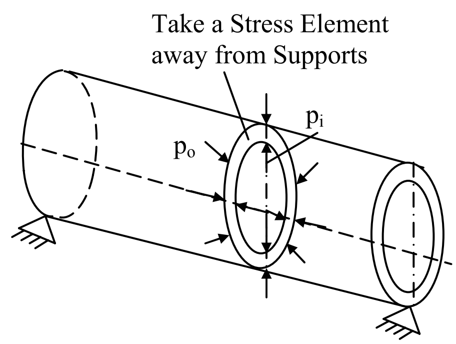
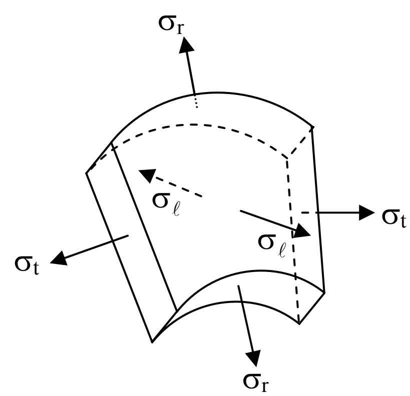
4-15 Stresses in Pressurized Cylinders

Cylindrical pressure vessels, hydraulic cylinders, air tanks and pipes carrying pressurized fluids develop tangential, radial and longitudinal (axial) stresses (σ_t, σ_r and σ_ℓ). A typical pressurized cylinder with inside and outside pressures of p_i and p_o is shown below:



The 3-D stress element taken from the wall of the cylinder shown above is enlarged as:



These stresses are given as:

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_\ell = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

In these equations, r_i and r_o are the inside and outside radii for the cross-section and r is the radial distance of the stress element on the cross-section from its center. *When the outside pressure is zero*, *i.e.* $p_o=0$,

$$\sigma_t = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

$$\sigma_\ell = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

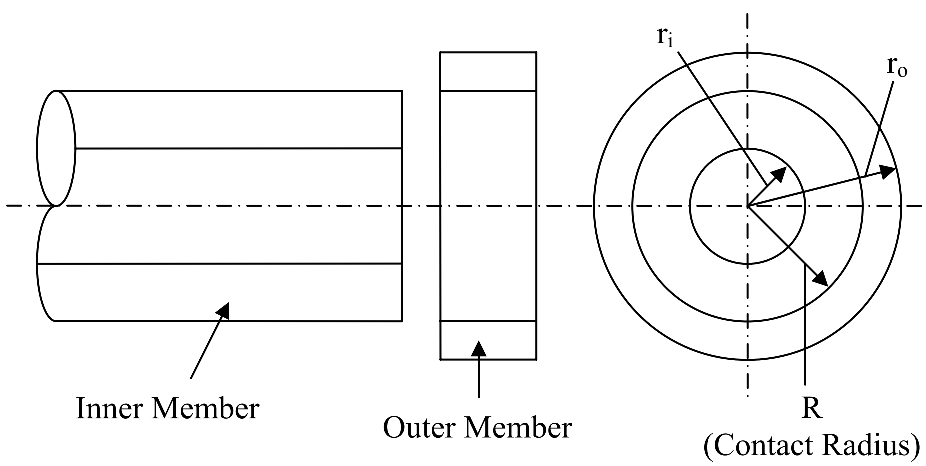
Thin-Walled Cylinders:

In the special case when the thickness of the cylinder t is much less than its average radius r, i.e. $r \geq 20t$, and assuming again that $p_o=0$, then the above equations greatly simplify to:

$$(\sigma_t)_{max} = \frac{p_i(d_i + t)}{2t}, \quad \sigma_r \approx 0 \text{ and } \sigma_\ell = \frac{p_i d_i}{4t}.$$

4-17 Press and Shrink Fits

When two cylindrical parts are assembled by shrinking or press fitting one part upon another, a contact pressure p is developed between the two parts along the contact surface at the radius of R. An example is the press fitting of a gear onto a shaft. The below figure illustrates the process:



After the assemblage, the inner member shrinks an amount of δ_i and the outer member expands δ_o in the radial direction. Given the total interference $\delta = |\delta_o| + |\delta_i|$, we can find the contact pressure p from the following relation:

$$\delta = \frac{pR}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{pR}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)$$

where E_o and E_i are the modules of elasticity; ν_o and ν_i are the Poisson's ratios for the outer and inner members. If the materials of the outer and inner members are same, i.e. E_o=E_i and ν_o=ν_i, then the pressure p is given directly as

$$p = \frac{E\delta}{R}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)}\right].$$