Free Disposal Hull (FDH) models, and the branches scale efficiency was not examined in detail. The more recent study by Schaffnit et al\textsuperscript{13} found a different picture of branches’ returns to scale: the majority operated under CRS, and of the remaining branches most had DRS. The study by Athanassopoulos\textsuperscript{16} found substantially different returns to scale characteristics according to branches’ profile.

Application of DEA—Input and output measures

The correct definition of the inputs and outputs for bank branches is not straightforward and controversy remains in the literature, giving rise to alternative approaches. Most banking studies have tended to adopt either the ‘production’ or the ‘intermediation’ approach.

The production approach emphasises the commercial activities at the branch, where they act as services providers for account holders. The output is represented by loans, savings and account activity as measured by the number of transactions processed. It is common to group these transactions according to the type, complexity or function, which helps the interpretation of the results obtained. The inputs considered are physical inputs such as capital and labour. Interest costs and revenue are excluded from this approach since only physical inputs are needed to perform transactions or provide other types of services.

Under the intermediation approach, financial institutions are thought of as primarily intermediating funds between savers and investors. The inputs and outputs are measured in monetary units. The inputs include both interest and non-interest costs. The outputs are measured by the total balance or revenue of loans and investments. Deposits may be either treated as inputs or outputs, depending on the objectives of the analysis (for a discussion of this topic see Colwell and Davis\textsuperscript{17}).

As the primary aim of this efficiency assessment was the development of a methodology for the evaluation and improvement of branches’ commercial activity, the analysis adopted the structure of the production approach. The empirical results were derived from the analysis of 168 bank branches in 1996. These branches deal with individuals and small business accounts and their activities are reasonably homogeneous. They were scattered across the country, although the two main Portuguese cities have a higher concentration of branches. Table 1 shows the summary statistics of the sample. The inputs and outputs are measured as follows:

**Inputs:**
- Number of employees in the branch.
- Floor space of the branch (in m\textsuperscript{2}).
- Operational costs (costs of supplies and other services, in thousand escudos).
- Number of external ATMs.

**Outputs:**
- Number of general service transactions performed by branch staff.
- Number of transactions in external ATMs.
- Number of all types of accounts at the branch.
- Value of savings (in thousand escudos).
- Value of loans (in thousand escudos).

Although most previous studies on branches’ efficiency included in the output set only the number of the various types of transactions processed, we chose to use a more comprehensive output set which allows for the characteristics of a particular branch’s activities. The number of

<table>
<thead>
<tr>
<th>Inputs/outputs</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of employees</td>
<td>9.1</td>
<td>8.9</td>
<td>3.9</td>
<td>2.8</td>
<td>21</td>
</tr>
<tr>
<td>Floor space</td>
<td>276</td>
<td>232</td>
<td>165</td>
<td>55</td>
<td>966</td>
</tr>
<tr>
<td>Operational costs</td>
<td>16.2</td>
<td>14.6</td>
<td>9.2</td>
<td>5</td>
<td>105</td>
</tr>
<tr>
<td>No. external ATMs</td>
<td>1.1</td>
<td>1.0</td>
<td>0.4</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>No. transactions done by staff</td>
<td>85,549</td>
<td>77,136</td>
<td>43,359</td>
<td>14,416</td>
<td>212,566</td>
</tr>
<tr>
<td>No. transactions in ext. ATMs</td>
<td>74,109</td>
<td>64,854</td>
<td>48,509</td>
<td>0</td>
<td>373,176</td>
</tr>
<tr>
<td>No. accounts</td>
<td>6,044</td>
<td>5,198</td>
<td>4,103</td>
<td>379</td>
<td>21,619</td>
</tr>
<tr>
<td>Value of savings</td>
<td>3,279</td>
<td>2,331</td>
<td>3,003</td>
<td>166</td>
<td>19,080</td>
</tr>
<tr>
<td>Value of loans</td>
<td>1,115</td>
<td>899</td>
<td>1,003</td>
<td>23</td>
<td>7,105</td>
</tr>
</tbody>
</table>
transactions performed by branch staff includes an aggregation of 20 different types of general service transactions which most commonly occur at the branches, for example, cheques processed, purchase of foreign currency, etc. The number of transactions in ATMs represents the volume of services provided through automated means. Instead of including a direct measure of the number of more complex transactions, for example, number of loans negotiated, pension funds set up, etc, stock measures that reflect the relevant outcomes of these transactions were included. These variables indirectly reflect the ability to attract and maintain a large customer base, measured by the number of accounts of all types in the branch, and the ability to attract and maintain high values of savings and loans, measured by their balances. By including these measures of total balances, the efficiency concept incorporates some features of the intermediation approach, broadening the scope of the efficiency assessment.

