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A NUMERICAL STUDY OF FLOW AND HEAT TRANSFER IN ROTATING RECTANGULAR CHANNELS (AR = 4) WITH 45° RIB TURBULATORS BY REYNOLDS STRESS TURBULENCE MODEL

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ABSTRACT
Computations were performed to study three-dimensional turbulent flow and heat transfer in a rotating smooth and 45° ribbed rectangular channels for which heat transfer data were available. The channel aspect ratio (AR) is 4:1, the rib height-to-hydraulic diameter ratio \((e/D_h)\) is 0.078 and the rib-pitch-to-height ratio \((P/e)\) is 10. The rotation number and inlet coolant-to-wall density ratios, \(\Delta \rho/\rho\), were varied from 0.0 to 0.28 and from 0.122 to 0.40, respectively, while the Reynolds number was fixed at 10,000. Also, two channel orientations \((\beta = 90° \text{ and } 135° \text{ from the rotation direction})\) were investigated with focus on the high rotation and high density ratios effects on the heat transfer characteristics of the 135° orientation. These results show that, for high rotation and high density ratio, the rotation induced secondary flow overpowered the rib induced secondary flow and thus change significantly the heat transfer characteristics compared to the low rotation low density ratio case. A multi-block Reynolds-Averaged Navier-Stokes (RANS) method was employed in conjunction with a near-wall second-moment turbulence closure. In the present method, the convective transport equations for momentum, energy, and turbulence quantities are solved in curvilinear, body-fitted coordinates using the finite-analytic method.

NOMENCLATURE

- \(T\) local coolant temperature
- \(T_w\) coolant temperature at inlet
- \(T_e\) local wall temperature
- \(W_s\) bulk velocity in streamwise direction
- \(\alpha\) rib angle
- \(\beta\) channel orientation measured from direction of rotation
- \(\rho\) density of coolant
- \(\Delta \rho/\rho\) inlet coolant-to-wall density ratio, \((T_w - T_e)/T_w\)
- \(\Omega\) rotational speed
- \(\theta\) dimensionless temperature, \((T - T_e)/(T_w - T_e)\)
- \(\mu\) dynamic viscosity of coolant

1. INTRODUCTION
1.1 Motivation: To improve thermal efficiency, gas-turbine stages are being designed to operate at increasingly higher inlet temperatures. A widely used method for cooling turbine blades is to bleed lower-temperature gas from the compressor and circulate it within and around each blade. The coolant typically flows through a series of straight ducts connected by 180° bends and roughened with ribs or pin fins to enhance heat transfer. These cooling ducts may not only be square in cross section or normal to the rotational direction of the blade. In fact, the aerodynamic shape of the turbine blade dictates the use of cooling channels that are rectangular in cross section (with different aspect ratios) and are at an angle, \(\beta\), from the direction of rotation. Rotation of the turbine blade cooling passages adds another complexity to the problem. It gives rise to Coriolis and buoyancy forces that can significantly alter the local heat transfer in the internal coolant passages from the non-rotating channels. The presence of rib turbulators adds a further complexity since these ribs produce complex flow fields such as flow separation, reattachment and secondary flow between the ribs, which produce a high turbulence level that leads to high heat transfer coefficients.

1.2 Literature Review: Experimental Studies. The complex coupling of the Coriolis and buoyancy forces with flow separation/reattachment by ribs has prompted many investigators to...
study the flow and temperature fields generated in heated, rotating ribbed wall passages. Most experimental studies on internal cooling passages have focused on non-rotating ducts. See, for example, Han and Park [1], Han et al. [2] Ekkard and Han [3] and Liu et al. [4] and the references cited there. Experimental studies on rotating ducts have been less numerous. Wagner et al. [5], Dutta and Han [6], Soong et al. [7] and Azad et al. [8] investigated rotating ducts with smooth and ribbed walls. Wagner et al. [9], Johnson et al. [10 and 11], Parsons et al. [12] and Zhang et al. [13] reported studies on rotating square channels with normal and angled ribs. Azad et al. [8] also investigated the effect of channel orientation on rotating ribbed two pass rectangular channel. Griffith et al. [14] studied the effect of channel orientation on rotating smooth and ribbed rectangular channels with channel aspect ratio of 4:1. They investigated a broad range of flow parameters including Reynolds number (Re = 5000-40000), rotation number (Ro = 0.04-0.3) and coolant to wall density ratio (\(\Delta \rho / \rho = 0.122\). Their experimental results provided a database for the present work.

