Minimizing and maximizing compressor and turbine work respectively

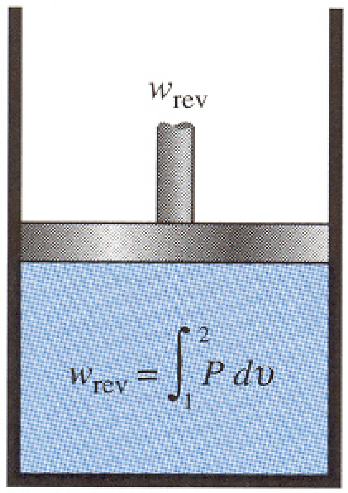
Reversible steady-flow work

- In Chapter 3, **<u>Work Done during a Process</u>** was found to be

$$W_b = \int_1^2 P dv$$

- It depends on the path of the process as well as the properties at the end states.

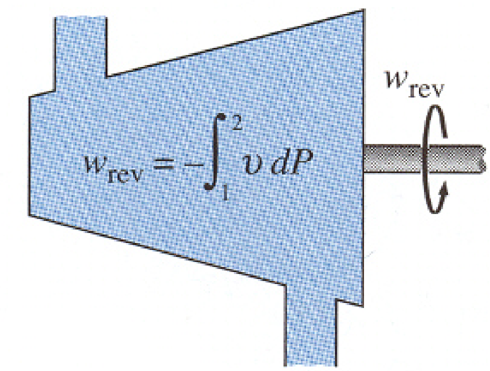
<u>Work Done during a Process</u>



(*b*) Closed system

Work Done During a steady state process

In a steady state process, usually there are no moving boundaries



(a) Steady-flow system

- It would be useful to be able to express the work done during a steady flow process, in terms of system properties
- Recall that steady flow systems work best when they have no irreversibilities

Consider general form of the Energy Balance for steady flow steady state processes

$$\dot{Q} - \dot{W} + \dot{m}_i \left[h_i + \frac{\vec{V}_i^{\,2}}{2} + g(z_i) \right] = \dot{m}_e \left[h_e + \frac{\vec{V}_e^{\,2}}{2} + g(z_e) \right]$$

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$

$$w_{rev} = -\int_1^2 v\,dP - \Delta ke - \Delta pe$$

For devices dealing with compressible fluids, like turbines and compressors, *v* is not constant, but the KE and PE are negligible. Hence

$$w_{rev} = -\int_1^2 vdP - \Delta ke - \Delta pe$$

$$w_{rev} = -\int_1^2 vdP$$

In order to integrate, we need to know the relationship between v and P.

Important observation

Note that the work term is smallest when v is small, so for a pump (which uses work) you want v to be **small**. For a turbine (which produces work) you want v to be **large.**

$$w_{rev} = -\int_1^2 v\,dP$$

Minimizing the Compressor Work

- The best way, is to keep the specific volume as low as possible during the compression process, by **cooling** it

Maximizing the turbine Work

- The best way, is to keep the specific volume as high as possible during the expansion process, by **heating** it

Effect of cooling the compressor

- To understand how the cooling affects the work, let us consider three processes:
  - ❖ Isentropic process (No cooling)
  - ❖ Polytropic process (some cooling)
  - ❖ Isothermal process (maximum cooling)
- Assume also that all three processes
  - ❖ Have the same inlet and exit pressures.
  - ❖ Are internally reversible
  - ❖ The gas behaves as an ideal gas
  - ❖ Specific heats are constants.

1- Isothermal process

$$w_{rev,in} = \int_1^2 v dP$$

Consider an ideal gas, at constant T

$$v = \frac{RT}{P}$$

$$w_{rev,in} = RT \ln\left(\frac{P_2}{P_1}\right)$$

Remember, this is only true for the isothermal case, for an ideal gas

2- Isentropic process

Isentropic means reversible and adiabatic (Q=0) i.e. No cooling is allowed

Recall from isentropic relations for an ideal gas

$$Pv^k = C$$

$$v = C^{1/k} P^{-1/k}$$

plug in and integrate

$$w_{rev,in} = \int_1^2 v dP$$

$$w_{rev,in} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

Remember, this equation only applies to the isentropic case, for an ideal gas, assuming constant specific heats

