

KING FAHD  
UNIVERSITY OF PETROLEUM & MINERALS

**ME 316: Thermofluids Laboratory**

Experiment # 4

COMPRESSIBLE FLOW IN A CONVERGENT-DIVERGENT NOZZLE

**1) OBJECTIVES**

- a) To introduce the main features of compressible flow.
- b) To measure the pressure variation in a convergent-divergent nozzle.

**2) INTRODUCTION**

A flow is called compressible when the density variations in the flow field are considerable. Such flow is called high-speed flow and the flow velocity becomes comparable to the speed of sound. Examples are flow of steam in a steam turbine nozzle, flow of exhaust gases through the nozzle of a jet engine. The most important dimensionless parameter in high-speed flow is the Mach number,  $M$ , which is defined as the ratio between the velocity of flow and the speed of sound ( $M=V/C$ ). Flow at speeds less than the speed of sound ( $M<1$ ) is called subsonic and at speeds higher than the speed of sound is called supersonic ( $M>1$ ).

A high-speed flow nozzle may have different geometrical shapes and, in general, it may be either convergent ( $dA<0$ ) or convergent-divergent ( $dA<0$  in the first part and  $dA>0$  in the second part) as shown in Figure 1. The section of minimum area in the C-D nozzle is called the throat. The pressure and velocity changes in a nozzle are strongly dependent on its geometrical shape. The governing equations of fluid motion are the mass, momentum and energy conservation equations together with the equation of state (or relation between properties) and the second law of thermodynamics.

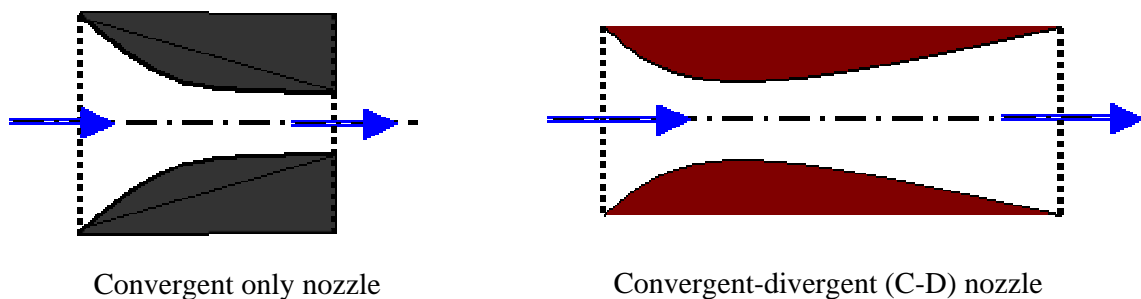


Figure 1. The general shapes of convergent and convergent-divergent nozzle.

### 3) THEORETICAL BACKGROUND

Considering flow of an ideal gas in the flow passage of general shape shown in Figure 2 and assume that the flow is steady and one-dimensional. We also assume that the flow is adiabatic and frictional effects are negligibly small (isentropic flow). The governing equations can be written as

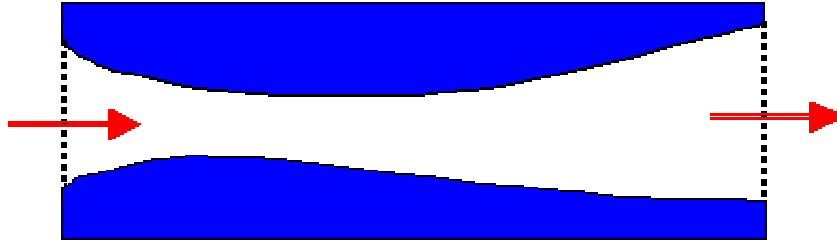


Figure 2. Flow in a flow passage of general shape.

