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# PIEZOELECTRO-MAGNETIC BEHAVIOR OF SMART STRUCTURES

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## ABSTRACT

A piezoelectro-magnetic medium is formed by bonding a piezoelectric and magnetostrictive composite on a cantilever beam. Piezoelectro-magnetic materials are often utilized as intelligent media, in the design of distributed sensors and actuators of smart structures. A Spectral Finite Element model is presented for evaluating the structural response of flexible structures. A cantilever beam mounted with piezoelectric and piezomagnetic sensors is considered as the case study. Both sensor responses for dynamic force input are presented in this paper.

## NOMENCLATURE

| | |
|---|---|
| a | Length of the beam |
| c | Width of the beam |
| w | Height of the beam |
| $\delta_s$ | Thickness of sensors |
| Y | Youngs modulus of elasticity |
| **T** | Stress Vector |
| $[c]$ | Elasticity Matrix |
| **S** | Strain Vector |
| [h] | Piezoelectric constant matrix |
| **D** | Electric Displacement Vector |
| $[\ ]^t$ | Matrix Transposed |
| **E** | Electric Field Vector |
| $[\beta]$ | Di-electric Impermeability |
| $[S]$ | Elastic Compliance |
| $[\varepsilon]$ | Di-electric Matrix |
| [d] | Piezoelectricity Constant Matrix |
| $[g]$ | Effective Piezo Magnetic Strain |
| **B** | Magnetic Flux Density Vector |
| $[\mu]$ | Permeability Matrix |
| H | Magnetic Field Strength |
| $\nabla$ | Gradient Operator |
| $\phi$ | Electric Potential Vector |
| $A^s$ | Surface Area |
| $\Delta^s$ | Distance from the beam's neutral axis to the piezoelectric layer's central axis |
| $\omega$ | Circular Frequency |
| $w_3$ | Displacement in Vertical Direction |
| x | Horizontal Variable |
| y | Vertical Variable. |

## Introduction

An Intelligent/Smart structure responds to a variety of physical or environmental changes through a control system to maintain a desirable performance. For sensing and control of flexible structures such as beams, shells and plates, a piezoelectric material is desirable as it is sensitive to the dynamic characteristics of the structures on which it is mounted.

Tzou and Gadre (1) derived a multi-layered shell actuator theory for distributed vibration control of flexible shell structures. A novel technique to control the vibrations of N modes of a flexible link attached to a rigid robot using piezoceramic actuators and fiber optic sensors employing impedance control technique was developed by Kalaycioglu et al (2). Successful simulation and experimental results were obtained for the vibration control of Titan -II robot arm. Tzou (3) presented a new intelligent shell structure composed of conventional elastic shell, a distributed piezoelectric sensor and a distributed piezoelectric actuator. A new generic theory for intelligent shell system was developed and it was concluded that the distributed sensor was capable of sensing all shell vibration modes and distributed actuator controlling all shell modes.

The finite element method (FEM) is established as a powerful numerical technique that provides solutions to many complicated engineering problems and is widely used in modern engineering designs and analysis. Finite element formulation of piezoelectric media was presented by Sunar and Rao (4) for piezoelectric control of flexible structures subjected to structural vibrations. The effectiveness of piezoelectric actuators was studied for various locations. Sunar and Rao (5) reported the increasing trend of research in piezoelectrical sensing and actuation. Piezoelectric polymer PVDF and Piezo ceramics such as $BaTiO_3$ and PZT's are contemporary piezoelectric materials used in various applications. Han et al (6) obtained numerical and experimental results for reducing the vibrational level of light weight composite structures using active vibration control methods. Using the classical laminated composite beam theory and Ritz method an analytical model of laminated composite beam with piezoelectric sensors and actuators was developed.

Sunar et al (7) demonstrated that a thermopiezomagnetic medium can be formed by bonding together a piezoelectric and magnetostrictive composite. Finite element equations for thermopiezomagnetic media were obtained using linear constitutive equations in Hamilton's principle together with finite element approximations. It was also shown that an electrostatic field applied to piezoceramic layer causes strain in structure that in turn produces magnetic field in the magnetoceramic layer.

