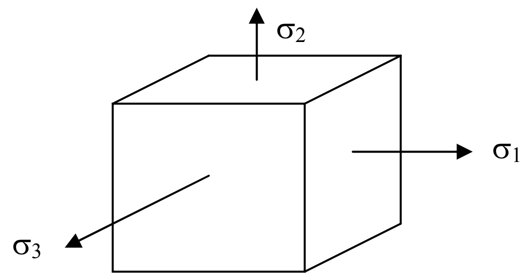
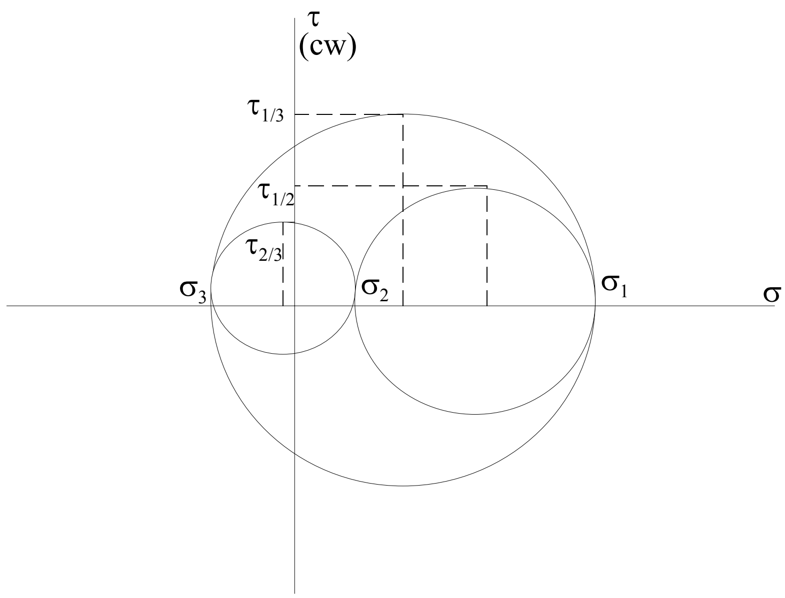
4-7 General 3-D Stress

In general for a *3-D stress*, there are 3 principal stresses: σ_1, σ_2 and σ_3. A 3-D stress element with the 3 principal stresses are shown below:



The Mohr's circle for the above stress element would look like this:



The extreme shear stresses are:

$$\tau_{1/2} = (\sigma_1 - \sigma_2)/2,\ \ \tau_{1/3} = (\sigma_1 - \sigma_3)/2 \ \text{ and } \tau_{2/3} = (\sigma_2 - \sigma_3)/2.$$

The maximum shear stress is:

$$\tau_{max} = \tau_{1/3}$$

Even for a *2-D plane stress analysis*, one can say that there are 3 principal stresses. σ_1 and σ_2 are found using the equations given before, but $\sigma_3=0$. There are 3 scenarios, which are:

1) $\sigma_1, \sigma_2 > 0, \sigma_3=0$; 2) $\sigma_1, \sigma_2 < 0, \sigma_3=0$ and 3) $\sigma_1 > 0, \sigma_2 < 0, \sigma_3=0$.

Assuming $\sigma_1>\sigma_2$, for the first case: $\tau_{max} = \tau_{1/3} = (\sigma_1-\sigma_3)/2 = \sigma_1/2$, for the second case: $\tau_{max} = \tau_{2/3} = (\sigma_2-\sigma_3)/2 = \sigma_2/2$ and for the third case: $\tau_{max} = \tau_{1/2} = (\sigma_1-\sigma_2)/2$.