$$\text{Mass Conservation} \quad \dot{m} = \rho AV = \text{Const.} \quad (1)$$

$$\text{Energy Conservation} \quad h + \frac{V^2}{2} + gz = \text{Const.} \quad (2)$$

$$\text{Isentropic flow} \quad \delta q = 0 = Tds = du + pdv = dh - \frac{dp}{\rho} \Rightarrow dh = \frac{dp}{\rho} \quad (3)$$

$$\text{Speed of sound} \quad C^2 = (E_v / \rho)_s = (dp / d\rho)_s \quad (4)$$

Using the above equations, one can deduce the relationships governing the variations of velocity, pressure and density with the change in the flow passage cross-sectional area. These relationships can be written as (for details see Appendix A),

$$\text{Velocity variation} \quad \boxed{\frac{dV}{V} = \left( \frac{-1}{1-M^2} \right) \frac{dA}{A}} \quad (5)$$

$$\text{Pressure variation} \quad \boxed{dp = \rho V^2 \left( \frac{1}{1-M^2} \right) \frac{dA}{A}} \quad (6)$$

$$\text{Density variation} \quad \boxed{\frac{d\rho}{\rho} = \frac{M^2}{1-M^2} \frac{dA}{A}} \quad (7)$$

The above equations are correct for the two cases of subsonic and supersonic flows subject to the above assumptions (flow is steady, one-dimensional, adiabatic and frictionless). Now, let us consider the case of subsonic flow ( $M < 1$ ) in C-D nozzle. In the converging part, the term ( $dA/A$ ) is negative and this results in  $dV$  positive [by Eq. (5)] and both  $dp$  and  $d\rho$  negative [by Eqs. (6-7)]. The opposite occurs in the diverging part of the nozzle and the variation of  $V$ ,  $p$  and  $\rho$  along the nozzle become as shown in Figure 3. This mode of operation of the

nozzle is called the venturi mode since the velocity and pressure variations are similar to that of a venturi meter.

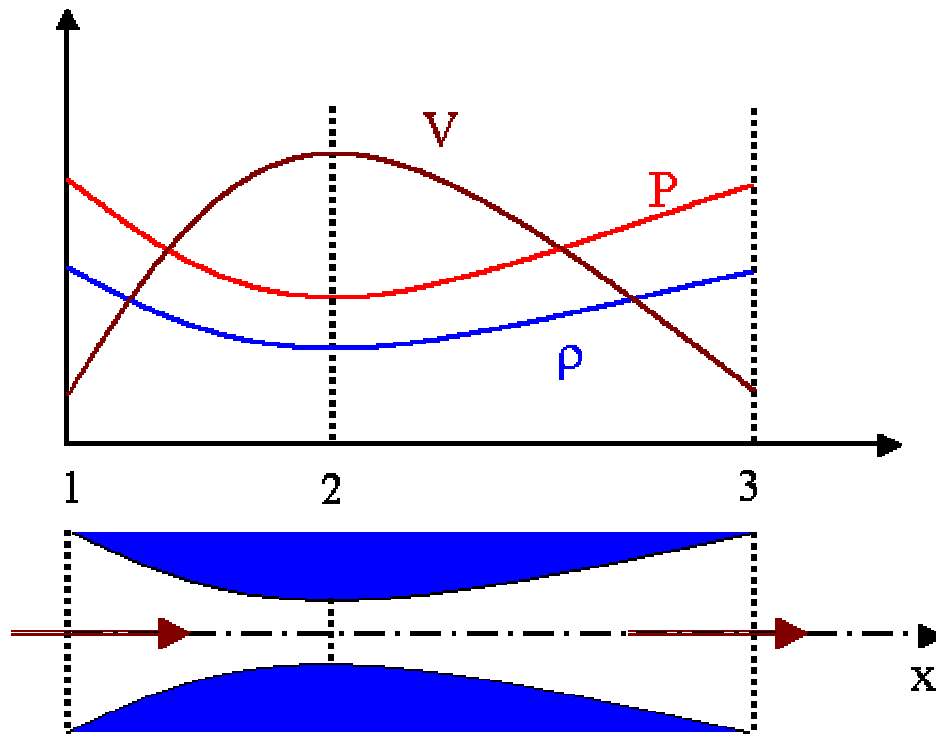


Figure 3. Variation of velocity, pressure and density along the nozzle in the venturi mode of operation.