A spectral FEM for cantilever sandwitch beams excited via a pair of piezoelectric actuators was presented by Wang et al (8). The spectral FEM was formulated in frequency domain using dynamic shape functions based on the exact displacement solutions for wave propagation methods. Compared to conventional FEM, spectral FEM showed improved computational efficiency and accuracy. Doyle (9) showed the spectral formulation for a dynamic FEM such that the distributed mass is modelled exactly allowing elements to span from joint to joint, thus saving computing power and time. Doyle (10) presented the analytical solution for wave propagation in constant thickness Euler beam.

In many physical problems it is observed that mechanical, electrical and magnetic fields co-exist together. The piezoelectric & magnetostrictive layers can be bonded together to form composites that exhibit magneto-electric effect. As a result of this effect an electrical signal is obtained from the piezoelectric layer due to the application of a magnetic field to the magnetostrictive layer. Conversely, the magneto strictive layer can be magnetized because of the application of an electric field to the piezoelectric layer.

## Piezoelectro-magnetism

The constitutive equations are presented for a piezoelectro-magnetic medium where mechanical, electrical and magnetic fields interact with each other. The three fundamental equations representing the direct and converse piezoelectric and magnetic effects are as follows:

$$\mathbf{S} = [s]\mathbf{T} + [g]^t\mathbf{B} + [d]^t\mathbf{E} \quad (1)$$

$$\mathbf{D} = [d]\mathbf{T} + [b]^t\mathbf{B} + [\varepsilon]^t\mathbf{E} \quad (2)$$

$$\mathbf{H} = -[l]^t\mathbf{S} + [\mu]^{-1}\mathbf{B} - [b]\mathbf{E} \quad (3)$$

The alternative form of equation (2) is given by

$$\mathbf{E} = [\beta]\mathbf{D} + [\alpha]\mathbf{B} - [h]\mathbf{S} \quad (4)$$

A beam mounted with a piezoelectric sensor as the bottom layer

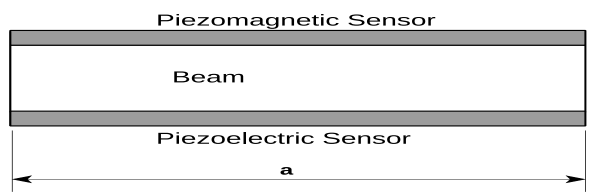


Figure 1. Composite Beam Model

and a piezomagnetic sensor as the top layer is considered as an example as shown in Fig. 1. For the piezoelectric sensor, the following relation is given

$$\mathbf{E} = -\nabla\phi \quad (5)$$

Following the procedure presented in Ref. (3), the voltage across the piezoelectric sensor can be obtained by integrating the electric field over the thickness

$$\phi = -\int_{\delta_s} E_3 dy \quad (6)$$

Substitution of equation(4) results

$$\phi = \delta_s[h_{31}S_1 - \beta_{33}D_3] \quad (7)$$

where $\delta_s$ denotes the piezoelectric sensor thickness and $S_1$ is the normal strain in the x direction. Hence the electric displacement $D_3$ is given by

$$D_3 = \frac{1}{\beta_{33}}\left[h_{31}S_1 - \phi/\delta_s\right] \tag{8}$$

The open circuit voltage $\phi^s$ is obtained by setting the charge to zero

$$\phi^s = \frac{\delta_s}{A^s}\int_{A^s}\left[h_{31}S_1\right]dA^S \tag{9}$$

For Euller-Bernoulli beam case the sensing equation of piezoelectric layer reduces to

$$\phi^s = -\frac{c\delta_s}{A^s}\int_x\left[h_{31}\Delta^s\frac{\partial^2 w_3}{\partial x^2}\right]dx = -\frac{c\delta_s}{A^s}h_{31}\Delta^s\frac{\partial w_3}{\partial x} \tag{10}$$

## Piezomagnetism

The magnetic field strength H, given by equation(3) with the absence of **B** and **E** terms reduces to

$$H_3 = -l_{31}S_1 \tag{11}$$

where $l_{31}$ is a constant for the piezomagnetic sensor (top layer).

## Spectral Finite Element Technique

The use of conventional finite element analysis for dynamic problems requires larger number of elements than needed for the equivalent static problem. To model the mass distribution exactly for the dynamic problem, the FEM using spectral formulation by Doyle (10) is used in this paper. The major significance of the method is that it allows elements to span from joint to joint, thus the subdivision of the beam into many elements is no longer needed.

In this approach the transient problem is converted to simple psuedo-static problem by transferring the equations of motion in frequency domain using the Fast Fourier Transformation (FFT). The resultant equations contain space as the only independent variable leading to the ordinary diffrential equations (ODE) that are solved at each frequency component using the FEM. The response is reconstructed in time domain using the Inverse Fast Fourier Transformation (IFFT).