### 1.3 Literature Review: Numerical Studies

In addition to the experimental studies mentioned above, several studies have been made to predict numerically the flow and heat transfer in radially rotating smooth and ribbed ducts. Stephens et al. [15, 16] studied inclined ribs in a straight non-rotating square duct. Stephens and Shih [17] investigated the effect of angled ribs on the heat transfer characteristics in a rotating two-passage duct using a low-Re number \(k-\omega\) turbulence model. They studied the effects of Reynolds number, rotation numbers, and buoyancy parameters. Prakash and Zerkle [18], employing a high Reynolds number \(k-\varepsilon\) turbulence model with wall function, performed a numerical prediction of flow and heat transfer in a ribbed rectangular duct (90° rib) with and without rotation. However, their calculations used periodicity and neglected buoyancy effects. They suggested that a low Reynolds number turbulence model is necessary to simulate real gas turbine engine conditions and a Reynolds stress model is required to capture anisotropic effects. Bonhoff et al. [19] calculated the heat transfer coefficients and flow fields for rotating U-shaped coolant channels with angled ribs (45°). They used a Reynolds stress turbulence model with wall functions in the FLUENT CFD code. Using the periodicity of the flow, lacovides [20] computed flow and temperature fields in a rotating straight duct with 90° ribs. Two zonal models of turbulence were tested: a \(k-\varepsilon\) with a one-equation model of \(k\) transport across the near-wall region and a low-Re differential stress model. He concluded that the differential stress model thermal computations were clearly superior to those of the \(k-\varepsilon\)-one-equation model.

Using the same model and method of Chen et al. [21, 22], Jang et al. [23, 24] studied flow and heat transfer behavior in a non-rotating two-pass square channels with 60° and 90° rib, respectively. Their results were in good agreement with Ekkard and Han’s [3] detailed heat transfer data which validated their code and demonstrated the second-moment closure model superiority in predicting flow and heat transfer characteristics in the ribbed duct. In a later study, Jang et al. [25] predicted flow and heat transfer in a rotating square channel with 45° angled ribs by the same second-moment closure model. Heat transfer coefficient prediction was well matched with Johnson et al. [11] data for both stationary and rotating cases. Al-Qahani et al. [26] predicted flow and heat transfer in a rotating two-pass rectangular channel with 45° angled ribs by the same second-moment closure model of Chen et al. [21, 22]. Heat transfer coefficient prediction was compared with the data of Azad et al. [8] for both stationary and rotating cases. It predicted fairly well the complex three-dimensional flow and heat transfer characteristics resulting from the angled ribs, sharp 180° turn, rotation, centrifugal buoyancy forces and channel orientation.

In practice, the aerodynamic shape of the turbine blade dictates the use of cooling channels that are rectangular in cross section and are at an angle \(\beta\) from the direction of rotation. The effect of rotation, channel orientation and large channel aspect ratio on the secondary flow and heat transfer in rectangular channels may vary from the square channels. None of the previous studies predicted the characteristics of fluid flow and heat transfer in rotating rectangular channels that have an aspect ratio, \(AR\), of 4:1 whether perpendicular or at an angle from the direction of rotation.

The objective of this study is to use the second moment RANS method of Chen et al. [21, 22] to (1) predict the three-dimensional flow and heat transfer for rotating smooth and ribbed one-pass rectangular ducts \((AR = 4:1)\) and compare with the experimental data of Griffith et al. [14] and (2) to investigate the effect of high rotation and high density ratios on the secondary flow field and the heat transfer characteristics in a ribbed duct at 135° orientation.

### 2. DESCRIPTION OF PROBLEM

A schematic diagram of the geometry is shown in Figure 1. It has a rectangular cross section with channel aspect ratio, \(AR\), of 4:1. Two geometries are investigated, one with smooth walls and the other one with ribs. Two of the four side walls, in the rotational direction, are denoted as the leading and trailing surfaces, respectively, while the other two side walls are denoted as the top and bottom surfaces. The channel hydraulic diameter, \(D_h\), is 0.8 in (2.03 cm). The distance from the inlet of the channel to the axis of rotation (Y-axis) is given by \(R_c/D_h = 20.0\) and the length of the channel is given as \(L/D_h = 22.5\). The channel consists of unheated starting smooth length \((L_i/D_h = 9.92)\), heated smooth or ribbed section \((L_2/D_h = 7.58)\) and unheated exit smooth section \((L_3/D_h = 5.00)\). The arc length \(S\) is measured from the beginning of the heated section to the end of it. In the ribbed section, the leading and trailing surfaces are roughened with nine equally spaced ribs of square cross section. The rib height-to-hydraulic diameter ratio \((e/D_h)\) is 0.078 and the rib-pitch-to-height ratio \((P/e)\) is 10. All ribs are inclined at an angle \(\alpha = 45°\) with respect to the flow. Two channel orientations are studied: \(\beta = 90°\) corresponding to the mid-portion of a turbine blade and \(\beta = 135°\) corresponding to the trailing edge region of a blade. A summary of the cases studied is given in Table 1.

