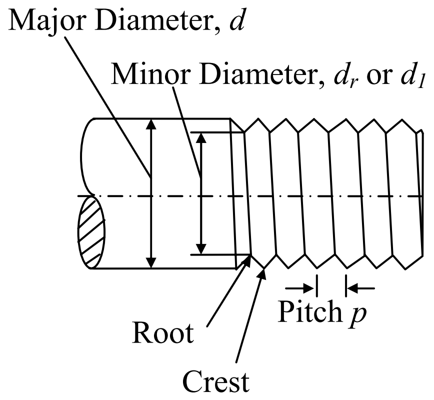
CHAPTER 8
SCREWS, FASTENERS, NONPERMANENT JOINTS

This chapter deals with the design and analysis of nonpermanent fasteners such as bolts, power screws, cap screws, setscrews, keys and pins.

8-1 Standards and Definitions

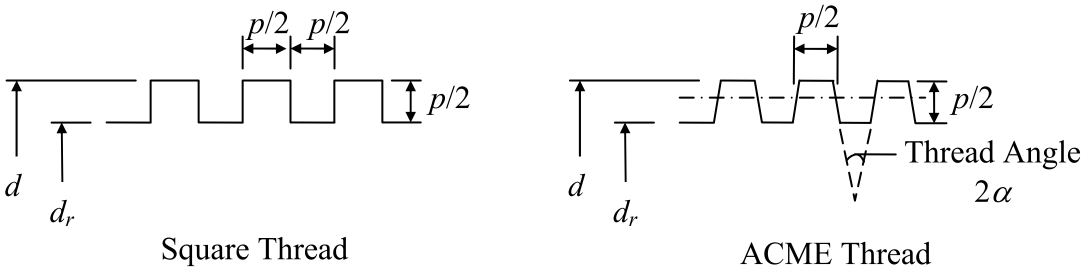
A screw thread used for fastening and related terminology are shown below in the figure:



Another important dimension not shown in the figure is the lead l, which is the distance that the nut advances along the screw axis when it is given one complete turn. We have $l = np$, where n is the number of threads for multiple-threaded screws. Standard screws are single-threaded, and hence for them $l = p$. There are two standards for screw threads, which are Unified (UN, Table 8-2, pg. 399) for inch-types and Metric (M, Table 8-1, pg. 398) for millimeter-types. In each type, there are Course (C) and Fine (F) threads. Furthermore, the MJ threads for the Metric and UNR threads for the Unified have rounded roots to improve their fatigue strengths. As an example, a M12 × 1.75 thread would mean a Metric thread with a major diameter of d=12 mm and a pitch of p=1.75 mm. Also, a $\frac{5}{8}$ in – 18 UNRF would indicate a Unified Fine thread with a rounded Root having a major diameter of $d=\frac{5}{8}$ in = 0.625in and 18 threads in one inch of axial length.

8-2 Power Screws

The power screws are used for power transmission, and although the basic terms are same as those of the fastening screws, nevertheless they have different threads. There are two types of threads: Square and ACME, which are shown below:



Important considerations for the power screws are the torques needed for raising a load (T_R) and lowering a load (T_L), which are calculated for a square thread as

$$T_R = \frac{F\,d_m}{2}\left(\frac{\pi\,f\,d_m + l}{\pi\,d_m - f\,l}\right) \text{ and } T_L = \frac{F\,d_m}{2}\left(\frac{\pi\,f\,d_m - l}{\pi\,d_m + f\,l}\right)$$

where d_m is the mean diameter of the power screw given for a square thread as $d_m = d - p/2$. Also, F is the load to be raised or lowered and f is the coefficient of friction between the screw and nut. A power screw is said to be *self-locking* if the nut does not roll back by itself after the load is raised. For the self-locking of a screw, we should have

$$T_L > 0\text{, or, from the above equation } \pi f\,d_m > l.$$

We also define an efficiency e for raising the load

$$e = \frac{T_0}{T_R}$$

where T_0 is the torque for raising the load without the friction, i.e. $T_0 = T_R\big|_{f=0} = \frac{F\,l}{2\pi}$, and hence

$$e = \frac{F\,l}{2\pi\,T_R}.$$

The torques for raising and lowering the load for an ACME thread are found by multiplying the friction terms by $\frac{1}{\cos\alpha} = \sec\alpha$, and thus

$$T_R = \frac{F\,d_m}{2}\left(\frac{\pi\,f\,d_m \sec\alpha + l}{\pi\,d_m - f\,l\sec\alpha}\right) \text{ and } T_L = \frac{F\,d_m}{2}\left(\frac{\pi\,f\,d_m \sec\alpha - l}{\pi\,d_m + f\,l\sec\alpha}\right).$$