## Comparison and Validity of Spectral FEM Model

An analytical study is performed by solving the beam equation with the application of boundary conditions for a cantilever case. First the standard beam equation is converted using the Fourier Transformation to yield the following equation

$$\frac{d^4\hat{w}_3}{dx^4} - \gamma^4\hat{w}_3 = 0 \tag{12}$$

where

$$\gamma^2 = \omega\sqrt{\frac{YI}{\rho A}}$$

The above equation is an ordinary differential equation that is dependent only on x and has the time derivative incorporated in $\hat{w}_3$. Force is introduced as the boundary condition in the above equation. The remaining boundary conditions are as follows: At free end i.e. at x=a,

$$YI\frac{d^3\hat{w}_3}{dx^3} = F$$

and

$$\frac{d^2\hat{w}_3}{dx^2} = 0$$

At fixed end i.e. at x=0,

$$\hat{w}_3 = \frac{d\hat{w}_3}{dx} = 0$$

With the application of the above boundary conditions in equation (12) the analytical electrical response is obtained using equation (10).

The so obtained analytical electrical response is compared with the Spectral FEM response at the center of the beam and there is great agreement in the results between the two as shown in Fig. 2. Also a convergence test for particular time equivalent to 200 $\mu s$ and at the center of the beam is performed using the Spectral FEM by varying the number of elements and the results are shown in Fig. 3. It is observed that the result obtained with two elements is same as that obtained with ten elements. Hence it is valid to run the program with two elements rather than more number of elements as it saves valuable computer time in obtaining the results. Fig. 4 shows the computer time variation with different number of elements where the simulations are performed on Pentium-I with 133 MHz speed. From the graph it is evident that for two elements it takes about 34 Sec whereas with ten elements it takes about 200 Sec, thereby saving considerable computer time.