When the nozzle operates at its design point, the flow at the nozzle exit section is supersonic ( $M_3 > 1$ ) while being subsonic at inlet ( $M_1 < 1$ ) and sonic at the throat ( $M_2 = 1$ ). In most applications, the flow enters the nozzle at low speeds and the velocity increases towards the throat until reaching sonic velocity at the throat. In the diverging section (2→3), the term  $dA/A$  becomes positive and the flow becomes supersonic ( $M > 1$ ). This leads to having  $dV$  positive [by Eq.(5)] and both  $dp$  and  $d\rho$  negative [by Eqs. (6-7)]. In this case, the diverging part of the nozzle acts as a nozzle (i.e. velocity increases and pressure decreases in direction of flow) opposite to what happened in the venturi mode. Accordingly, the variations of  $V$ ,  $p$  and  $\rho$  along the nozzle axis become as shown in Figure 4 when operating at the design condition. In order to achieve this mode of operation, the back pressure,  $p_3$ , should be much lower than the inlet pressure,  $p_1$ , as shown in the figure.

The throat section of the nozzle is called the critical section when operating at the design condition since the Mach number there reaches unity. All fluid properties at this section are labeled with a star (i.e.  $p_2 = p^*$ ,  $\rho_2 = \rho^*$  and  $A_2 = A^*$ ). The Mach number at the nozzle exit depends strongly on the nozzle area ratio ( $A_3/A^*$ ).

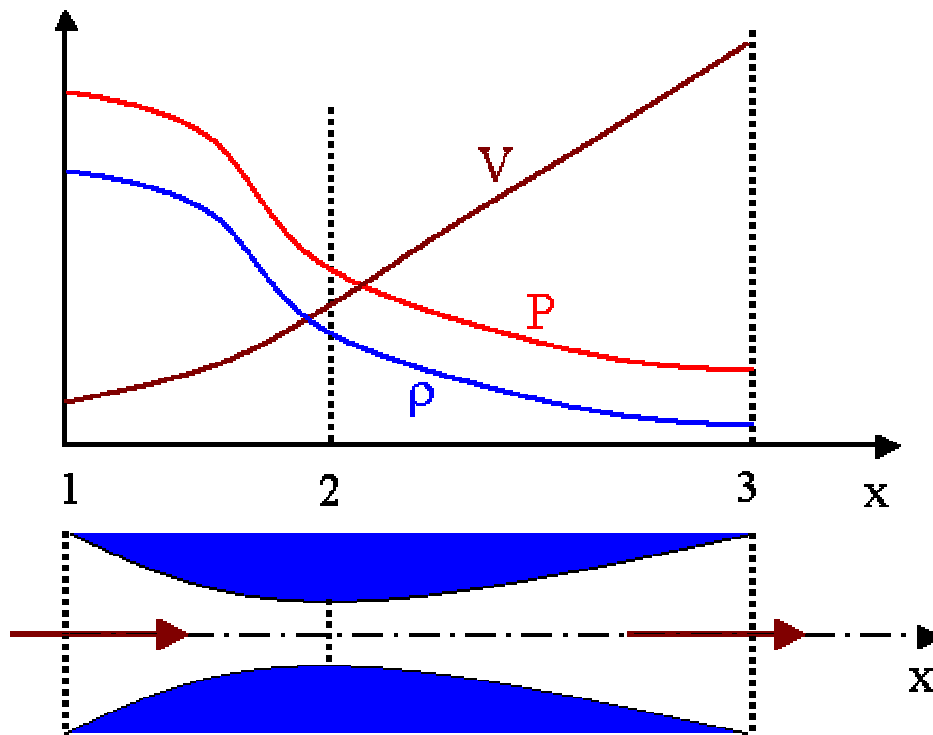


Figure 4. Variation of velocity, pressure and density along the nozzle when operating at design condition.