4-8, 9 Elastic Strain and Stress

For a bar subject to a tensile axial load of F, the bar elongates with an amount of δ, then the axial elastic strain ε is defined as

$$\varepsilon = \delta/L$$

where L is the original length of the bar. The axial stress σ is simply

$$\sigma = F/A$$

where A is the cross-sectional area of the bar. The Hooke's Law is given as

$$\sigma = E\varepsilon$$

where E is the modulus elasticity of the material of the bar, which is given as 207 GPa for the carbon steel. Refer to Table A-5, p. 963, in the Appendix A of the textbook, for the physical constants of materials. From the above relations, one can obtain the elongation of the bar to be

$$\delta = (FL)/(AE).$$

Also, the shear stress τ for a bar subjected to a direct shear force of F (scissor action)

$$\tau = F/A$$

for which the Hooke's Law is expressed as

$$\tau = G\gamma$$

where γ is the shear angle and G is the modulus of rigidity related to E as

$$E = 2G(1+\nu)$$

in which the symbol ν stands for the Poisson's ratio defined as

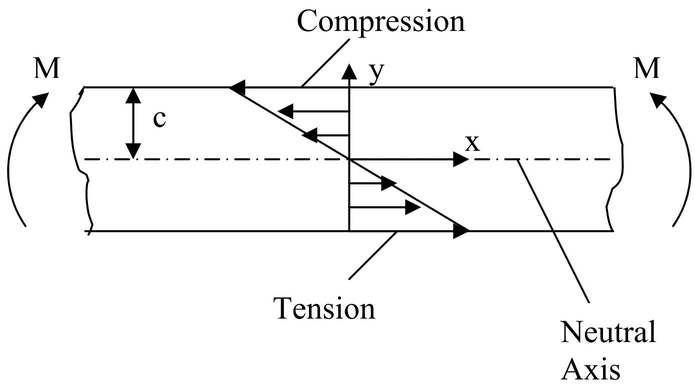
$$\nu = -\text{Lateral Strain/Axial Strain.}$$

The Poisson's ratio can be taken as 0.3 for the carbon steel, again refer to Table A-5 in the textbook.

For the general elastic stress-strain relations (Hooke's Laws) for the uniaxial (1-D), biaxial (2-D) and triaxial (3-D) stress cases, refer to Table 4-2, p. 124, in the textbook.

4-10 Normal Stresses for Beams in Bending

A cut beam subject to a positive bending moment M is shown below:



The normal stress at a distance y from the neutral axis is given by:

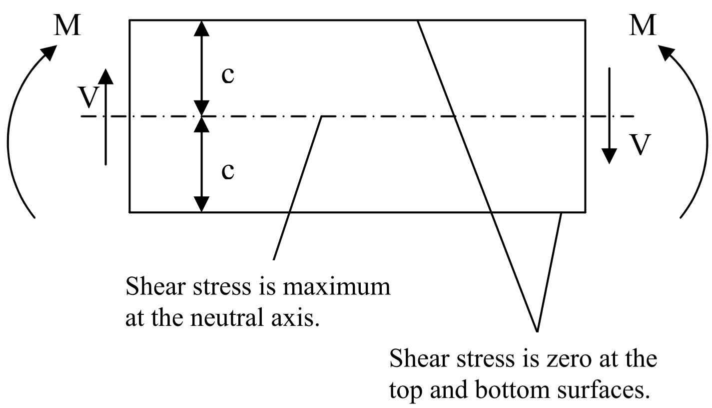
$$\sigma_x = -(M/I)y$$

where I is the area moment of inertia or the second moment of area about the z axis. The stress at the top surface is compression whose value is $\sigma_x = -(M/I)c$ and at the bottom surface the stress is tension given by $\sigma_x = (M/I)c$. The moment of inertia for a solid round cross-section is: $I=\pi d^4/64$ and therefore the normal stress is given by: $\sigma_x = 32M/(\pi d^3)$. For a rectangular cross-section of width b and height h, $I = bh^3/12$ and $\sigma_x = 6M/(bh^2)$.

Note: Review Examples 4-5 and 4-6 in textbook, pages 127-130.