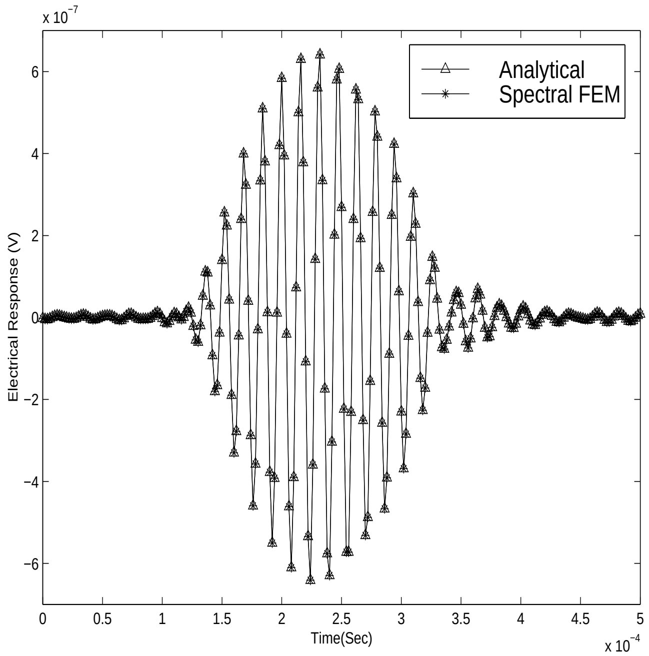


Figure 2. Comparison of Analytical and Spectral FEM Electrical Response

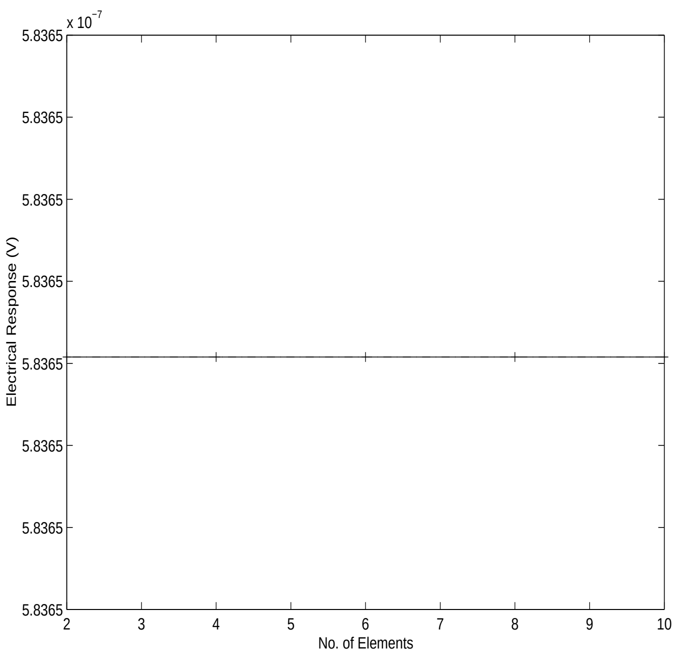


Figure 3. Convergence Test with respect to Electrical Response

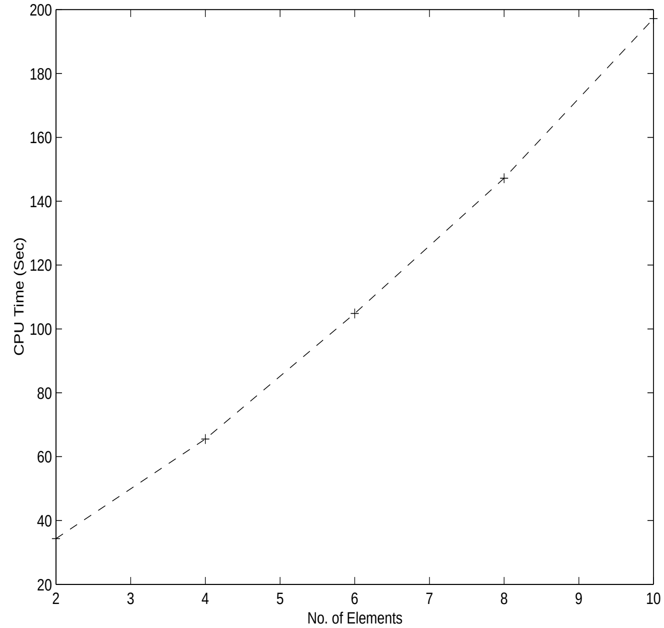


Figure 4. CPU Time required with varying Number of Elements for Spectral FEM

## 1 Numerical Example

A composite beam as shown before in Fig. 1 is considered as an example to illustrate the use of piezoelectro-magnetic equations. The dimensions of the beam are taken as length: a = 1 m, height: w = 0.0007 m, width: c = 0.1 m. The thickness of the sensors is taken as $\delta_s$ = 0.0002 m. The beam is composed of a piezoelectric sensor made of $BaTiO_3$ and piezomagnetic sensor made of $CoFe_2O_4$. The material properties are given in Table 1. The beam is subjected to a modulated signal input of the form F $= Cos(4000000t) \times Sin(t \times 1000000/70)$ N for a time interval t = 0 to 0.00024 Secs. The beam is divided into four elements each of about 0.25 m and Spectral FEM is used to obtain the Electrical and Magnetic response. The Electrical Response is shown in Fig. 5 which indicates that as we move to the free end of the beam there is an increase in its magnitude. The Magnetic Response at the fixed end of the beam is shown in Fig. 6. It is observed from simulations that the Magnetic Response is maximum at the fixed end and tends to decrease as we move towards the free end of the beam.

## Conclusion

The constitutive equations of piezoelectro-magnetic are used to show the interaction between electrical, mechanical and magnetic effects. A piezoelectro-magnetic beam is formed by bonding together layers of piezoelectric and magnetostrictive

Table 1. Material Properties

| | |
|---|---|
| Beam (Steel) | $Y$ (N/m$^2$) : $2.068 \times 10^{11}$<br>$\rho(kg/m^3)$: 7830 |
| Piezo Ceramic ($BaTiO_3$) | $h_{31}$ (C/m$^2$): $6.8 \times 10^8$ |
| Magneto Ceramic ($CoFe_2O_4$) | $l_{31}$ (N/Wb or A/m): $2.86 \times 10^8$ |