3- Polytropic process

$$w_{rev,in} = \int_1^2 vdP$$

$$Pv^n = C$$

Back in Chapter 3 we said that in a polytropic process Pv^n is a constant

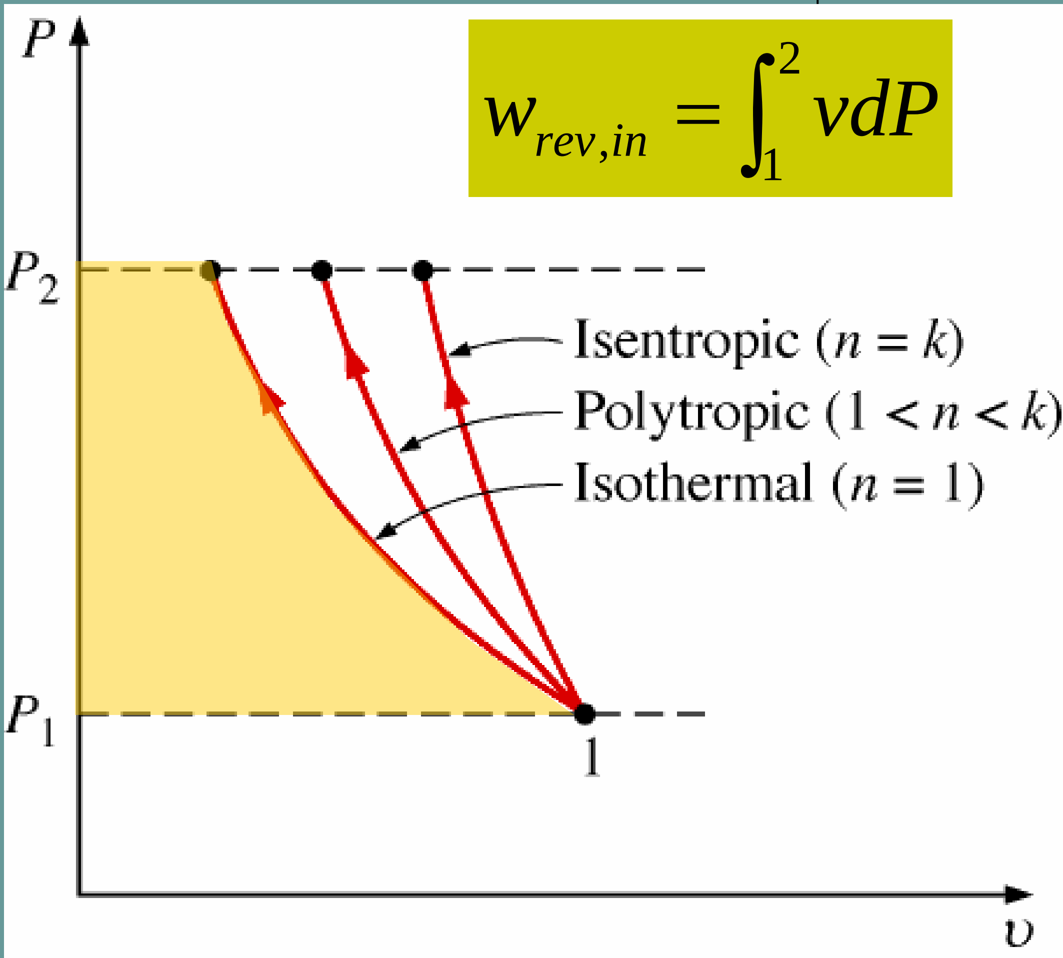
This is exactly the same as the isentropic case, but with n instead of k!!

$$w_{rev,in} = \frac{v_2P_2 - v_1P_1}{1 - 1/n} = \frac{R(T_2 - T_1)}{1 - 1/n} = \frac{nR(T_2 - T_1)}{n-1}$$

$$w_{rev,in} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Let us plot the three processes on a P-v Diagram for the same final and initial pressures

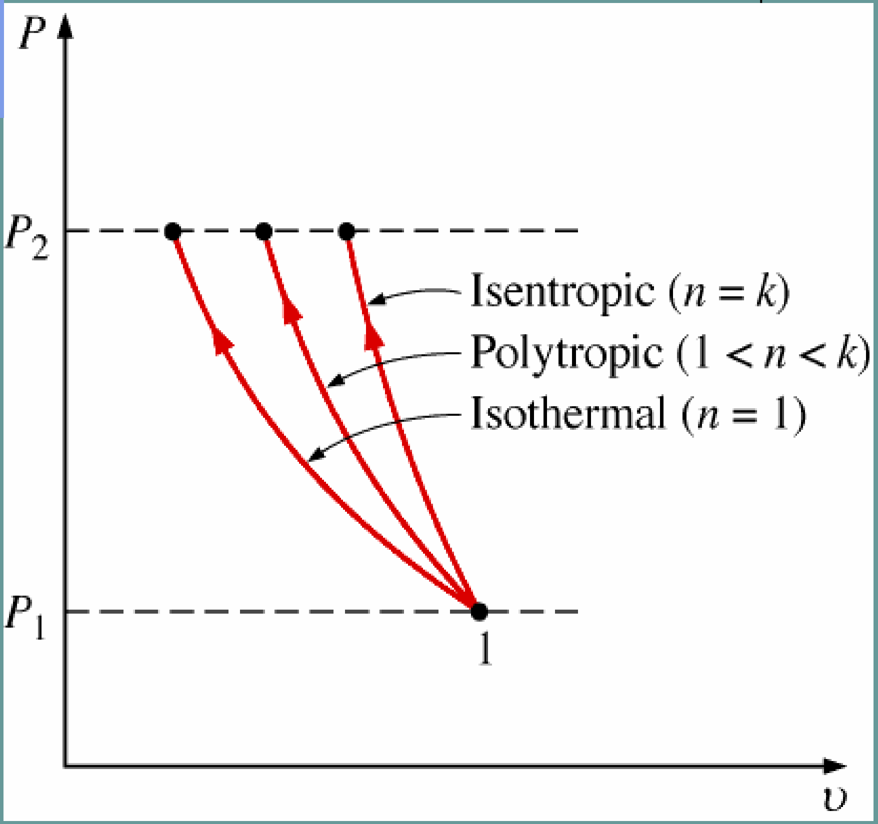
The area to the left of each line represents the work, vdP