4-12 Shear Stresses for Beams in Bending

The shear force V for a beam in bending causes shear stresses, which are maximum at the neutral axis, and zero at the top and bottom surfaces. The cut beam is shown below:



For a rectangular cross-section, the shear stress on a stress element at a distance y from the neutral axis is given by:

$$\tau_{xy} = 3V(1-y^2/c^2)/(2A)$$

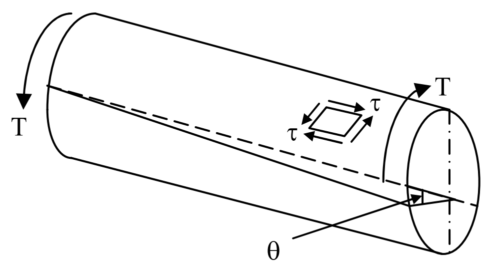
Hence, it is zero at the top and bottom surfaces where $y=\pm c$; and it is maximum at the neutral axis with a value of $\tau_{xy} = 3V/(2A)$. *For a solid round cross-section*,

$$\tau_{xy} = 4V(1-y^2/c^2)/(3A)$$

And again, it is zero at the top and bottom surfaces where $y=\pm c$; and it is maximum at the neutral axis with a value of $\tau_{xy} = 4V/(3A)$. For different cross-sections, see Table 4-3, p.136, in the textbook.

4-13 Torsion

The twisting torque T on a round bar or shaft causes angular displacements and shear stresses as shown below:



The angular displacement or angle of twist θ is given as:

$$\theta = TL/(G\,J)$$

where L is the length of the bar and J is the polar moment of inertia or second polar moment of area about the centroid of the cross-section. For a solid round cross-section with a diameter of d, J is given as $J=\pi d^4/32$. The shear stress caused by the torque T *on a round bar* is given by:

$$\tau = T\rho/J$$

where ρ is the radial distance of the stress element from the center of the cross-section. This shear stress is maximum at the outside surface, which is at a distance of $\rho = r$ from the center. This maximum stress is

$$\tau = Tr/J = 16T/(\pi d^3)$$

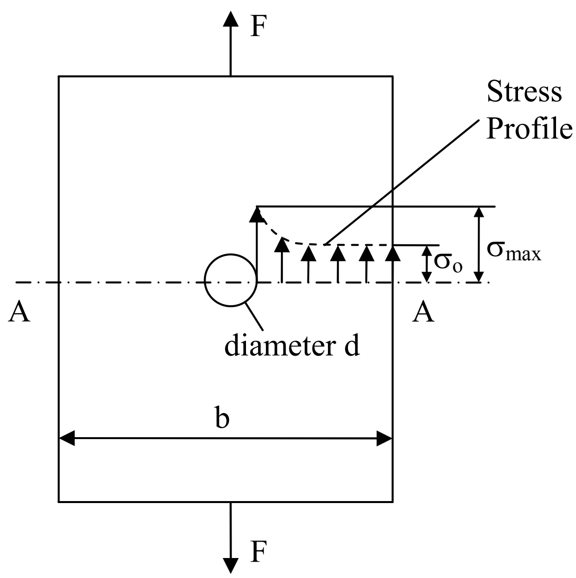
for a solid round cross-section at the outside surface. The maximum shear stress for a rectangular cross-section of $b\times c$, where b is the longer side, i.e. $b>c$:

$$\tau = T(3 + 1.8c/b)/(bc^2).$$

Note: Review Examples 4-7 (p.133), 4-8 (p.139) and 4-9 (p. 141) in the textbook.

4-14 Stress Concentration

The elastic stress across the cross-section of a machine element is uniform in the case of a bar in tension, or linear as in the case of a beam in bending. Many times, machine elements are required to have holes, notches, grooves etc. due to various reasons. For example, a shaft may be drilled a hole because of mounting a gear onto it. Such discontinuities disturb the stress distribution in machine elements and cause stress concentrations. See, for example, the following plate with a hole of diameter d subjected to tension:



At the section of A-A, the nominal stress σ_o is

$$\sigma_o = \frac{F}{t(b-d)}$$

where t is the thickness of the plate. But, due to the stress concentration at the edge of the hole, this stress rises to σ_{max}, which is given as

$$\sigma_{max} = K_t \, \sigma_o$$

where K_t is called the theoretical stress concentration factor for normal stress and $K_t \geq 1$. We have a similar scenario for the case of shear stress where:

$$\tau_{max} = K_{ts} \, \tau_o$$

where K_{ts} is the theoretical stress concentration factor for shear stress and again $K_{ts} \geq 1$. The values of K_t and K_{ts} depend on the type of loading, i.e. tension, bending, torsion, and also on the geometry. The values for several cases are given in the Appendix of the textbook, Table A-15, pages 982-988.