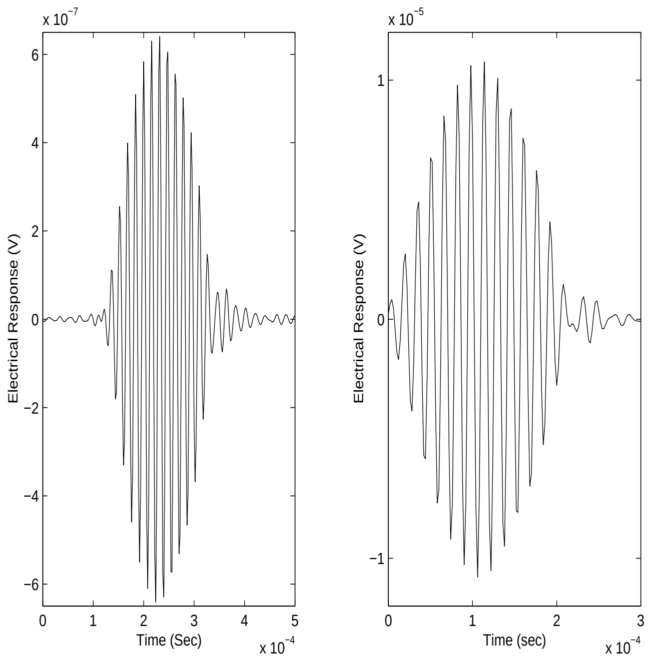


Figure 5. Electrical Response at the Middle and Free End of the Beam

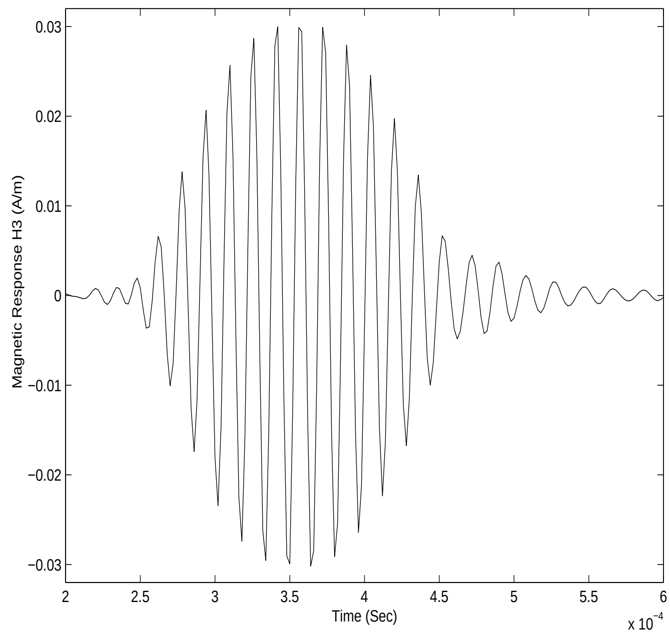


Figure 6. Magnetic Response at the Fixed End of the Beam

ceramics that act as sensors to dynamic force input. Spectral FEM technique is used for obtaining electrical and magnetic responses.


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## REFERENCES


Tzou, H.S. and Gadre, M. Theoretical analysis of multilayered thin shell coupled with piezoelectric shell actuators for distributed vibration controls. *Journal of Sound and Vibration*, 132:433–450, 1989.

Kalaycioglu, S., Giray, M. and Asmer, H. Vibration control of flexible manipulators using smart structures. *Journal of Aerospace Engineering*, 11:90–94, 1998.

Tzou, H.S. A new distributed sensor and actuator theory for intelligent shells. *Journal of Sound and Vibration*, 153(2):335–349, 1992.

Sunar, M. and Rao, S.S. Distributed modeling and actuator location for piezoelectric control systems. *AIAA Journal*, 34:2209–2211, 1996.

Sunar, M. and Rao, S.S. Recent advances in sensing and control of flexible structures via piezoelectric materials technology. *ASME Applied Mechanics Review*, 52(1):1–16, 1999.

Han, J., Rew, K. and Lee, In. Experimental study of active vibration control of composite structures with a piezo-ceramic actuator and a piezo-film sensor. *Smart Materials and Structures*, 6:549–558, 1997.

Sunar, M., Al-Garni, A.Z., Ali, M.H. and Kahraman, R. Finite element modeling of thermopiezomagnetic smart structures. *Submitted to AIAA Journal*.

Wang, G. and Wereley, N.M. Spectral finite element analysis of piezo-actuated sandwich beams with a viscoelastic core. *AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, 2681–2694, 1999.

Doyle, J.F. A spectrally formulated finite element for longitudinal wave propagation. *International Journal of Analytical and Experimental modal Analysis*, 3:636–645, 1987.

Doyle, J.F. *Wave Propagation in Structures*. Springer-Verlag.

Wansworth and Brooks, first edition, 1989.

 