<table>
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<tr>
<th>Case #</th>
<th>Surface</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>Expt.</th>
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<td>-</td>
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<td>-</td>
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<td>9</td>
<td>Ribbed</td>
<td>0.28</td>
<td>0.400</td>
<td>135°</td>
</tr>
</tbody>
</table>

Table 1: Summary of cases studied, \(Re = 10,000\).
3. COMPUTATIONAL PROCEDURE

3.1 Overview

The Reynolds-Averaged Navier-Stokes equations in conjunction with a near wall Reynolds stress turbulence model are solved using the chimera RANS method of Chen et al. [21, 22]. The governing equations with the second-moment closure turbulence model were described in detail by Chen et al. [21, 22] and will not be repeated here. The flow is considered to be incompressible since the Mach number is quite low. However, the density in the centrifugal force terms is approximated by $\rho = \rho_r T_r / T$ to account for the density variations caused by the temperature differences. $\rho_r$ and $T_r$ are the density and temperature at the inlet of the cooling channel. In general, the density is also a function of the rotating speed because the centrifugal force creates a pressure gradient along the duct. In the experiments of Griffith et al. [14], the maximum pressure variation between the channel inlet and the exit is approximately 0.0113 atm for the highest rotation number of 0.28 (i.e., $\beta = 550$ rpm) considered in the present study. This gives a maximum density variation of only about 1.1% from the inlet to the exit of the duct at the highest rotation number. It is therefore reasonable to omit the density variation caused by the pressure gradients induced by the channel rotation.

3.2 Boundary conditions

A uniform velocity profile was used at the inlet of the duct ($Z = 0$). The unheated length ($L_1$) was long enough for the velocity profile to be fully developed turbulent profile before the heating start-point ($Z = L_1$). At the exit of the duct, zero-gradient boundary conditions were specified for the mean velocity and all turbulent quantities, while linear extrapolation was used for the pressure field. The coolant fluid at the inlet of the duct is air at uniform temperature $T = T_o$ (i.e., $\theta = (T - T_o) / (T_w - T_o) = 0$). The wall temperature of the unheated sections is kept constant at $T = T_o$ ($\theta = 0$) while the wall temperature of the heated section is kept constant at $T = T_w$ ($\theta = 1$).

3.3 Computational grid details

Figure 2 shows the computational grid around the ribs for the ribbed duct. The grid was generated using an interactive grid generation code GRIDGEN [27]. It was then divided into five overlapped chimera grid blocks (three for the case of smooth duct) to facilitate the implementation of the near-wall turbulence model and the specification of the boundary conditions. To provide adequate resolutions of the viscous sublayer and buffer layer adjacent to a solid surface, the minimum grid spacing in the near-wall region is maintained at $10^{-3}$ of the hydraulic diameter which corresponds to a wall coordinate $y^+$ of the order of 0.5. The number of grid points in the streamwise direction from inlet to outlet is 50 for the smooth case and 394 for the ribbed duct. Whether smooth or ribbed, the number of grid points in the cross-stream plane is $33 \times 75$. The number of grid points and their distributions in the present smooth and ribbed ducts were obtained based on extensive grid-refinement studies that were performed in Chen et al. [21, 22], Jang et al. [23-25] and Al-Qahtani [26] for similar channels of a square and rectangular cross sections. The interested reader is referred to these references for the details of the grid refinement studies performed on the similar smooth and ribbed channels. In all calculations, the root-mean-square (rms) and maximum absolute errors for both the mean flow and turbulence quantities were monitored for each computational block to ensure complete convergence of the numerical solutions and a convergence criterion of $10^{-3}$ was used for the maximum rms error.