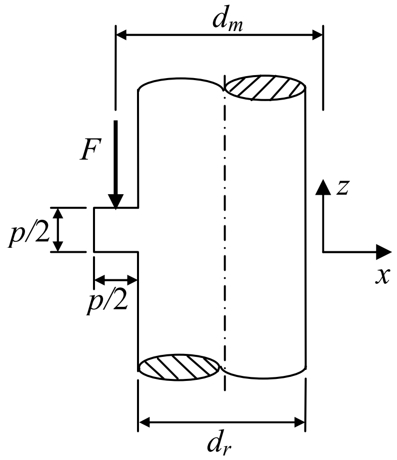
For both the square and ACME threads, if there is a collar attached to the screw for the load to sit on, then the friction on this collar should also be included for finding the torques of raising and lowering the load. We now have the total torque T for raising the load

$$T = T_R + T_c = T_R + \frac{F f_c d_c}{2}$$

where f_c is the coefficient of friction between the collar and the load, and d_c is the diameter of friction circle. For lowering the load, the total torque T is given by

$$T = T_L + T_c.$$

The stresses on a power screw during raising or lowering a load are 3-dimensional. These stresses are given for a square thread as:



1) Axial compressive stress: $\sigma_z = -\dfrac{4F}{\pi d_r^2}$,

2) Torsional shear stress: $\tau_{yz} = \dfrac{16T}{\pi d_r^3}$, and

3) Bending stress: $\sigma_x = \dfrac{6F}{\pi d_r n_t p}$. This bending stress can also be approximated as $\sigma_x = \dfrac{6(0.38F)}{\pi d_r p}$, which assumes that only the 1st tooth between the nut and screw takes the 38% of the load. Hence, we can compute an equivalent von Mises stress σ' from these 3 stresses using the equation (6-14) in Chapter 6, pg. 262, as

$$\sigma' = \frac{1}{\sqrt{2}}\left[\sigma_x^2 + \sigma_z^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{yz}^2\right]^{1/2}$$

This equivalent stress σ' can be used to find a factor of safety for the power screw to decide on its strength. There is another stress, bearing stress σ_B, that is used to judge if the force F is crushing the surface of the thread. This stress is calculated as

$$\sigma_B = \frac{F}{\pi d_m n_t p / 2} = \frac{2F}{\pi d_m n_t p} .$$

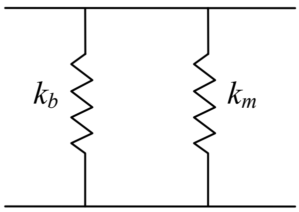
Note: Review Example 8-1, pgs. 405-407.

8-3 Standards and Definitions

Various hexagonal-head bolts and cap screws are shown in Figures 8-9 and 8-10, pg. 409, in the textbook. Several machine screw head types are illustrated in Figure 8-11, pg. 410. Dimensions of hexagonal bolts and cap screws are given in the appendix of the textbook, Tables A-29 and A-30, pgs. 1007 and 1008. Hexagonal nuts are shown in Figure 8-12, pg. 410, and their dimensions are provided in the appendix, Table A-31, pg. 1009.

8-4 Modeling of Screw Joints – Fastener (Bolt or Screw) Stiffness

A screw joint or connection is idealized through parallel springs with a bolt stiffness (spring constant) of k_b and member stiffness of k_m as shown in the figure below:



The bolt stiffness k_b is found using the length of the screw within the grip of the connection. The model of the bolt is thought as two springs in series, one spring corresponding to the unthreaded portion and the other spring for the threaded portion of the bolt. So, we have

$$\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t}$$

where k_d and k_t are the stiffnesses of the unthreaded and threaded portions of the screw, which are defined as

$$k_d = \frac{A_d E}{l_d} \text{ and } k_t = \frac{A_t E}{l_t}$$

where

A_d = cross-sectional area of unthreaded portion = $\frac{\pi}{4} d^2$ (d is major diameter),
l_d = length of unthreaded portion,
A_t = tensile stress area found in Tables 8-1 and 8-2 (pgs. 398 and 399), and
l_t = length of threaded portion of the screw within the grip.

Hence, we have from the above equations:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} .$$

8-5 Modeling of Screw Joints – Member Stiffness

The calculation of the stiffness k_m for the members clamped by a bolt is not as straightforward. Members are clamped in series, hence we have

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$

The stiffness for each member in the above equation (k_1, k_2, and so on) can be found from

$$k = \frac{0.5774\,\pi\,E\,d}{ln\dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$$

where t is the thickness of the member, d is the major bolt diameter and D=1.5d. If there is a soft gasket among the members, then we can set as $k_m = k_g$, where k_g is the gasket stiffness. If there are only two clamped members that are identical, i.e. they have the same thickness of t and are made up of the same material, then the resultant member stiffness is obtained directly from

$$k_m = \frac{0.5774\,\pi\,E\,d}{2\,ln\left(5\dfrac{(0.5774l + 0.5d)}{(0.5774l + 2.5d)}\right)}$$

where l=2t. In the case of two identical members we can also use the equation

$$\frac{k_m}{Ed} = A\,e^{Bd\,/\,l}$$

where the constants A and B can be obtained from Table 8-8, pg. 416, in the textbook.