Note that neither technology within the branch nor the state of its premises has been considered in the model, as the equipment and branch image is fairly homogenous in this network. The business potential of branches’ location, which depends on competition and socio-economic conditions within the catchment area, was not included in the analysis due to data unavailability. However, some input may be necessary for attracting customers in difficult locations and therefore the model may underestimate the efficiency of branches located in such areas. This issue is addressed in the following section.

Efficiency and profitability

The efficiency analysis used input oriented models, consistent with the expressed managerial aim of attaining efficiency through a rationalisation of resources at existing branches. The results of the DEA analysis, using model (1), indicated that the average technical efficiency for the entire sample is 78% and that only 34 branches (20%) are operating efficiently, that is, there is scope for efficiency improvements in 134 branches. There is also a significant efficiency spread, for example, the standard deviation is 18.7% and some branches have rather low efficiency values (close to 20%).

In order to obtain an enhanced picture of branches’ performance, the relation between the DEA efficiency measure and the EAS profitability measure currently used in the bank was explored. The joint use of the efficiency and profitability measures can highlight the potential performance improvements that management might be able to exploit, leading to higher profits. This analysis is based on the ‘efficiency-profitability matrix’ proposed by Dyson et al. and Boussofiane et al. As the efficiency measure is an index and the profitability measure used in the bank is scale dependent, it is necessary to convert the profitability measure into an index so that larger branches are not favoured in the profit dimension of the matrix. The profit index is obtained dividing the profit before indirect costs by the total costs (interest and non-interest costs). The efficiency-profitability matrix is shown in Figure 1.

The efficiency profitability matrix was divided in four quadrants, where different profiles of branches are likely to exist. The precise boundary positions between quadrants is subjective. What is apparent, however, is that no matter where the boundaries are drawn, some branches which score well on efficiency have low profitability. This is despite the more general trend that higher efficiency is associated with higher profitability. Clearly, judging the performance of the branches on the basis of profitability alone would overlook some branches with high efficiencies. In fact, 100% efficient branches can be found across a wide range of profits, which indicates that high profitability is not exclusively related to high efficiency. As 65% of the branches are located in the ‘sleeper’ and ‘?’ quadrants, it is concluded that most branches have potential for profitability increase through efficiency improvements.

Branches located in the ‘star’ quadrant provide benchmarks for the network. Any improvement in efficiency will reinforce their viability, but the scope for profitability improvement may be limited, as there is no empirical evidence indicating that the operating practices of these branches can be substantially improved. Any benchmarking practices used to indicate potential for improved performance must be looked for in other bank branch networks.

Branches in the ‘dog’ quadrant are operating efficiently but are relatively low on profitability. This may be due to an unfavourable environment, in which case their viability should be questioned, as the branches’ profit may be critically affected by the presence of competition and low business potential in the catchment area. The reasons for this low profitability should be carefully studied to see whether some improvement to the profitability levels is possible through the adoption of a different product mix.

Branches located in the ‘question mark’ quadrant have the potential for both greater efficiency and profitability. Some of these branches may be able to improve their efficiency as an attempt to move towards the ‘star’ quadrant.

![Figure 1 Efficiency-profitability matrix.](image)
in order also to attain high profits. Others may be suffering from an unfavourable environment not captured in the DEA model. The branches should be evaluated on a case by case basis taking account of location and environment to identify those with the potential for improvement.

The ‘sleepers’ are profitable, yet inefficient. Their profitability is likely to be a consequence of favourable environment rather than good management. They should be prime candidates for an efficiency improvement effort leading to greater profits. The high value of the profit index of the branch isolated on the top of the ‘sleeper’ quadrant is due to a very high income received from commissions. This highlights the importance of using more than one performance measure in order to obtain a better assessment of the branch network, as it is not possible to account for all aspects of branches’ performance using a single performance indicator. The sole use of the profitability measure would identify this branch as a benchmark although from the analysis of the efficiency-profitability matrix it becomes evident that some performance enhancements can still be achieved through efficiency improvements.

Efficiency levels and branches’ size

As the major concern of the bank is the efficient use of resources, the measure of branches’ size should be the variable that best reflects the resources level of the branches. As the personnel costs account for approximately 75% of the total operational costs, the variable ‘number of employees’ used in the DEA model is considered the most appropriate measure of branches’ size. Figure 2 shows the relation between technical efficiency and size, and Figure 3 shows the relation between the profit index and size. Each of the four size groups considered contains approximately 40 branches.

Figure 3 shows that larger branches have higher efficiency and profits, which can be an indication of the existence of economies of scale in branches’ activities. These graphs prompted a detailed study of the returns to scale properties of branches’ activity.

It was, a priori, unclear if branches’ activities demonstrated constant or variable returns to scale. Banker has proposed hypothesis tests for determining the type of returns to scale of the DMUs’ activity. If the efficiency distributions obtained using the CRS and VRS models are similar, it means that scale inefficiency is almost non-existent, and thus there is not enough evidence to support the hypothesis that the DMUs’ activity exhibit VRS. In these cases, the differences in the shape of the production frontier using CRS and VRS models may be due to random variations and not to the intrinsic VRS properties of DMUs’ activities. The existence of VRS in branches’ activities was formally tested using the Kolmogorov–Smirnov test. The null hypothesis was rejected, at a 5% significance level, which indicates that the branches operate under variable returns to scale.