4. RESULTS AND DISCUSSION

As summarized in Table 1, computations were performed for one Reynolds number (10,000), rotation numbers ranging from 0 to 0.28 and inlet coolant-to-wall density ratios $\Delta \rho / \rho$ ranging from 0.122 to 0.40 with two channel orientations of $\beta = 90^\circ$ and $135^\circ$. The Nusselt numbers presented here were normalized with a smooth tube correlation by Dittus-Boelter/McAdams (Rohsenow and Choi [28]) for fully developed turbulent non-rotating tube flow:

$$Nu = 0.023 \, Re^{0.8} \, Pr^{0.4}$$

4.1 Velocity and Temperature Fields

Before discussing the detailed computed velocity field, a general conceptual view about the secondary flow patterns induced by angled ribs and rotation is summarized and sketched in Figure 3. The parallel angled ribs in the non-rotating duct (Figure 3a) produce symmetric counter rotating vortices that impinge on the top surface. The Coriolis force in the $\beta = 90^\circ$ rotating duct (Figure 3b) produces two additional counter-rotating vortices that push the cooler fluid from the core to the

Figure 1. Geometry

Figure 2. Numerical Grid

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trailing surface. For the $\beta = 135^\circ$ rotating duct (Figure 3c), the Coriolis force produces two long vortices parallel to the ribbed surfaces and a third small vortex near the corner of the top-trailing surfaces. The effect of this rotation secondary flow is to combine destructively (opposite directions) with the rib induced secondary flow along the whole leading and trailing surfaces. This is an important concept that will help explain some of the coming flow and heat transfer characteristics.

4.1.1 Smooth Duct.

At two axial stations as defined in Figure 1a, Figures 4 through 6 show the calculated secondary flow vectors and constant temperature contours for the smooth cases as mentioned in Table 1. Note that these axial stations are viewed from upstream of the channel. It can be seen from Figure 4a that secondary corner vortices are generated as a result of the Reynolds stress anisotropy. It can be noticed from the corresponding temperature contour plots that the cooler fluid is located in the core region of the channel cross section. Further downstream (Figure 4b), the level of the secondary corner vortices is the same and the fluid in the duct core is heated more.

In Figure 5, the Coriolis forces produce a cross-stream two vortex flow structure (Figure 5a) that pushes the cold fluid from the core toward the trailing surface and then brings it back along the inner and outer surfaces to the leading surface. This means that the thermal boundary layer starts at the trailing surface, grows along the two side surfaces and ends at the leading surface. This results in small temperature gradient near the leading surface (hence lower heat transfer coefficients) and steeper one near the trailing surface (hence higher heat transfer coefficients) as seen from the corresponding contour plot of Figure 5a. Moreover, the cooler heavier fluid near the trailing surface will be accelerated by the centrifugal buoyancy force while the hotter lighter fluid near the leading surface will be decelerated to maintain the continuity in the streamwise direction. The Coriolis forces, in the $\beta = 135^\circ$ smooth duct (Figure 6a), produce a secondary flow that pushes the cold fluid away from the corner of the leading and top surfaces. This produces two counter rotating vortices with the one near the leading surface stronger than the one near the trailing surface. It can also be noticed that a small vortex is generated at the corner of the top and trailing surfaces. As a result of this secondary flow, the fluid is pushed toward the bottom surface at which part of the secondary flow will move back along the trailing surface while the other part moves along the leading surface such that they meet again at the leading corner. This means that the thermal boundary layer starts at the bottom surface, grows along the trailing and leading surfaces and ends at the leading corner. This can be seen from the corresponding temperature contour plots where high temperature contours are located near the leading corner.
4.1.2 Ribbed Duct.

At several axial stations as defined in Figure 2a, Figures 7 through 10 show the calculated secondary flow vectors and constant temperature contours for the ribbed cases as mentioned in Table 1. Figure 7 shows the calculated secondary flow vectors and constant temperature contours for the non-rotating case (case 4). Since the ribs are oriented at a negative 45° angle, the fluid adjacent to the top and ribbed surfaces will reach the ribs first and change direction along the ribbed surfaces toward the bottom surface (Figure 7a). It then returns back to the top surface along the centerline of the inclined cross-stream plane. In the same figure, one can also notice the early stages of two symmetric counter-rotating vortices, which become two full symmetric counter-rotating vortices in the midsection of any two ribs (Figure 7b). Along the streamwise direction, the size of these two vortices oscillates from the largest in the middle of each inter-rib distance to the smallest on the rib tops (Figure 7c). This pattern keeps repeating until the last rib (Figure 7d and 7e). The effect of the secondary flow on the temperature field is convecting the cooler fluid from the top surface and along the ribbed surfaces towards the bottom surface. It then moves back to the top surface which results in steep temperature gradients and high heat transfer coefficients on both the top and ribbed surfaces as seen in the corresponding temperature contours.