Note, that it takes the maximum work in isentropic compression while it takes minimum work for an isothermal compression

So as an engineer, you should **compress gas isothermally,** in order to consume minimum work.

However, for a turbine, we need to produce the maximum work. So, a turbine should expand **isentropically** (adiabatically and reversibly). That is why we assume Q = 0 in the 1st low analysis of a turbine.

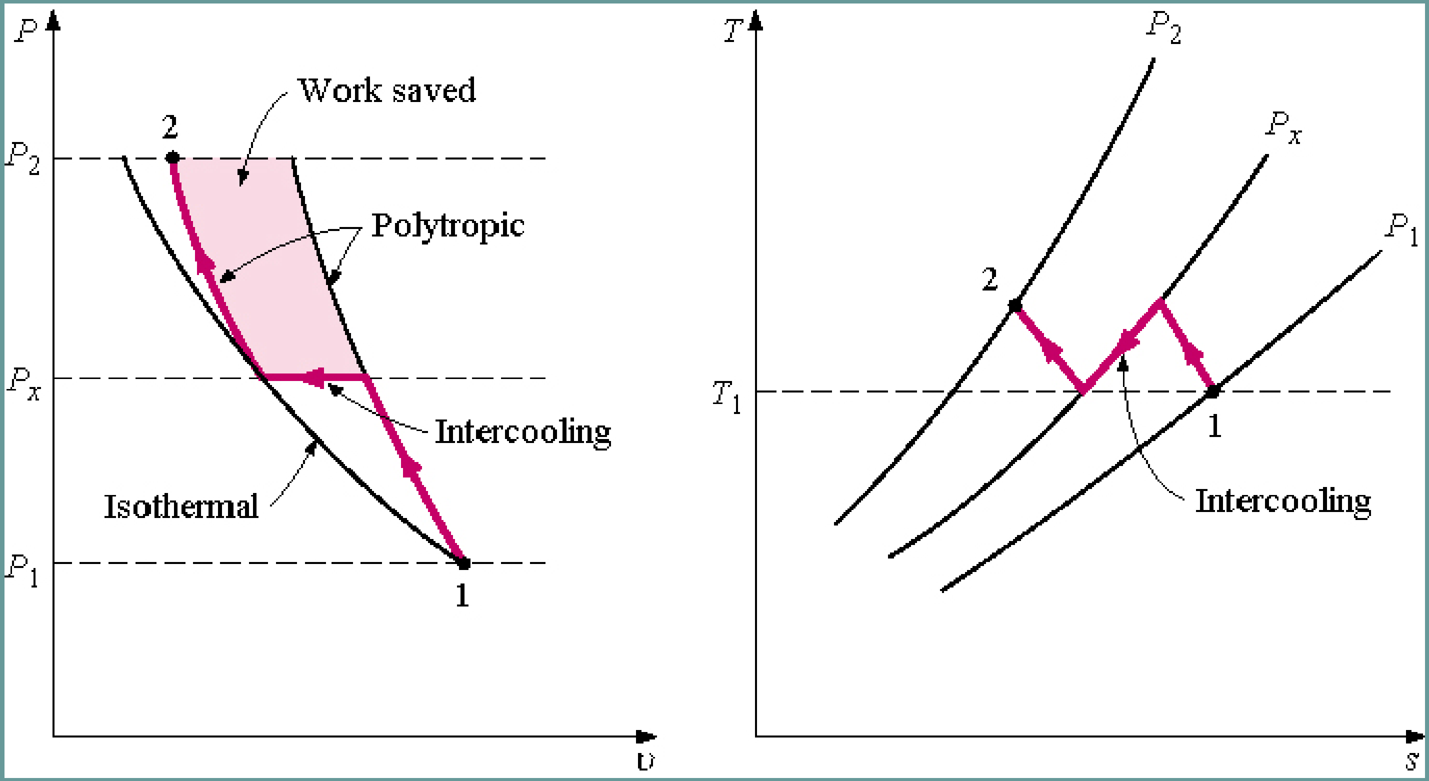


Multistage compression with intercooling

- One common way is to use cooling jackets around the casing of the compressor.
- However, this is not sufficient in some cases.
- Instead, multistage compression is more common, with cooling between steps.
- The gas is compressed in stages and cooled to the initial temperature after each stage.
- This is done by passing it a heat exchanger called “intercooler”.
- Multistage cooling is attractive in high pressure ratio compression.