In order to explore the relation between pure technical efficiency and scale efficiency with branch size, both input and output orientations of the efficiency assessment were examined. Under VRS, input and output oriented efficiency assessments are fundamentally different concepts that can lead to different efficiency measures for inefficient branches. Table 2 shows the summary statistics of pure technical efficiency and scale efficiency for all branches in the sample. Figure 4 and Figure 5 show the average pure technical and scale efficiencies of the different groups of branches, both with input and output orientations.

Table 2  Pure technical efficiency (PTE) and scale efficiency (SE) results

<table>
<thead>
<tr>
<th></th>
<th>Input orientation</th>
<th>Output orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTE</td>
<td>Mean</td>
<td>89.8%</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>9.8%</td>
</tr>
<tr>
<td></td>
<td>No. efficient branches</td>
<td>49</td>
</tr>
<tr>
<td>SE</td>
<td>Mean</td>
<td>86.1%</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>16.2%</td>
</tr>
<tr>
<td></td>
<td>No. efficient branches</td>
<td>34</td>
</tr>
</tbody>
</table>

Figure 2  Technical efficiency and size.

Figure 3  Profit index and size.
If an input orientation consistent with the goals expressed by branch managers is adopted, then the pure technical efficiency improvements should be primarily sought on branches with a number of employees between 6 and 11 (Figure 4). However, after achieving pure technical efficiency the smaller branches will still exhibit significant scale inefficiencies (Figure 5). This indicates that after achieving pure technical efficiency, the resulting scale size of branches does not allow the maximisation of productivity due to the inherent returns to scale properties of branches’ activities.

If an output orientation is adopted, which assumes that there is still scope for the growth of branches’ business, then the pure technical efficiency improvements should be primarily sought among smaller branches, with a number of employees between 3 and 8. Although the resulting activity levels free of pure technical inefficiencies would still have scale inefficiencies, their magnitude would be smaller than if an input orientation were adopted. It should be noted that independently of the efficiency orientation adopted, scale inefficiencies are almost non-existent in larger branches.

Returns to scale and branches’ size

The returns to scale properties of a DMU are determined by the shape of the VRS frontier. For DMUs not operating on the VRS frontier, their returns to scale can only be determined after the elimination of pure technical inefficiency through the projection towards the efficient frontier. Depending on the chosen projection, the DMUs can end up on different facets of the frontier, where the classification regarding returns to scale can be contradictory. This is illustrated in Figure 6, which shows a set of DMUs using one input to produce a single output.

The DMUs A, B, C and D form the VRS efficient frontier. The segments AB, BC and CD represent the increasing returns to scale (IRS), constant returns to scale and decreasing returns to scale (DRS) subsets of the efficient frontier, respectively. Let us consider the case of DMU E. With an input oriented model, this DMU would be classified as exhibiting CRS, as it would be projected onto the CRS part of the frontier. However, with an output orientation, it would exhibit DRS. Thus, the returns to scale characterisation of an inefficient DMU depends both on its location on the production possibility set and the direction of the projection towards the frontier.

Assuming that the DMUs can only be projected to a position on the frontier that does not represent either an increase in input usage or decrease in the outputs, for example the shaded area shown for DMU E, the following six groups within the PPS can be distinguished according to the returns to scale characterisation of the DMUs:

- **II**: DMUs exhibit IRS independently of model orientation.
- **CC**: DMUs exhibit CRS independently of model orientation.
- **DD**: DMUs exhibit DRS independently of model orientation.
- **IC**: DMUs may exhibit IRS or CRS, depending on model orientation.
- **CD**: DMUs may exhibit CRS or DRS, depending on model orientation.
- **ID**: DMUs may exhibit IRS, CRS or DRS, depending on model orientation.

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- **IC**: DMUs may exhibit IRS or CRS, depending on model orientation.
- **CD**: DMUs may exhibit CRS or DRS, depending on model orientation.
- **ID**: DMUs may exhibit IRS, CRS or DRS, depending on model orientation.
The partition of the PPS in six regions was originally proposed by Fare et al.\textsuperscript{21} without empirical applications. Fukuyama\textsuperscript{22} used this classification to explore the differences in the input-based and output-based predictions on returns to scale of credit associations in Japan. By adopting this classification method for characterising bank branches’ returns to scale, the number of branches in each size group is as follows: 91 belong to group II, 34 to CC, 16 to DI, 10 to IC and 17 to ID. None of the existing branches belongs to group CD. Figure 7 shows the returns to scale groups for branches of different sizes. Whilst branches located in the II region have up to 15 employees, all branches located in the DD region have nine or more employees. The branches with constant returns to scale (from CC region), and therefore with optimal scale size, are predominantly in the largest size groups.