#### 4) APPARATUS

The apparatus to be used is the Plint & Partners nozzle flow apparatus (model # TE 27). Figure 5 represents a general view of the apparatus and Figure 6 shows schematic diagram of various components. The setup consists of the following:



Figure 5. A general view of the nozzle flow apparatus

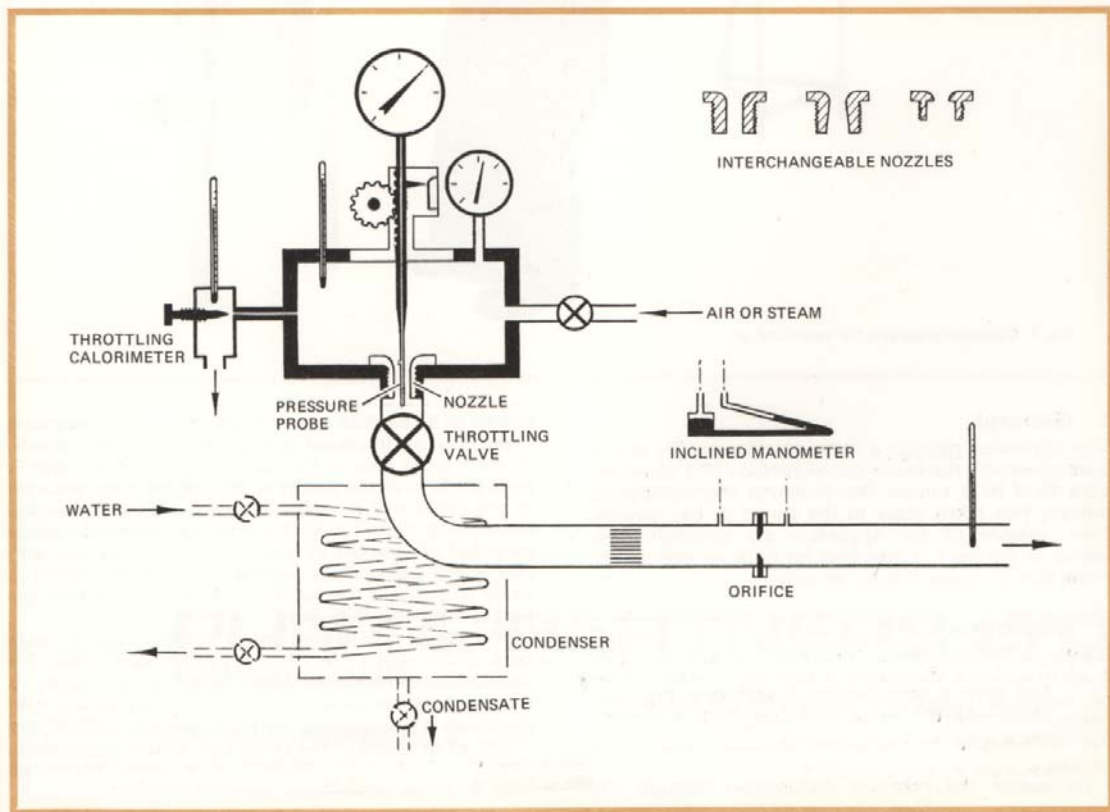


Figure 6. Schematic layout of the nozzle apparatus

- i) Compressed Air Reservoir: The reservoir is provided with compressed air from a set of air compressors located outside the laboratory.
- ii) The Inlet Air Chest: The inlet chest is supplied with air from the compressed air reservoir through a throttling valve to regulate the flow into the chest. The maximum allowed pressure in the chest is 600 kPa. The chest also carries a mercury thermometer in an oil pocket, a pressure gauge for indicating the chest pressure.
- iii) The Convergent-Divergent Nozzle: A Laval C-D nozzle is screwed into a seating at the center of the inlet air chest. The exact nozzle geometry is shown in Figure 7.
- iv) Static Pressure Probe: A static pressure probe is positioned along the nozzle axis and allowed to move up or down through a traversing mechanism as shown in Figure 5. The probe transmits the local pressure to a high-grade pressure gauge mounted on the probe carrier. The probe is traversed in increments of 2mm by rotating a calibrated dial. A pointer attached to the probe carrier moves over a replica of the nozzle profile in order to indicate the position of the measuring point in the nozzle.
- v) The Discharge Section: The nozzle discharges into a vertical tube of large bore fitted with a throttling valve by which the nozzle back pressure may be regulated. Downstream of the throttle valve, the fluid flows through a long straight pipe and a flow straightener to an orifice flowmeter with  $D/2$  pressure taps (designed in accordance with British Standard Specifications). The pressure difference across the orifice plate is indicated by an inclined manometer and the orifice discharge coefficient is 0.62. A thermometer is provided downstream of the orifice for measuring air temperature.