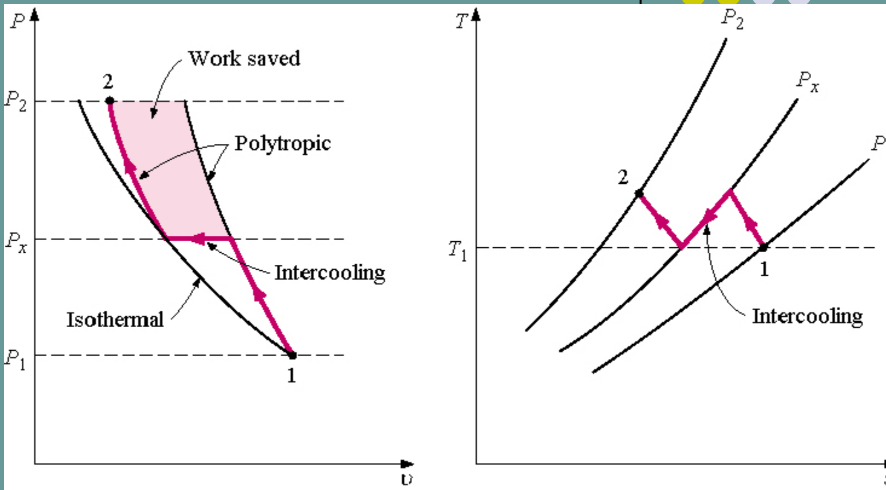
Two stage Compressor

The *colored area on the P-υ diagram* represents the work saved as a result of two-stage compression with inter-cooling.



Minimizing the work input for a two stage Compressor

The size of the colored area (the saved work input) on previous slide varies with the value of the intermediate pressure P_x.



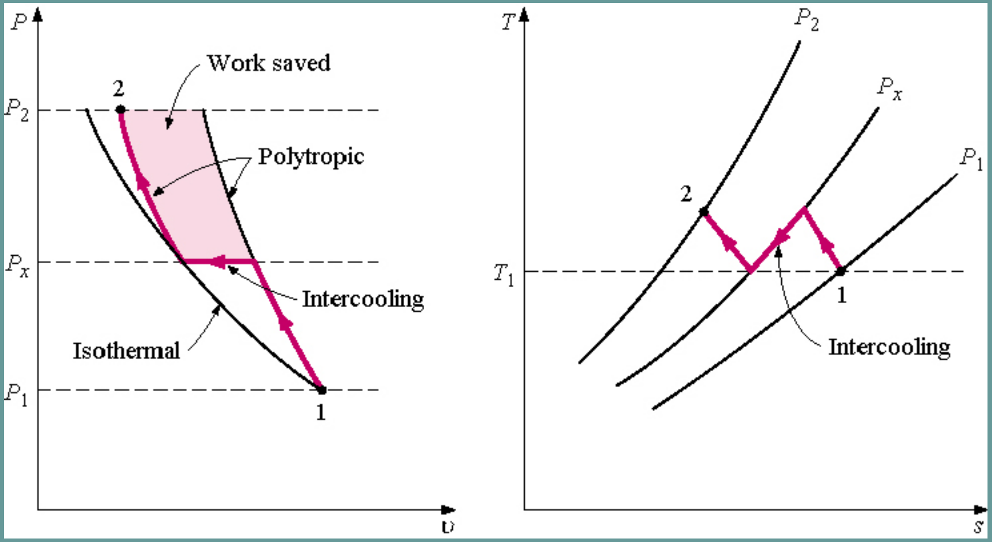
The total work input for a two-stage compressor is the sum of the work inputs for each stage of compression.

$$W_{comp,in} = W_{comp\,I,in} + W_{comp\,II,in}$$

$$= \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_x}\right)^{(n-1)/n} - 1\right]$$

The only variable is P_x.

The P_x value that will minimize the total work is determined by differentiating the above expression with respect to P_x. And setting the result to zero.



This gives

$$\left(\frac{P_x}{P_1}\right) = \left(\frac{P_2}{P_x}\right)$$

That is to minimize the compression work during two stage compression, the pressure ratio a cross each stage of the compressor must be the same.

$$w_{comp\ I,in} = w_{comp\ II,in}$$