King Fahd University for Petroleum and Minerals

Mechanical Engineering Department

Failure – Static Loading, Lecture 1

Machine Design I
ME307

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Steady Load Failure Theories

Ductile Materials

Uniaxial Stress/Strain Field

- Maximum-Normal-Stress
- Maximum-Normal-Strain
- Maximum-Shear-Stress
- Distortion-Energy
  - Shear-Energy
  - Von Mises-Hencky
  - Octahedral-Shear-Stress
- Internal-Friction
- Fracture Mechanics

Brittle Materials

Multiaxial Stress/Strain Field

Many theories have been put forth – some agree reasonably well with test data, some do not.

Maximum Normal Stress Theory

Postulate: Failure occurs when one of the three principal stresses equals the strength.

σ_1, σ_2, and σ_3 are principal stresses $\quad \sigma_1 > \sigma_2 > \sigma_3$

Failure occurs when either

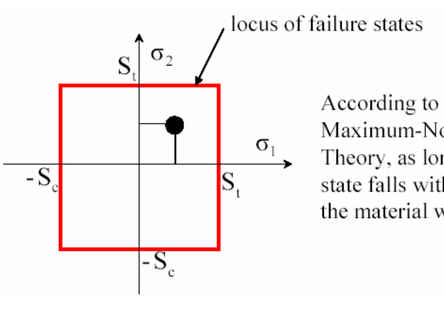
$\sigma_1 = S_t$ Tension

$\sigma_3 = -S_c$ Compression

$S_t \equiv$ Strength in Tension

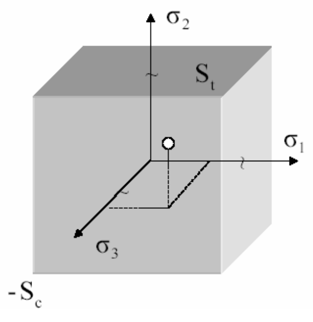
$S_c \equiv$ Strength in Compression

Maximum Normal Stress Theory (biaxial)



According to the Maximum-Normal-Stress Theory, as long as stress state falls within the box, the material will not fail.

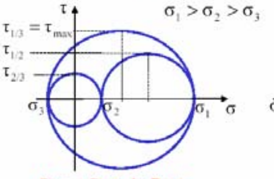
Maximum Normal Stress Theory (triaxial)

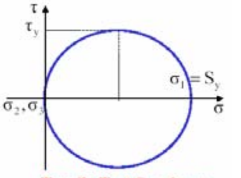


According to the Maximum-Normal-Stress Theory, as long as stress state falls within the box, the material will not fail.

Maximum Shear-Stress Theory
(Tresca Criterion)

Postulate: Yielding begins whenever the maximum shear stress in a part becomes equal to the maximum shear stress in a tension test specimen that begins to yield.

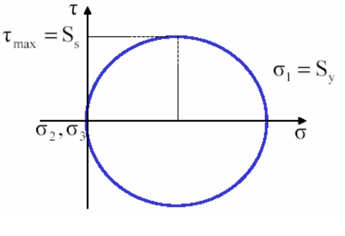




Maximum Shear-Stress Theory (cont'd)

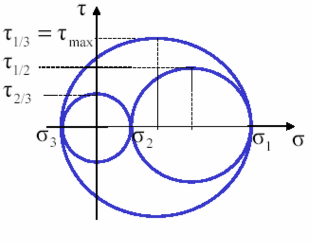
$$S_s = 0.5 S_y$$

The shear yield strength is equal to one-half of the tension yield strength.



Maximum Shear-Stress Theory (cont'd)

Stress State in Part



$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$

$$\tau_{1/3} = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sigma_1 > \sigma_2 > \sigma_3$$

Maximum Shear-Stress Theory (cont'd)

$$S_s = \frac{S_y}{2}$$

From Mohr's circle for a tensile test specimen

$$\tau_{1/3} = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

From Mohr's circle for a three-dimensional stress state.

$$\boxed{S_y = \sigma_1 - \sigma_3}$$

Maximum Normal Strain Theory

Postulate: Yielding occurs when the largest of the three principal strains becomes equal to the strain corresponding to the yield strength.

$$E\varepsilon_1 = \sigma_1 - \nu(\sigma_2 + \sigma_3) = \pm S_y$$
$$E\varepsilon_2 = \sigma_2 - \nu(\sigma_1 + \sigma_3) = \pm S_y$$
$$E\varepsilon_3 = \sigma_3 - \nu(\sigma_1 + \sigma_2) = \pm S_y$$

$E \equiv$ Young's Modulus

$\nu \equiv$ Poisson's Ratio

Maximum Normal Strain Theory Cont'd

Effective Stress or Von Mises Stress

$$\sigma' = \left(\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right)^{1/2}$$

Yielding occurs when

$$\boxed{\sigma' \geq S_y}$$

Maximum Normal Strain Theory Cont'd

In general form (normal and shear stresses)

$$\sigma' = \left(\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{2}\right)^{1/2}$$

Biaxial Stress

$$\sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$$