Figure 7. Secondary flow and temperature \( \theta = (T - T_o)/(T_w - T_o) \) for non-rotating ribbed duct, Ro = 0.00.

Figure 8 shows the cross-stream velocity vectors and temperature contours for case 5 (Ro = 0.14 and \( \beta = 90^\circ \)) at the same planes as in the non-rotating ribbed duct (case 4). As the flow approaches the first rib, this Coriolis force induced secondary flow starts to distort the secondary flow started by the inclined ribs. This effect can be clearly seen by comparing Figures 8a through 8e with Figures 7a through 7e.

From this comparison, the following conclusions can be drawn. (1) The magnitude of the Coriolis force induced secondary flow is weaker than the rib induced secondary flow. (2) In the midsections of each of two ribs, the rib induced vortex near the bottom surface is distorted slightly in the midsection of rib 1 and 2 (Figure 8b) but this distortion increases as the fluid proceeds downstream the duct (Figure 8d). (3) On the ribs (Figure 8c), both vortices shrink in size and get distorted only near the bottom. This pattern repeats itself until the last rib (Figure 8e). The general effect of the Coriolis force induced secondary flow is to distort the rib-induced vortices. Consequently, the temperature contours are shifted toward the trailing surface, which affects the heat transfer coefficients from both the leading and trailing surfaces as seen from the corresponding temperature contour plot.

Figure 8. Secondary flow and temperature \( \theta = (T - T_o)/(T_w - T_o) \) for rotating ribbed duct, Ro = 0.14 and \( \beta = 90^\circ \).

Figure 9 shows the cross-stream velocity vectors and temperature contours for the low rotation low density ratio \( \beta = 135^\circ \) (case 6) at the same planes as in cases 5 and 6. Comparing Figure 9 with Figure 8, the following can be noticed. Just before the ribbed section, the rotation induced secondary flow is still dominant as can be seen from comparing Figures 9a and 8a. However, from rib 1 on, this low rotation induced secondary flow is dominated by the rib induced secondary flow. A careful comparison between the secondary flow fields of case 6 and case 5 (e.g. Figure 9d with Figure 8d) shows that there is only minor change in the net effect of the secondary flow fields. This minor change appears more clearly in the temperature field. By comparing the temperature contours in Figure 9 with Figure 8, we notice that the cooler fluid is pushed back toward the leading surface, reducing the steep temperature gradients on the trailing surface.
As we increase the rotation number and density ratio, the strength of the rotation-induced secondary flow increases and gradually overcomes the rib induced secondary flow (recall Figure 3c). By reaching a rotation number of 0.28 and a density ratio of 0.40 as shown in Figure 10 (case 9), the rotation-induced secondary flow is found to be dominant over the rib induced secondary flow especially downstream of the channel. This is very clear by comparing the corresponding axial stations in Figures 10 and 9. This important result is very clear by comparing the rotation-induced secondary flow (recall Figure 3c). By reaching a rotation number of 0.28 and a density ratio of 0.40 as shown in Figure 10, the rotation-induced secondary flow is found to be dominant over the rib induced secondary flow especially downstream of the channel.

Figure 9. Secondary flow and temperature $[\theta = (T - T_a)/(T_w - T_a)]$ for rotating ribbed duct, $\psi = 0.14$, $\Delta \rho/\rho = 0.122$ and $\beta = 135^\circ$.

Figure 10. Secondary flow and temperature $[\theta = (T - T_a)/(T_w - T_a)]$ for rotating ribbed duct, $\psi = 0.28$, $\Delta \rho/\rho = 0.40$ and $\beta = 135^\circ$.

4.2 Detailed Local Heat Transfer Coefficient Distribution

For various rotation numbers and density ratios, Figures 11 and 12 show the local Nusselt number ratio contours of the ribbed leading and trailing surfaces, respectively. The non-rotating case in Figure 11a (12a for the trailing surface) will be used as a baseline for comparison and discussion. Figures 11b through 11e (12b through 12e for the trailing surface) show the Nusselt number ratios contours on the ribbed heated section. First, the effect of the channel orientation on the Nusselt number ratios is discussed via comparing Figures 11b and 11f (12b and 12f for the trailing surface). Second, the effect of increasing the rotation number on the $\beta = 135^\circ$ Nusselt number ratios is discussed via Figures 11a through 11e (12a through 12c for the trailing surface). Third, the effect of increasing the density ratio on the $\beta = 135^\circ$ Nusselt number ratios is discussed via Figures 11c through 11e (12e through 12f for trailing surface).