If the market allows, a recommended strategy for this network would be to attain pure technical efficiency through an output oriented perspective, that is to increase activity levels through the efficient utilisation of the current resources. However, if there is not enough demand to support the expansion of branches’ business, an input oriented perspective has to be adopted. In this case, emphasis should be given to the attraction of a large client base for the new branches, so that their size can be increased in the near future to exploit the existing economies of scale. Also the analysis indicates that mergers with the branches from recently acquired networks to form larger units could improve productivity. This can occur if two branches that are below efficient scale and near each other are consolidated.

**Target setting**

An important aspect of a DEA efficiency assessment is the set of input-output targets that would render an inefficient DMU efficient. Two scenarios were considered for target setting:

- Selection of appropriate benchmarks for eliminating pure technical inefficiency, under input minimisation;
- The correction of both pure technical and scale inefficiencies, through the adoption of the most productive scale size.

**Targets with selection of appropriate benchmarks**

The standard VRS model (Banker \textit{et al.}) is free to chose any peer to build the composite branch against which the assessed branch is compared, as long as the envelopment constraints are satisfied, that is the convex combination of inputs (outputs) is at or below (above) the levels of the branch evaluated, and no extrapolation is allowed, so that \( \sum_{j=1}^{n} \lambda_j = 1 \). Occasionally, some of the peers used to build the composite branch may have a scale size very different from that of the assessed branch. In these cases, the peer branches may not be suitable benchmarks, as their size, and consequently the operating practices, may not be easily transferable to the branch assessed.

As a branch’s scale size can significantly affect its efficiency, the standard VRS model (Banker \textit{et al.}) was modified in order to preclude from the peer set branches that are either too large or too small to be considered benchmarks for the assessed branch. In this study it was considered that the peers should not differ from the current size of the branch evaluated by more than two employees. The detailed formulation of this enhanced DEA model with peer restriction is given in Appendix 2.

When using the model (A2) in Appendix 2, the process of target setting should be seen as a two stage procedure. Firstly, the branch is motivated to reach a best practice in a group of branches of similar size, with peers and targets identified from (A2); Secondly, the branches should be directed to the overall best practice frontier, whose peers and targets are obtained from the standard VRS model.

As expected, the use of model (A2) led to a general increase of the average efficiency score (the value obtained was 94\%, whereas the value for the standard VRS model is 90\%) and less demanding input reduction targets, shown in Table 3.

Overall, the inputs used least efficiently are floor space and operational costs, which could be reduced to 87\%. However, the floor space may be difficult to change in

<table>
<thead>
<tr>
<th>Inputs/outputs</th>
<th>Observed</th>
<th>Targets</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of employees</td>
<td>1,523</td>
<td>1,466</td>
<td>96%</td>
</tr>
<tr>
<td>Floor space</td>
<td>46,412</td>
<td>40,333</td>
<td>87%</td>
</tr>
<tr>
<td>Operational costs</td>
<td>2,713</td>
<td>2,365</td>
<td>87%</td>
</tr>
<tr>
<td>No. external ATMs</td>
<td>185</td>
<td>175</td>
<td>95%</td>
</tr>
<tr>
<td>No. transactions done by staff</td>
<td>14,372,273 15,114,062</td>
<td>105%</td>
<td></td>
</tr>
<tr>
<td>No. transactions in ext. ATMs</td>
<td>12,450,336 14,459,526</td>
<td>116%</td>
<td></td>
</tr>
<tr>
<td>No. accounts</td>
<td>1,015,456</td>
<td>1,058,878</td>
<td>104%</td>
</tr>
<tr>
<td>Value of savings</td>
<td>550,891</td>
<td>591,303</td>
<td>107%</td>
</tr>
<tr>
<td>Value of loans</td>
<td>187,236</td>
<td>202,689</td>
<td>108%</td>
</tr>
</tbody>
</table>
existing branches, so this information should mainly be used as a guideline for planning new branches. Labour is the input best used, as the potential reduction to the number of employees is the smallest. However, it can still be reduced by approximately 4%, which corresponds to a surplus of 57 employees. In relation to the number of ATMs, only one branch in the Lisbon region could reduce its number of external ATMs from 3 to 2 without constraining demand. This ATM could be more efficiently utilised if it were relocated. Overall, these input reductions would still allow an increase of approximately 5% to the branches’ business levels.

**Most productive scale size targets**

The targets derived above aim at the elimination of pure technical inefficiency by reducing input levels. However, scale inefficiencies would still prevail. In order to explore further improvements in branches’ performance, an alternative set of targets that eliminates both pure technical and scale inefficiencies is possible. These targets are based on the notion of most productive scale size (MPSS) introduced by Banker.