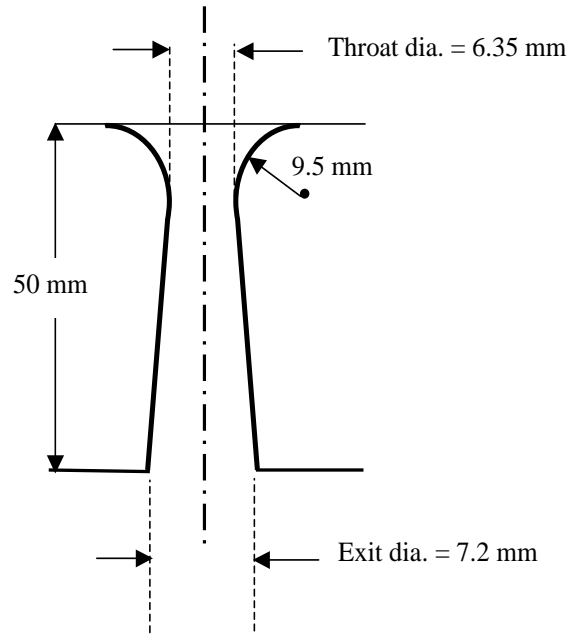


Figure 7. The main dimensions of the convergent-divergent nozzle.

## 5) TEST PROCEDURE AND CALCULATIONS

### a) Test Procedure

1. Ensure that the inclined manometer is correctly leveled and zeroed.
2. Before starting the flow process, record the inlet air temperature and the barometer reading.
3. Open the back pressure valve and traverse the search tube (pressure probe) to its upper limit. The pressure gauge should indicate the inlet chest pressure. This pressure may be set to the desired value by adjusting the inlet throttling valve.
4. The selected inlet chest pressure should remain constant during the course of the experiment. Should there be any tendency for the inlet chest pressure to change in the course of the experiment (this may occur when the apparatus is being supplied with air by a compressor of insufficient capacity) a student should be placed in charge of the inlet throttling valve, with the task of maintaining a constant inlet chest pressure by adjusting the throttling valve as necessary. In this task he makes use of the small inlet chest pressure gauge.
5. Take the reading of the inclined manometer at the beginning of the experiment.
6. Use the traversing mechanism to record the pressure at several locations along the nozzle axis. The probe is traversed in increments of 2mm by rotating a calibrated dial.
7. Take the reading of the inclined manometer at the end of the experiment and use the average for calculating the mass flow rate.
8. At the end of the traverse, the search tube should be returned to the upper position and the chest pressure rechecked.

## b) Calculations

1. The orifice flowmeter is used to calculate the airflow rate through the nozzle. The pipe diameter,  $D_1=76.2$  mm and the orifice diameter,  $D_2= 50$  mm.
2. The discharge coefficient of the orifice,  $C_d = 0.62$ .
3. Calculate the ideal air volume flow rate from the equation (for details see the handout of Experiment # 1)

$$Q_{ideal} = A_2 V_2 = \frac{A_2}{\sqrt{1 - (A_2 / A_1)^2}} \sqrt{2 \left( \frac{\Delta p}{\rho} \right)} \quad (8)$$