In Figure 11a, the highest Nusselt number ratios were obtained on the top of the ribs, and the lower Nusselt number ratios were obtained right before and after the ribs. Between any two ribs, the Nusselt number ratios are highest near the top surface and decrease as we move towards the bottom surface. This is due to the rib induced secondary flow that moves from the top surface (and parallel to the ribbed walls) to the bottom surface.

Effect of channel orientation on the leading and trailing surfaces: For fixed rotation number and density ratio ($\psi = 0.14$ and $\Delta \rho/\rho = 0.122$), Figures 11b and 11f show the Nusselt number ratios contours on the leading side for $\beta = 135^\circ$ and $90^\circ$, respectively. Comparing these figures with the non-rotating leading side (Figure 12a), it is noticed that the Nusselt number ratios decreased in both cases with the decrease in the $\beta = 135^\circ$ case being the most (a 19% decrease compared to a 10% decrease in the $90^\circ$ case). Figures 12b and 12f show the Nusselt number ratios contours on the trailing side for $\beta = 135^\circ$ and $90^\circ$, respectively. Comparing these figures with the non-rotating trailing side (Figure 12a), it is noticed that the Nusselt number increased in both cases with the increase in $\beta = 135^\circ$ being the least (a 1% increase compared to a 5% increase in the $90^\circ$ case). The reason why the Nusselt number ratios in the $\beta = 135^\circ$ case decreased more on the leading side and increased less on the trailing side compared to $\beta = 90^\circ$ case can be understood in light of the conceptual secondary flow diagram in Figure 3. The rotation induced vortex in the $\beta = 135^\circ$ configuration move along the full face of the leading or trailing surfaces. However, the rotation induced vortex in the $\beta = 90^\circ$ configuration moves along only one half the face of the leading or trailing surfaces. With this in mind, we notice in Figure 3 that the two secondary flows produced by rotation and angled ribs for the rotating $\beta = 135^\circ$ duct combine destructively (opposite direction) and thus reduce heat transfer on both the leading surface and the trailing surface. On the other hand, the two secondary flows produced by rotation and angled ribs for the rotating $\beta = 90^\circ$ duct combine to (i) constructively (same direction) enhance heat transfer for only one half of each of the leading and trailing surfaces and (ii) destructively (opposite direction) reduce heat transfer for the other half of each of the leading and trailing surfaces.
Effect of increasing the rotation number on the leading surface: In Figure 11b, the rotation number is increased to 0.14 while the density ratio is kept fixed at 0.122. As discussed before, this causes the Nusselt number ratios to decrease by 19% compared to the non-rotating case (Figure 11a). But when the rotation number was increased to 0.28 (Figure 11c), the Nusselt number ratios decreased only by 10% compared to the non-rotating case. Moreover, it is noted that the high Nusselt number ratios regions are shifted to the middle of the ribbed surface. This is because of the rotation induced secondary flow getting stronger and gradually overcomes the rib induced secondary flow.

Effect of increasing the density ratio on the leading surface: In Figure 11d, the rotation number is kept fixed at 0.28 while the density ratio is increased to 0.20. It is seen from this figure that the high Nusselt number ratios regions are moved further toward the bottom surface. Increasing the density ratio further to 0.40 (Figure 11e), we notice that the high Nusselt number ratios regions are now existing next to the bottom surface with a total decrease of only 4% compared to the non-rotating case.

Effect of increasing the rotation number on the trailing surface: Figure 12 shows the same information as in Figure 11 but for the trailing surface. Figure 12a (Ro = 0.00) will be used as the baseline for comparison and discussion. As discussed before, increasing the rotation number to 0.14 (Figure 12b) causes the Nusselt number ratios to increase only by 1% compared to the non-rotating case. In Figure 12c, the rotation number is increased further to 0.28 while the density ratio is kept fixed at 0.122. This causes the Nusselt number ratios to increase by 6% compared to the non-rotating case. Also, it is seen from this figure that the high Nusselt number ratios regions are spreading toward the bottom surface.