The estimation of MPSS seeks to obtain the scale size that maximises the productivity of each branch. In order to maximise the productivity, a branch should increase its scale size if IRS were prevailing, and decrease the scale size if DRS were prevailing. A production possibility \((X, Y) \in \Phi\) is a MPSS for its input and output mix, if and only if for all \((\delta X, \varepsilon Y) \in \Phi\) we have \(\delta \geq \varepsilon\). From the optimal solution of model (1), formulae (2) can be used to estimate a MPSS target for \(\text{DMU}_0\), where \(\hat{\gamma}\) represents the value of a variable at the optimal solution, and \((x^{\text{MPSS}}_0, y^{\text{MPSS}}_0)\) represents the MPSS inputs and outputs of \(\text{DMU}_0\) (see Banker).

\[
(x^{\text{MPSS}}_0, y^{\text{MPSS}}_0) = \left( \frac{\delta^*_0 x_0 - s^{\text{MPSS}}_0, \delta^*_0 y_0 + s^{\text{MPSS}}_0}{\sum_{j=1}^{n} \lambda^*_j, \sum_{j=1}^{n} \lambda^*_j} \right),
\]

To illustrate how the MPSS targets are obtained, consider again Figure 6. The MPSS based method would re-scale the CRS target for \(\text{DMU}_0\) from its projection at \(F^C\) by expanding it to point B or C. The advantage of this scaling is that the resulting target has a scale of operation more comparable to existing efficient DMUs with constant returns to scale. However, in the presence of multiple optimal solutions to the DEA model, the target scale may not be unique, as illustrated for point F above, where both DMUs B and C are a MPSS. Therefore, maximum productivity can be achieved through the projection to any point on the facet defined by the segment BC.

This study developed a method to choose between the multiple MPSS targets of any given branch. It is argued that the MPSS targets should be as close as possible to the branch’s pure technically efficient target. Therefore, the branches with IRS are set the smallest feasible MPSS target (namely, point B in Figure 6) and branches with DRS are set the largest feasible MPSS target (namely, point C in Figure 6). The branches with CRS are already at the MPSS and so no further movements within the frontier are needed. The method used to obtain the MPSS targets according to the above criteria is detailed in Appendix 3. Table 4 shows the aggregate targets for the branch network that eliminate both pure technical and scale inefficiencies through the adoption of the MPSS target closest to the branch’s pure technical efficient target, derived with an input orientation.

In terms of the entire network, the MPSS targets indicate that an increase of approximately 6% in the number of employees and external ATMs, keeping the operational costs at their current level, could support an increase in the output variables of more than 30%, if branches operated efficiently.

**Conclusions**

This study is part of an on-going study of the efficiency of branches from a Portuguese bank. The use of the DEA efficiency measure to complement the profitability measure currently used at the bank has provided important insights on how branches’ performance can be improved. The use of an ‘efficiency-profitability matrix’ indicated which branches can still increase their profit through efficiency improvements. It was found that branches’ efficiency has a positive effect on profits, although high profitability is not necessarily directly related to high efficiency.

The analysis of the relation between branch size and efficiency indicated that most branches have significant scale inefficiencies mainly due to increasing returns to scale. Therefore, only a small number of branches should focus on the rationalisation of resources to attain higher productivity levels. Overall, a recommended strategy for the network growth would be expansion of branches’ business activities keeping the existing input levels.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>MPSS targets for the branch network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs/outputs</td>
<td>Observed</td>
</tr>
<tr>
<td>No. of employees</td>
<td>1,523</td>
</tr>
<tr>
<td>Floor space</td>
<td>46,412</td>
</tr>
<tr>
<td>Operational costs</td>
<td>2,713</td>
</tr>
<tr>
<td>No. external ATMs</td>
<td>185</td>
</tr>
<tr>
<td>No. transactions done by staff</td>
<td>14,372,273</td>
</tr>
<tr>
<td>No. transactions in ext. ATMs</td>
<td>12,450,336</td>
</tr>
<tr>
<td>No. accounts</td>
<td>1,015,456</td>
</tr>
<tr>
<td>Value of savings</td>
<td>550,891</td>
</tr>
<tr>
<td>Value of loans</td>
<td>187,236</td>
</tr>
</tbody>
</table>
However, if the local business potential will not support an increase in business at the existing branches, then mergers with bank branches from the recently acquired financial institutions may be advisable. Productivity gains could occur if two branches that are below efficient scale and near each other are consolidated. Also, the recently opened branches should focus on attracting a large client base rapidly, in order to overcome the scale efficiency penalties due to operating with a small number of employees.

The definition of branches’ targets has followed a resource rationalisation perspective, consistent with managerial goals. Particular emphasis was given to the selection of benchmark branches whose operating practices are easily transferable to the inefficient branches. For this purpose, an enhanced DEA model with restricted peer selection was used. The analysis was extended further to enable the achievement of the branches’ most productive scale size. In order to keep branches’ size as close as possible to the pure technical efficient targets previously obtained, a method to choose between the alternative MPSS targets for each branch was developed. This consisted of choosing the smallest MPSS for branches with IRS and the largest MPSS for branches with DRS. This ensures an elimination of inefficiencies as smooth as possible for the organisation, as it requires less effort in training and relocation of resources. Also, at the branch level, keeping the scale size similar to the original levels enables, to some extent, the maintenance of current practices. This facilitates a ‘trouble free’ adaptation to the new efficiency culture.