Note: Review Example 8-2, pgs. 416 and 417.

8-6 Bolt Strength – Material Properties for Bolts

A bolt strength is specified by its proof and tensile strengths, which are listed in Tables 8-9, 8-10 (UN bolts) and 8-11 (M bolts), pgs. 418 through 420 in the textbook. A proof load is the maximum load or force that a bolt can withstand without becoming plastic. Then the proof strength is defined as the proof load divided by the bolt's tensile stress area.

8-7 and 8-9 Static Analysis of Bolts

The scenario is as follows. We first apply a preload of F_i to the bolt, which subjects the bolt to tension and clamped members to compression. Afterwards, the joint is put to service, where both the bolt and members are subjected to an external load of P. This P is taken by the bolt as P_b and by the members as P_m. Remember from the previous Section 8-4 that the bolt and members are connected as two parallel springs of k_b and k_m. We can find the following relations

$$P_b = \frac{k_b}{k_b + k_m} P = C\,P \text{ and } P_m = \frac{k_m}{k_b + k_m} P = (1 - C)\,P$$

where C is called the joint constant. Consequently we have

F_b = resultant bolt load = $P_b + F_i = CP + F_i$

and

F_m = resultant member load = $P_m - F_i = (1-C)P - F_i$

The members should always be compressed and hence we always need $F_m < 0$. The preload F_i in the above equations is assumed as

$$F_i = \begin{cases} 0.75\,A_t S_p & \text{for reused bolts or screws} \\ 0.90\,A_t S_p & \text{for permanent bolts or screws} \end{cases}$$

Now we can calculate the bolt stress as

$$\sigma_b = \frac{F_b}{A_t} = \frac{C\,P}{A_t} + \frac{F_i}{A_t}$$

Multiplying the external load P by a load factor of n and letting the bolt stress equal to its proof strength at the limit, we get

$$\frac{C\,nP}{A_t} + \frac{F_i}{A_t} = S_p$$

which we can solve it for the load factor as

$$n = \frac{S_p A_t - F_i}{C\,P}.$$

Remember that always $n > 1$ and it stands for a kind of factor of safety for the bolt strength. We should also check the member tightness with another factor of safety n_0 to guard against the joint separation. We have for the members at the limit of separation

$$(1-C)\, n_0\, P - F_i = 0$$

from which we obtain

$$n_0 = \frac{F_i}{(1 - C)P}.$$

And again we need $n_0 > 1$.

Note: Review Example 8-4, pgs. 427 and 428.

8-8 Required Bolt Torque

We can express the torque T required to produce a preload F_i by

$$T = KF_i d$$

where K is obtained for different bolt conditions from Table 8-15, pg. 424. In the absence of any information we can assume that $K = 0.2$.

Note: Review Example 8-3 (parts a and b), pgs. 424 and 425.

8-11 Fatigue Analysis

For the fatigue, we assume that the external load changes between 0 and P. Hence we have the maximum and minimum bolt forces as

$$(F_b)_{\max} = CP + F_i \text{ and } (F_b)_{\min} = F_i$$

Then the maximum and minimum bolt stresses are found as

$$(\sigma_b)_{\max} = \frac{CP}{A_t} + \frac{F_i}{A_t} = \frac{CP}{A_t} + \sigma_i \text{ and } (\sigma_b)_{\min} = \frac{F_i}{A_t} = \sigma_i$$

Thus, we have the alternating and midrange stresses in the bolt as

$$\sigma_a = \frac{|(\sigma_b)_{\max} - (\sigma_b)_{\min}|}{2} = \frac{CP}{2A_t}$$

and

$$\sigma_m = \frac{(\sigma_b)_{\max} + (\sigma_b)_{\min}}{2} = \frac{CP}{2A_t} + \sigma_i = \sigma_a + \sigma_i$$