Effect of increasing the density ratio on the trailing surface: In Figure 12d, the rotation number is kept fixed at 0.28 while the density ratio is increased to 0.20. It is seen in this figure that the high Nusselt number ratios regions are pushed slightly more toward the bottom surface. Increasing the density ratio further to 0.40 (Figure 12e) causes the Nusselt number ratios to increase by 12% compared to the non-rotating case. It is also seen from this figure that, upstream of the channel, the high Nusselt number ratios are moved toward the bottom surface while downstream they dominate most of the inter-rib regions.
4.3 Spanwise-Averaged Heat Transfer Coefficients and Comparison with Experimental Data

4.3.1 Smooth Duct.

In Figure 13, comparisons of the spanwise-averaged Nusselt number ratios \(\left(\frac{Nu}{Nu_0}\right)\) were made with the experimental data of Griffith et al. [14]. In order to compare the effects of the channel orientation on the heat transfer, Figure 13 shows the Nusselt number ratios for the three smooth cases: 1, 2 and 3. In this Figure, the inlet coolant-to-wall density ratio was held constant at value of 0.122. The effect of the model orientation can be seen by comparing the \(\beta = 135^\circ\) Nusselt number ratios with the \(\beta = 90^\circ\) ones. It can be seen that the \(\beta = 135^\circ\) Nusselt number ratios are higher on the leading surfaces, and lower on the top surfaces. This can be explained in terms of the secondary flow patterns and temperature contours shown in Figures 5 and 6. For the \(\beta = 90^\circ\) case, the cold fluid reaches the leading surface after it passes over the trailing surface and both of the two side surfaces. On the other hand, the cold fluid in the \(\beta = 135^\circ\) case moves directly to the bottom surface at which it splits and comes back along the leading and trailing surfaces. When the channel orientation was changed from \(\beta = 90^\circ\) to \(\beta = 135^\circ\), more cold fluid was flowing to the leading surface while the trailing surface received less cold fluid. This has led to higher heat transfer on the leading surface and lower heat transfer on the trailing surface for the \(\beta = 135^\circ\) case. For the top surface, the lower Nusselt number ratios observed in the \(\beta = 135^\circ\) rotating case can be attributed to the fact that most of the top surface behaves as a leading surface in the sense that the fluid is moving away from this surface. Similarly, the \(\beta = 135^\circ\) bottom surface behaves as a trailing surface with high heat transfer since it receives the cold fluid directly from the duct core. Comparisons with the experimental values reveal the following: (1) for the non-rotating case, the matching between the experimental and prediction is good on all surfaces, (2) fair agreement on the leading, trailing, top and bottom sides is achieved for the rotating cases of \(\beta = 90^\circ\) and \(135^\circ\).

4.3.2 Ribbed Duct.

Figures 14 and 15 show the spanwise-averaged and regional-averaged Nusselt number ratios \(\left(\frac{Nu}{Nu_0}\right)\) for the ribbed cases 4 (\(\beta = 90^\circ\)) and 5 (\(\beta = 135^\circ\)). The rotation number and the inlet coolant-to-wall density ratio were held constant at values of 0.14, and 0.122, respectively. Note that the experimental regional-averaged Nusselt number in Griffith et al. [14] is based on the projected area of each copper plate rather than the true heat transfer surface area which includes the \(45^\circ\) rib-increased area. However, the predicted regional-averaged Nusselt Number is based on the true heat transfer area for the test surfaces with \(45^\circ\) ribs which is 1.25 times the projected area. Therefore, the experimental data were divided by 1.25 to reasonably compare with our regional-averaged Nusselt number, except for the inner and outer surfaces where there were no ribs. The predicted Nusselt number ratios on the leading and trailing surfaces are in good agreement with Griffith et al. [14] data for the non-rotating case (Figure 14) while relatively close to the experimental data in the rotating case (Figure 15). Downstream of the channel, the predicted Nusselt numbers on the top and bottom surfaces are mildly over-predicted and under-predicted, respectively. This may be partly attributed to the fact that the predicted Nusselt number ratios are based on a uniform wall temperature boundary condition while the experimental ones are based on a uniform wall heat flux boundary condition.

Figure 13. Effect of rotation and channel angle on Nusselt number distribution for smooth duct, Re = 10,000.

Figure 14. Calculated and measured Nusselt number ratio distribution for non-rotating ribbed duct, Re = 10,000.

