CHAPTER 18
SHAFTS

A shaft is a rotating member, usually of circular cross-section, used to transmit power or motion. A shaft analysis and design usually have 2 concerns: 1) deflection and slope (Chapter 5), 2) stress and strength (Fatigue, Chapter 7).

1) Deflection and Slope (18-2 Geometric Constraints)
The shafts are treated as beams to find their deflections and slopes. There are several methods for deflection and slope analysis, such as Castigliano, integration and superposition. We've learned all these methods in Chapter 5, out of which the superposition method is used in the textbook in Chapter 18 to come up with equations of deflection and slope for a shaft that has many applied forces (F_i's) and applied moments (M_i's) and is supported at the two ends with bearings (see equations in pgs. 927 and 928, in the textbook). Based on allowable slopes at the bearings of this shaft, two diameters are found through equations (18-1) and (18-2), pg. 928, from which we should choose the greater one for the shaft diameter. We can repeat this process for different uniform-diameter shaft loadings and supports using the superposition principle and Table A-9, pgs. 969-976.

Note: Review Examples 18-1 and 18-2, pgs. 928-931.

To find deflections and slopes for stepped shafts, we cannot directly use Table A-9, because it is for the uniform-diameter shaft. We have to refer to methods like Castigliano or integration to take care of different diameters of the shaft.

2) Stress and Strength (18-3 Strength Constraints)
We have to perform a fatigue analysis for the shaft using the DE-Gerber or DE-Elliptic Criterion. The procedure for the two criteria are very similar and hence we can concentrate on the DE-Gerber Criterion. In general, the shaft is subjected to the alternating and midrange bending moments and torques, i.e. M_m, M_a, T_m and T_a. Thus, with the appropriate fatigue stress concentration factors, for a solid round shaft of diameter d, we have

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3}, \quad \sigma_m = K_f \frac{32 M_m}{\pi d^3}, \quad \tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \text{ and } \tau_m = K_{fs} \frac{16 T_m}{\pi d^3}.$$

We then define the equivalent alternating and midrange stress components as

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{16}{\pi d^3}\sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2} = \frac{16 A}{\pi d^3} \text{ and}$$
$$\sigma'_m = \sqrt{\sigma_m^2 + 3\tau_m^2} = \frac{16}{\pi d^3}\sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2} = \frac{16 B}{\pi d^3}.$$

Remember the Gerber fatigue method

$$\frac{S_a}{S_e}+\left(\frac{S_m}{S_{ut}}\right)^2=\frac{n_f\,\sigma_a'}{S_e}+\left(\frac{n_f\,\sigma_m'}{S_{ut}}\right)^2=\frac{16n_f A}{\pi d^3 S_e}+\left(\frac{16n_f B}{\pi d^3 S_{ut}}\right)^2=1$$

which we can solve it for d or $1/n_f$. Solving for d yields

$$d=\left(\frac{8n_f A}{\pi S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3}.$$

Or, solving for $1/n_f$ gives

$$\frac{1}{n_f}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}.$$

Usually, for the shafts we have: $M_m = 0$ and $T_a = 0$, for which case $A=2K_f M_a$ and $B=\sqrt{3}\,K_{fs}\,T_m$ and hence the above equations become

$$d=\left(\frac{16n_f K_f M_a}{\pi S_e}\left\{1+\left[1+3\left(\frac{K_{fs}\,T_m\,S_e}{K_f\,M_a\,S_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3}$$

and

$$\frac{1}{n_f}=\frac{16K_f M_a}{\pi d^3 S_e}\left\{1+\left[1+3\left(\frac{K_{fs}\,T_m\,S_e}{K_f\,M_a\,S_{ut}}\right)^2\right]^{1/2}\right\}.$$

For the DE-Elliptic Criterion, the equations are given in pg. 936 of the textbook.

Note: Review Example 18-3, pgs. 937, 938.

18-8 Shaft Design

General steps for a shaft design consider the slope and deflection limits and fatigue concerns. You are encouraged to review this section in the textbook in pgs. 950 and 951. The steps provided by the textbook are straightforward and easy to follow.