Now we can use one of the fatigue methods to find the fatigue factor of safety n_f for the bolt. If we use the Gerber approach

$$\frac{n_f \sigma_a}{S_e} + \left(\frac{n_f \sigma_m}{S_{ut}}\right)^2 = 1$$

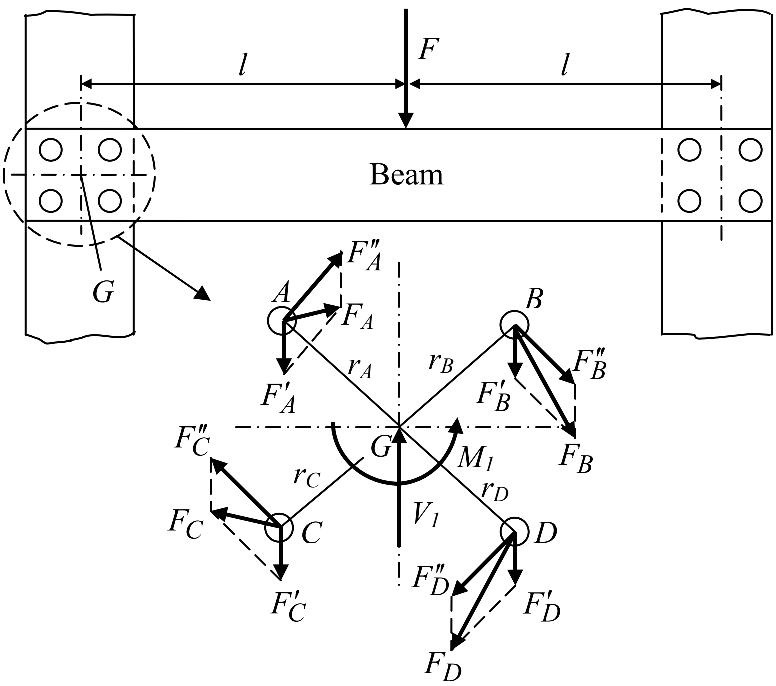
which can be solved for n_f. The endurance limit or endurance strength S_e in the above equation for the bolt can be obtained directly from Table 8-17, pg. 430, in the textbook. Remember that we get the tensile strength S_{ut} for the bolt from Tables 8-9 through 8-11, pgs. 418-420. Or, we can find the fatigue factor of safety from $n_f = S_a / \sigma_a$, where S_a is given as

$$S_a = \frac{1}{2S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right].$$

Note: Review Example 8-5 (only concern about n, n_0 and n_f), pgs. 432-435.

8-12 Shear Joints

When a group of bolts or pins are subjected to shear, we have to find the resultant shear forces and stresses on the bolts and identify the critical bolt or bolts taking the maximum shear stress. A typical situation is shown below:



There are four bolts or pins at the left side of the beam, which are resisting the reaction force of V_1 and the moment of $M_1 = Fl$. Reactions V_1 and M_1 are put in the centroid of the left bolt group, G. We have two types of shear forces in each bolt, one is the primary shear force (F'_A, F'_B, F'_C and F'_D) and the other is the secondary shear force (F''_A, F''_B, F''_C and F''_D). The primary shear forces are due to the shear force of V_1 and hence are equal to

$$F'_A = F'_B = F'_C = F'_D = \frac{V_1}{n}$$

where n is the number bolts in the bolt group (it is 4 in this case). The secondary shear forces exist to resist the bending moment M_1 and thus we have

$$F''_A = \frac{M_1 r_A}{r_A^2 + r_B^2 + r_C^2 + r_D^2} \text{ and so on.}$$

As shown in the figure, we have to find out the resultant shear force in each bolt (F_A, F_B, F_C, and F_D) and divide it by the cross-sectional area to calculate the shear stress. It looks like in the figure that the bolts B and D are taking the maximum shear forces. Hence, assuming that all the bolts are of the same size (diameter), the bolts B and D are the critical ones, because they get the maximum shear stresses.

Note: Review Example 8-7, pgs. 439 and 440.

8-13, 14 Setscrews, Keys and Pins

These machine elements are used to mount a hub (disk, gear, pulley, etc) on a shaft. Setscrews rely on compression to develop a clamping force between the hub and the shaft. This clamping force generates a friction force along the contact surface, which holds the hub axially on the shaft. This is in a way similar to a press and shrink fit. Several types of setscrews are shown in Figure 8-27 and setscrew sizes are given in Table 8-18, pg. 441, in the textbook.

Keys and pins are also used for mounting hubs onto shafts (see Figure 8-28, pg. 442). Keys are categorized according to their cross-sections: rectangular, square and round keys. There are also special keys: Gib-head and Woodruff keys. The keys are placed inside grooves between the shaft and hub to provide axial positioning of the hub on the shaft, whereas the pins are inserted inside transverse holes in shafts and hubs. The pins are also classified with respect to their shapes: round and taper pins. Some key and pin dimensions are listed in Tables 8-19 and 8-20, pgs. 442 and 443.

Note: Review Example 8-8, pg. 446.