CHAPTER 6
FAILURE THEORIES – STEADY LOADING

The failure theories in this chapter explain mechanisms of failure for the machine elements subjected to loads and stresses that do not change with time. We now know how to calculate stresses due to tension, bending, torsion, etc, but how are we going to put them together to judge whether the machine element will fail or not? So we need some criteria to decide on the failure of the machine element and the failure theories just do that! These theories are classified into the ones for *ductile materials* and those for *brittle materials*.

1) Failure Theories for Ductile Materials

6-4 Maximum Shear Stress (MSS) Theory:

This theory is originated by the fact the yield and fracture lines formed during a simple tension test make 45° angle with the *x* axis of the bar. Since the maximum shear stress (τ_{max}) calculated for this tension test also makes the same 45° angle with the *x* axis, it is logical to think that τ_{max} is responsible for the failure or yielding of the bar. Hence, for the general stress case, the failure occurs when

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \text{ or } \sigma_1 - \sigma_3 \geq S_y$$

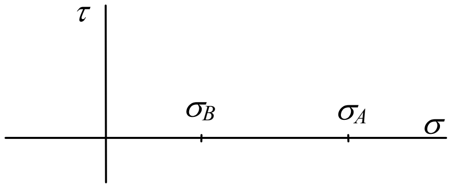
For the safety of the machine element we require

$$\sigma_1 - \sigma_3 = \frac{S_y}{n}$$

where *n* is the factor of safety and should be always greater one.

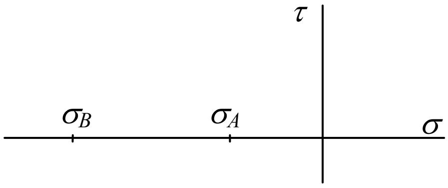
For <u>plane stress</u>, we have only 2 principal stresses: σ_A and σ_B, and we assume the third principal stress to be zero. Depending on the sign of σ_A and σ_B we have three cases:

Case 1: $\sigma_A \geq \sigma_B \geq 0$. As seen in the figure below, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$, and hence from the above equation: $\sigma_A = \frac{S_y}{n}$.



Case 2: $\sigma_A \geq 0 \geq \sigma_B$. From the figure below, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$, and hence from the above equation: $\sigma_A - \sigma_B = \frac{S_y}{n}$.

Case 3: $0 \geq \sigma_A \geq \sigma_B$. As seen in the figure below, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$, and hence from the above equation: $-\sigma_B = \dfrac{S_y}{n}$.



6-5 Distortion Energy (DE) Theory

This theory postulates that the failure of a machine element is because of its angular distortion. Hence the failure occurs when the distortion energy per unit volume reaches or exceeds the distortion energy per unit volume of the element during the simple tension test. This theory results in the following for the safety of the machine element:

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2} = \frac{S_y}{n}$$

where σ' is known as the von Mises stress, which can also be written as

$$\sigma' = \left[\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}{2}\right]^{1/2}.$$

For the *plane stress*, the von Mises stress becomes

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2} \text{ or } \sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$$

where σ_A and σ_B are the two principal stresses in plane as before.

Note: Review Example 6-1, pgs. 264-266, in the textbook.

6-6 Ductile Coulomb-Mohr (DCM) Theory

This theory is used if the yield strengths of the material for tension and compression are not the same, i.e. $S_{yt} \neq S_{yc}$. According to this theory we have:

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} = \frac{1}{n}$$

For the *plane stress*, there are 3 cases:

Case 1: $\sigma_A \geq \sigma_B \geq 0$, then $\sigma_A = \dfrac{S_{yt}}{n}$.

Case 2: $\sigma_A \geq 0 \geq \sigma_B$, then $\dfrac{\sigma_A}{S_{yt}} - \dfrac{\sigma_B}{S_{yc}} = \dfrac{1}{n}$.

Case 3: $0 \geq \sigma_A \geq \sigma_B$, then $-\sigma_B = \dfrac{S_{yc}}{n}$.

Note: Review Example 6-2, pg. 268, and Examples 6-3 and 6-4, pgs. 270-272, in the textbook.

2) Failure Theories for Brittle Materials

6-8 Maximum Normal Stress (MNS) Theory:

This theory states that the failure for a brittle material occurs when any of the three principal stresses equals or exceeds the ultimate strength of the material. If the principal stresses are ordered as $\sigma_1 \geq \sigma_2 \geq \sigma_3$, then for safety we need,

$$\sigma_1 = \frac{S_{ut}}{n} \quad \text{or} \quad -\sigma_3 = \frac{S_{uc}}{n}$$

where S_{ut} and S_{uc} are the ultimate tensile and compressive strengths for the material.

6-9 Brittle Coulomb-Mohr Theory and its Modifications

Brittle Coulomb-Mohr (BCM):
For the *plane stress*, we have:

If $\sigma_A \geq \sigma_B \geq 0$, then $\sigma_A = \dfrac{S_{ut}}{n}$.

If $\sigma_A \geq 0 \geq \sigma_B$, then $\dfrac{\sigma_A}{S_{ut}} - \dfrac{\sigma_B}{S_{uc}} = \dfrac{1}{n}$.

If $0 \geq \sigma_A \geq \sigma_B$, then $-\sigma_B = \dfrac{S_{uc}}{n}$.

Modified II-Mohr (M2M):
Again for the *plane stress*, we have:

If 1) $\sigma_A \geq \sigma_B \geq 0$ or 2) $\sigma_A \geq 0 \geq \sigma_B$ and $\left|\dfrac{\sigma_B}{\sigma_A}\right| \leq 1$ then $\sigma_A = \dfrac{S_{ut}}{n}$.

If $\sigma_A \geq 0 \geq \sigma_B$ and $\left|\dfrac{\sigma_B}{\sigma_A}\right| > 1$ then $\dfrac{n\sigma_A}{S_{ut}} + \left(\dfrac{n\sigma_B + S_{ut}}{S_{ut} - S_{uc}}\right)^2 = 1$.

If $0 \geq \sigma_A \geq \sigma_B$, then $-\sigma_B = \frac{S_{uc}}{n}$.

Note: Review Example 6-5, pg. 276, in the textbook.

After listing all the failure theories, we can now ask the question: how do we select a failure theory? The answer is given in the textbook in Figure 6-30, pg. 278, where a nice selection chart is shown.