4. Use the barometer reading and the recorded air temperature to determine the air density [may use the equation of state ( $\rho = p / RT$ )].
5. The actual mass flow rate can be obtained from the equation

$$\dot{m} = \rho Q_{actual} = C_d \rho Q_{ideal} = \frac{C_d A_2}{\sqrt{1 - (A_2 / A_1)^2}} \sqrt{2 \rho \Delta p} \quad (9)$$

## 6) PRESENTATION OF RESULTS

1. It is required to plot the pressure variation along the nozzle axis to show the continuous pressure decrease from the nozzle inlet to exit sections. Typical pressure variations are shown in Figure 8 for different modes of operation.
2. Calculate the mass flow rate through the nozzle.

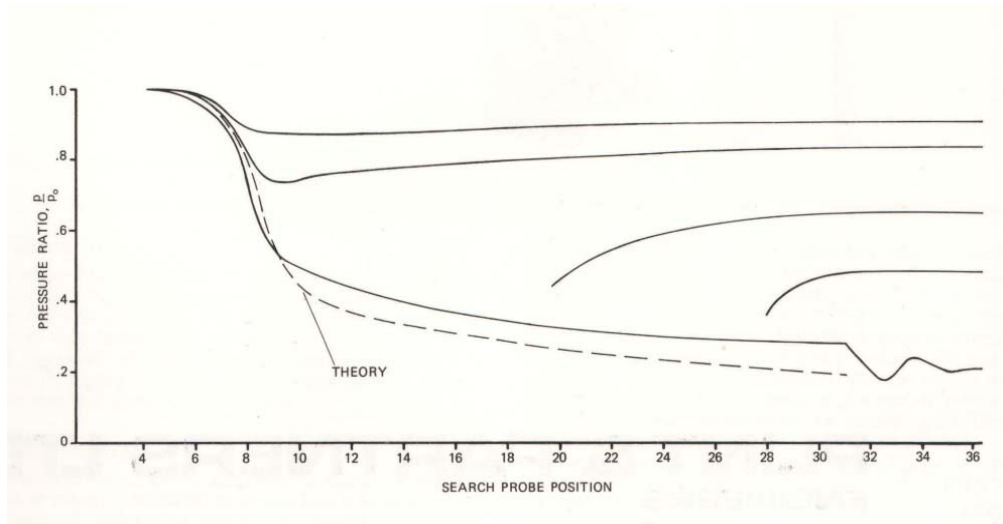


Figure 8. Typical pressure variations along the nozzle axis for different modes of operation.

## **7) IDEAS FOR FURTHER DISCUSSIONS**

1. The diameter of the pressure probe is 3.33 mm. What is the effect of the probe diameter on the measured pressure distribution? Does it really represent the pressure distribution in the nozzle shown in Figure 6 or a different one?
2. What is the percentage error involved in calculating the mass flow rate?
3. How would you explain the pressure drop downstream of the nozzle exit?
4. What can be done to improve the apparatus or the test procedure?



# APPENDIX A

## Isentropic Flow in Nozzles

The nozzle is an important part of steam and gas turbines, jet and rocket propulsion systems and many other thermal devices. The nozzle may have different geometrical shapes and in general it may be either convergent ( $dA < 0$ ) or convergent-divergent ( $dA < 0$  followed by  $dA > 0$ ). The flow properties within a nozzle change from one section to another and this change depends very much on the nozzle shape. The question now is how to determine the relationship between the nozzle geometry and the resulting flow properties.