Overall, the study reported proved the usefulness of DEA as a tool to inform bank managers both with respect to the optimal strategies regarding the development of the branch network and to set targets to improve both efficiency and profitability levels. From a practical perspective, this analysis was well received by bank management. It was decided to start planning an implementation phase. The DEA results were considered a powerful tool to complement both the EAS and SIM methods. The benchmarking properties of DEA, and the identification of peer branches within the network, were found particularly useful to set targets that are well adjusted to the profile of each branch. This will contribute to the acceptance of the results by branch staff.

Appendix 1 Published applications of DEA to bank branches

<table>
<thead>
<tr>
<th>Author and year</th>
<th>Inputs</th>
<th>Outputs</th>
<th>Orientation and eff. measure</th>
<th>Methods</th>
<th>Sample and country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-Faraj TN, Alidi AS,</td>
<td>No. of employees, % Employees with college degree</td>
<td>Average monthly net profit, Average monthly balance of current accounts</td>
<td>Input TE</td>
<td>DEA</td>
<td>15 bank branches from a commercial bank in US</td>
</tr>
<tr>
<td>Bu-Bshait KA,23</td>
<td>Average no. of years of experience, Location index, Highest authority rank index (%), Index for expenditure on decoration (%), Index for other operational expenses (%)</td>
<td>Average monthly balance of savings accounts, Average monthly balance of other accounts, Average monthly value of mortgages, Average monthly value of mortgages, No. of current accounts</td>
<td>Input TE</td>
<td>DEA</td>
<td>15 bank branches from a commercial bank in US</td>
</tr>
<tr>
<td>Athanassopoulos A16</td>
<td>Market efficiency, No. of transactions, Potential market, Sales representatives, Internal automatic facilities, No. branch outlets in the surrounding area</td>
<td>Liability sales, Loans and mortgages, Insurances and securities, Number of cards, Cost efficiency, Direct labour costs, Total technology facilities</td>
<td>Input TE SE</td>
<td>DEA, multivariate statistical analysis</td>
<td>580 bank branches from a commercial bank in UK</td>
</tr>
<tr>
<td>Athanassopoulos A24</td>
<td>Production approach, No. of employees, No. of transactions, No. of ATMs and teller machines, No. of computers terminals</td>
<td>No. of deposit accounts, No. of credit transaction, No. of debit transactions, No. of loan applications evaluated, No. of transactions involving commissions, No. of cards</td>
<td>Input TE DEA</td>
<td>DEA, and regression analysis</td>
<td>68 bank branches from a commercial bank in Greece</td>
</tr>
<tr>
<td>Drake L, Howcroft B17</td>
<td>No. of interview rooms, Square meters of branch space, Management grades, Clerical grades, Stationery costs</td>
<td>Till transactions, Lending products, Deposit products, Automated transfers, Clearing items, Ancillary business, Insurance business</td>
<td>Input TE PTE SE</td>
<td>DEA</td>
<td>190 bank branches from a UK clearing bank</td>
</tr>
</tbody>
</table>