Figure 15. Calculated and measured Nusselt number ratio distribution for rotating ribbed duct (Ro = 0.14), Re = 10,000.
The spanwise-averaged Nusselt number distributions on the leading and trailing surfaces of Figures 14 and 15 show periodic spikes. The higher spikes which occur on the ribs tops are caused by the flow impingement on the ribs, and the lower spikes (which occur right before and after the ribs) are caused by the flow reattachment between the ribs. The Nusselt number ratios are high in the regions between the ribs. The Nusselt number ratios increase until the last rib, which is similar to the results obtained in Jang’s et al. [25] 45°-ribbed square channel and Al-Qahtani’s et al. [26] 45°-ribbed rectangular channel (AR = 2). This phenomenon is caused by the rib-induced secondary flow becoming stronger along the duct as discussed in Figure 7. The Nusselt number distribution on the top surface of Figures 14 and 15 shows that it increases all the way to rib 9 as a result of the secondary flow that pushes the cold fluid towards the top surface. For the same reason, the Nusselt number distribution on the bottom surface is decreasing (although mildly) since it receives the heated fluid from the ribbed surfaces.

The higher spikes which occur on the ribs tops are caused by the flow that pushes the cold fluid towards the top surface. For the same reason, the Nusselt number distribution on the bottom surface is decreasing (although mildly) since it receives the heated fluid from the ribbed surfaces.

For case 5 ($\beta = 90^\circ$), the rotation-induced cross-stream secondary flow distorts the rib-induced vortices and consequently, rotation shifts the temperature contours and affects the heat transfer coefficients from both leading and trailing surfaces.

For case 6 ($\beta = 135^\circ$) the effect of increasing the rotation number (with fixed density ratio) is to monotonically increase the Nusselt number ratio on the trailing surface. On the leading surface, the Nusselt number ratio decreases first (case 6) and then increases (case 7).

5. CONCLUSIONS

A multi-block RANS method was employed to predict three-dimensional flow and heat transfer in a rotating smooth and ribbed rectangular channel with aspect ratio of 4:1 and for various rotation numbers and inlet coolant-to-wall density ratios. Two channel orientations are studied: $\beta = 90^\circ$ and $135^\circ$. The present near-wall second-moment closure model results were compared with the experimental data of Griffith et al. [14]. It predicted fairly well the complex three-dimensional flow and heat transfer characteristics resulting from the large channel aspect ratio, rotation, centrifugal buoyancy forces and channel orientation. The main findings of the study may be summarized as follows.

A) Smooth duct:

I. The Coriolis force induces secondary flow, in the $\beta = 90^\circ$ rotating case, which pushes the cold fluid from the leading to the trailing surface.

II. The Coriolis force induces secondary flow, in the $\beta = 135^\circ$ rotating case, which pushes the cold fluid from the leading corner to the bottom surface.

III. In the $\beta = 135^\circ$ rotating case, most of the top surface behaves as a leading side and thus the Nusselt number ratios on this surface are lower than the corresponding ones on the $\beta = 90^\circ$ rotating case. Similarly, most of the bottom surface behaves as a trailing side. Thus, the increase in the Nusselt number ratios is higher on the bottom surface when compared with their counterparts in the $\beta = 90^\circ$ rotating case.

B) Ribbed duct:

I. The inclined ribs start two counter-rotating vortices that oscillate in size along the streamwise direction. For case 4 (non-rotating), the secondary flow results in steep temperature gradients and high heat transfer coefficients on both the top and ribbed surfaces.

II. For case 5 ($\beta = 90^\circ$), the rotation-induced cross-stream secondary flow distorts the rib-induced vortices and consequently, rotation shifts the temperature contours and affects the heat transfer coefficients from both the leading and trailing surfaces.

III. The rib-induced vortices are slightly distorted by low rotation-induced secondary flow (case 6) but significantly changed by the high rotation high density ratio induced secondary flow. This results into reversing the flow on the leading surface and reduces significantly the magnitude of rib-induced secondary flow on the trailing surface.

IV. For case 6, 7, 8 and 9 ($\beta = 135^\circ$):

III. The rib-induced vortices are slightly distorted by low rotation-induced secondary flow (case 6) but significantly changed by the high rotation high density ratio induced secondary flow. This results into reversing the flow on the leading surface and reduces significantly the magnitude of rib-induced secondary flow on the trailing surface.

Figure 16. Effect of rotation and density ratio on 135° Nusselt number ratio distribution for Re = 10,000.
V. The effect of increasing the density ratio (with fixed rotation number) is to have higher and uniform Nusselt number ratio on the leading and trailing surfaces.

VI. From design point of view, it is clear that the rib angle and the direction of rotation should be chosen such that the secondary flows that are induced by the rib angle and rotation direction should combine constructively to give maximum heat transfer.

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