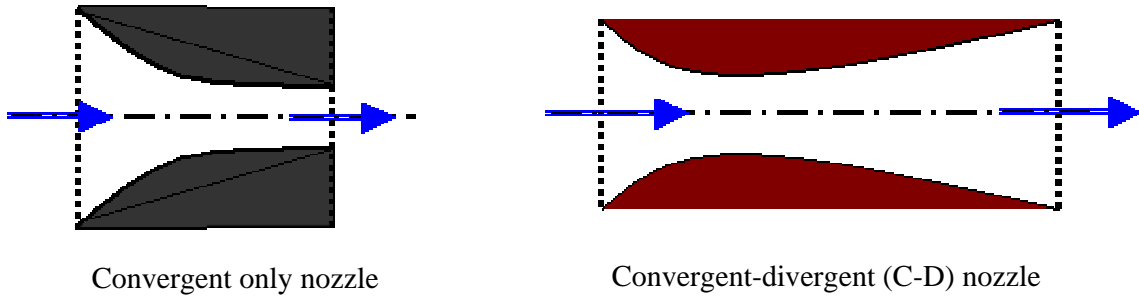


Figure A1. The general shapes of convergent and convergent-divergent nozzle.

### Relation Between Nozzle Shape and Flow Properties

Consider the case of steady compressible flow in a nozzle of arbitrary shape and apply the mass conservation principle, then

$$\dot{m} = \rho AV = \text{Const.}$$

Therefore  $d\dot{m} = d(\rho AV) = 0$

Or  $d\rho AV + \rho dA V + \rho A dV = 0$

Divide each side by  $\rho AV \Rightarrow \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$  (A1)

The energy equation for steady isentropic flow may be written in the form

$$h + \frac{V^2}{2} + gz = \text{Const.} \quad \text{and since } z \approx \text{Const.} \quad \Rightarrow$$



$$h + \frac{V^2}{2} = \text{Const.} \Rightarrow dh = -d(V^2/2) = -VdV \quad (\text{A2})$$

But  $\delta q = Tds = du + pdv = d(u + pv) - vdp = dh - \frac{dp}{\rho}$  and since  $\delta q = 0$  for isentropic flow, then

$$dh = \frac{dp}{\rho} \text{ and by using equation (A2), we can write } \frac{dp}{\rho} = -V dV \quad (\text{A3})$$

Substitute in Eq. (A1) to obtain

$$\frac{d\rho}{\rho} + \frac{dA}{A} - \frac{dp}{\rho V^2} = 0 \Rightarrow \frac{dp}{\rho} = V^2 \left( \frac{d\rho}{\rho} + \frac{dA}{A} \right) \Rightarrow \frac{dp}{d\rho} \frac{d\rho}{\rho} = V^2 \left( \frac{d\rho}{\rho} + \frac{dA}{A} \right)$$

But  $C^2 = (E_v / \rho)_s = (dp / d\rho)_s$ , therefore one can write the above equation in the form

$$C^2 \frac{d\rho}{\rho} = V^2 \left( \frac{d\rho}{\rho} + \frac{dA}{A} \right) \Rightarrow \frac{d\rho}{\rho} = \frac{V^2}{C^2} \left( \frac{d\rho}{\rho} + \frac{dA}{A} \right) = M^2 \left( \frac{d\rho}{\rho} + \frac{dA}{A} \right) \Rightarrow$$

$$\text{Rearrange to obtain } \Rightarrow \boxed{\frac{d\rho}{\rho} = \frac{M^2}{1-M^2} \frac{dA}{A}} \quad (\text{A4})$$

$$\text{Substitute in Eq. (A1)} \Rightarrow \frac{M^2}{1-M^2} \frac{dA}{A} + \frac{dA}{A} + \frac{dV}{V} = 0 \Rightarrow \boxed{\frac{dV}{V} = \left( \frac{-1}{1-M^2} \right) \frac{dA}{A}} \quad (\text{A5})$$

But one can use Eq. (A3) to write  $-\frac{dp}{\rho V^2} = \frac{dV}{V}$  that can be used in Eq. (A5) to obtain

$$-\frac{dp}{\rho V^2} = \left( \frac{-1}{1-M^2} \right) \frac{dA}{A} \Rightarrow \boxed{dp = \rho V^2 \left( \frac{1}{1-M^2} \right) \frac{dA}{A}} \quad (\text{A6})$$

Equations (A4-A6) establish important relationships between the flow passage geometry and the changes in the fluid density, velocity and pressure for the special case of steady, one-dimensional isentropic flow.