(continued)
<table>
<thead>
<tr>
<th>Author and year</th>
<th>Inputs</th>
<th>Outputs</th>
<th>Orientation and eff measure</th>
<th>Methods</th>
<th>Sample and country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gioskas D14</td>
<td>No. person-hours worked</td>
<td>DEA</td>
<td>Input DEA</td>
<td>1988 data on 17 bank branches from a Greek commercial bank</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Square meters of utilised branch space</td>
<td>Weighted no. deposit transactions</td>
<td>TE</td>
<td>loglinear function</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Operating costs (excluding labour costs)</td>
<td>Weighted no. credit transactions</td>
<td>PTE</td>
<td>estimation and correlation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total weighted no. of transactions</td>
<td>Loglinear function:</td>
<td>SE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lovell CAK, Pastor JT25</td>
<td>No input</td>
<td>17 performance targets set by the bank (i.e., demand, high yield demand, time and home purchase deposits, personnel, credit card and mortgage loans; line-of-credit accounts, national commercial discounts, portfolio management, pension plans, investment funds, insurance policies, no. persons with direct deposits and credit cards, co-signed loans and reciprocal of delinquencies).</td>
<td>Input DEA</td>
<td>1995 data on 545 bank branches from a Spanish bank</td>
<td></td>
</tr>
<tr>
<td>Oral M, Kettani O, Yolalan R11</td>
<td>Productivity assessment</td>
<td>Amount of standard time spent on all kinds of transactions</td>
<td>Input DEA</td>
<td>44 bank branches from a Turkish bank</td>
<td></td>
</tr>
<tr>
<td>Oral M, Yolalan R10</td>
<td>No. of personnel</td>
<td>Time spent on general service transactions</td>
<td>Input DEA</td>
<td>20 bank branches from a Turkish bank</td>
<td></td>
</tr>
<tr>
<td>Parkan C26</td>
<td>No. full-time equivalent employees</td>
<td>Basic transactions (weighted by standard time)</td>
<td>Output DEA</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Schaffnit C, Rosen D, Paradi JC13</td>
<td>No. tellers</td>
<td>No. counter transactions</td>
<td>Input DEA</td>
<td>1993 data on 291 Canadian bank branches</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. ledgers and accounting officers</td>
<td>No. counter sales</td>
<td>TE</td>
<td>statistical tests</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. typing staff</td>
<td>No. security transactions</td>
<td>PTE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. supervision personnel</td>
<td>No. deposit sales</td>
<td>SE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. credit staff</td>
<td>No. commercial loan sales</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. personal loan sales</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. term accounts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. commercial loan accounts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soteriou AC, Stavrinides Y29</td>
<td>No. hours worked by clerical personnel</td>
<td>No. more resource consuming transactions</td>
<td>Input DEA</td>
<td>14 bank branches from a US savings bank</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. hours worked by managerial personnel</td>
<td>No. medium/high resource consuming trans.</td>
<td>TE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. computer terminal hours used</td>
<td>No. least resource consuming transactions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sherman HD, Gold F27</td>
<td>No. full-time equivalent tellers</td>
<td>No. deposits, withdrawals and checks cashed</td>
<td>Input DEA</td>
<td>33 bank branches from a US bank</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rent paid</td>
<td></td>
<td>TE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total cost of supplies</td>
<td>No. deposit, withdrawal and checks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. bond, bank &amp; traveller checks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. night deposits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. mortgage and consumer loans</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Operating cost (excluding personnel and rent)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. new accounts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sherman HD, Ladino G28</td>
<td>No. full-time equivalent tellers</td>
<td>Service quality index</td>
<td>Input DEA</td>
<td>1994 data on 26 bank branches</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. full-time platform personnel</td>
<td></td>
<td>TE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No. full-time manager personnel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Square feet of office space</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Operating cost (excluding personnel and rent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 1 (continued)

<table>
<thead>
<tr>
<th>Author and year</th>
<th>Inputs</th>
<th>Outputs</th>
<th>Orientation and eff. measure</th>
<th>Methods</th>
<th>Sample and country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tulkens H15</td>
<td>Square meters of office space No. personnel accounts No. savings accounts No. business accounts No. credit application accounts</td>
<td>No. checking and saving accounts transactions No. automatic teller machine transactions No. international transactions No. brokerage activities No. credit operations No. new accounts opened No. special services (e.g., card issues) Miscellany (e.g., insurance transactions)</td>
<td>Input FDH TE PTE</td>
<td>DEA SE</td>
<td>from a Cyprus bank of a public bank and bank branches of a private bank in Belgium</td>
</tr>
<tr>
<td>Vassiloglou M, Giokas D30</td>
<td>No. employees No. hours worked by personnel Costs of supplies No. of `easiest' transactions Input DEA 20</td>
<td>No. transactions aggregated in 7 categories No. of 'easiest' transactions No. of 'medium-easy' transactions No. of 'medium-difficult' transactions No. of 'most difficult' transactions</td>
<td>Input DEA</td>
<td>TE 20</td>
<td>bank branches from a Greek bank</td>
</tr>
</tbody>
</table>

Appendix 2

**DEA model with restricted peer selection**

\[
\min \theta_0^P - \sum_{i=1}^{f} s_i^+ - \sum_{i=1}^{m} s_i^- \quad (A2)
\]

subject to

\[
x_0\theta_0^P - \sum_{j=1}^{n} x_j \lambda_j - s_i^- = 0, \quad i = 1, \ldots, m,
\]

\[
\sum_{j=1}^{n} y_{ij} \lambda_j - s_i^+ = y_{i0}, \quad r = 1, \ldots, t,
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

\[
\lambda_j = 0 \quad \text{if} \quad x_{\text{EMP}}^j \neq [x_{\text{EMP}} - 2, x_{\text{EMP}} + 2]
\]

\[
\lambda_j, s_i^-, s_i^+ \geq 0, \quad \forall j, r \text{ and } i.
\]

where

- \( \theta_0^P \) is the relative efficiency of branch \( 0 \);
- \( j \) is the branch under assessment from the set \( 1, \ldots, n \) bank branches;
- \( i \) represents a branch from the set \( 1, \ldots, m \);
- \( r \) represents an output from the set \( 1, \ldots, t \);
- \( x_{ij} \) is the observed level of the \( r \)th input at branch \( j \);
- \( y_{ij} \) is the observed level of the \( r \)th output at branch \( j \);
- \( x_{\text{EMP}}^j \) is the number of employees of branch \( j \);
- \( e \) is infinitesimal;
- \( s_i^- \) and \( s_i^+ \) are slack variables for input \( i \) and output \( r \), respectively;
- \( \lambda_j \) is the multiplier associated with branch \( j \).

Appendix 3

**Method to derive the MPSS target closest to a DMUs pure technical efficient target**

The description of this method will consider that the pure technical efficient targets are obtained using an input minimisation perspective. To start, suppose that an optimal solution with value \( \theta_0^P \) has been obtained from (1), with \( \sum_{j=1}^{n} \lambda_j^* < 1 \). To check the existence of alternative optimal solutions to (1), identify the type of returns to scale of DMU \( 0 \) and derive its mpss target, the following model is solved (see banker et al.):

\[
\max \sum_{j=1}^{n} \lambda_j + e \sum_{r=1}^{f} s_i^+ + e \sum_{i=1}^{m} s_i^- \quad (A3.1)
\]

subject to

\[
\sum_{j=1}^{n} x_0 \lambda_j + s_i^- = x_0 \theta_0^P, \quad i = 1, \ldots, m,
\]

\[
\sum_{j=1}^{n} y_{0r} \lambda_j - s_i^+ = y_{0r}, \quad r = 1, \ldots, t,
\]

\[
\sum_{j=1}^{n} \lambda_j \leq 1
\]

\[
\lambda_j, s_i^-, s_i^+ \geq 0, \quad \forall j, r \text{ and } i.
\]

Given the existence of an optimal solution with \( \sum_{j=1}^{n} \lambda_j^* < 1 \) in model (1), returns to scale at DMU \( 0 \) are either constant (if an only if \( \sum_{j=1}^{n} \lambda_j^* = 1 \) in (A3.1)) or increasing (if and only if \( \sum_{j=1}^{n} \lambda_j^* < 1 \) in (A3.1)). To obtain the mpss target closest to the pte target of DMU \( 0 \), the scaling factor (that is, \( \sum_{j=1}^{n} \lambda_j^* \)) obtained from (A3.1) is used. Note that this scaling factor is as high as possible if the DMU has IRS or equal to 1 if it has CRS. The
corresponding MPSS target is obtained using the projection formula (A.3.2), where the symbol \( ^\circ \) identifies the variables from model (A.3.1).

\[
(x_{0}^t, y_{0}^t) = \left( \frac{P_i u_{0}^j - \hat{s}_{i}^+ + \hat{s}_{i}^-}{\sum_{j=1}^{n} \hat{\lambda}_{j}^* - \sum_{j=1}^{n} \frac{\hat{\lambda}_{j}^*}{\hat{\lambda}_{j}^*}}, \frac{v_{0}^j + \hat{s}_{j}^+ - \hat{s}_{j}^-}{\sum_{j=1}^{n} \hat{\lambda}_{j}^*} \right).
\]  

Conversely, if the optimal solution of model (1) has \( \sum_{j=1}^{n} \hat{\lambda}_{j}^* > 1 \), to check the existence of alternative optimal solutions to (1), identify the type of returns to scale of the DMU\(_0\), and derive its MPSS target, the following model is solved:

\[
\min \sum_{j=1}^{n} \hat{\lambda}_{j} - \varepsilon \sum_{j=1}^{n} \hat{s}_{j}^+ - \varepsilon \sum_{j=1}^{n} \hat{s}_{j}^- \quad \text{subject to}
\]

\[
\sum_{j=1}^{n} x_{ij}^j \hat{\lambda}_{j} = x_{0j}^j, \quad i = 1, \ldots, m,
\]

\[
\sum_{j=1}^{n} y_{0j}^j - \hat{s}_{j}^- = y_{0j}, \quad r = 1, \ldots, t,
\]

\[
\sum_{j=1}^{n} \hat{\lambda}_{j} \geq 1
\]

\[
\hat{\lambda}_{j}, \hat{s}_{j}^+, \hat{s}_{j}^- \geq 0, \quad \forall j, r \text{ and } i.
\]

Given that the optimal solution of model (1) has \( \sum_{j=1}^{n} \hat{\lambda}_{j}^* > 1 \), DMU\(_0\) can only exhibit CRS (if an only if \( \sum_{j=1}^{n} \hat{\lambda}_{j}^* = 1 \) in (A.3.3)) or DRS (if and only if \( \sum_{j=1}^{n} \hat{\lambda}_{j}^* > 1 \) in (A.3.3)). The MPSS target closest to the pure technical efficient target of DMU\(_0\) can be obtained using the scaling factor obtained as the optimal solution to model (A.3.3). Note that this scaling factor is as small as possible if DMU\(_0\) has DRS or 1 if it has CRS. The MPSS target is derived using formula (A.3.2), considering that the symbol \( ^\circ \) identifies the variables from model (A.3.3).

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References


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