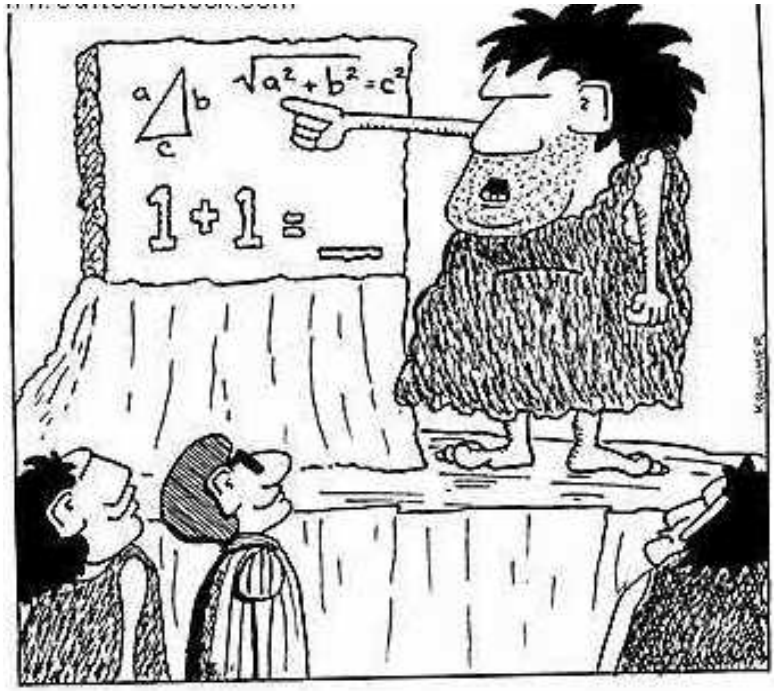
Yet Another Math Lecture



TODAY'S MATH LESSON IS .... HEY, WHO WROTE JUNK ON MY MATH BOARD ?

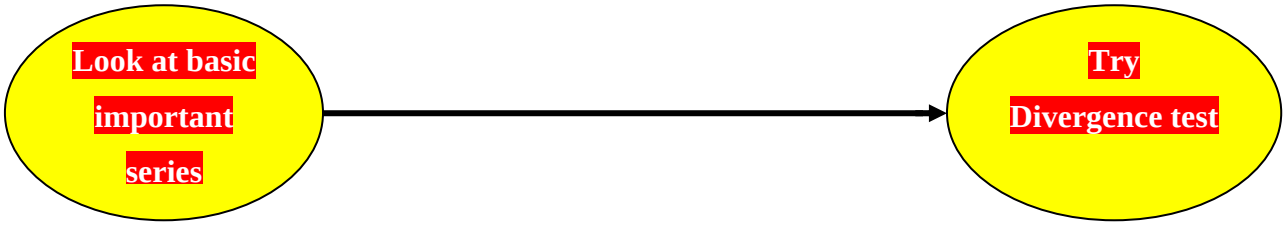
Note

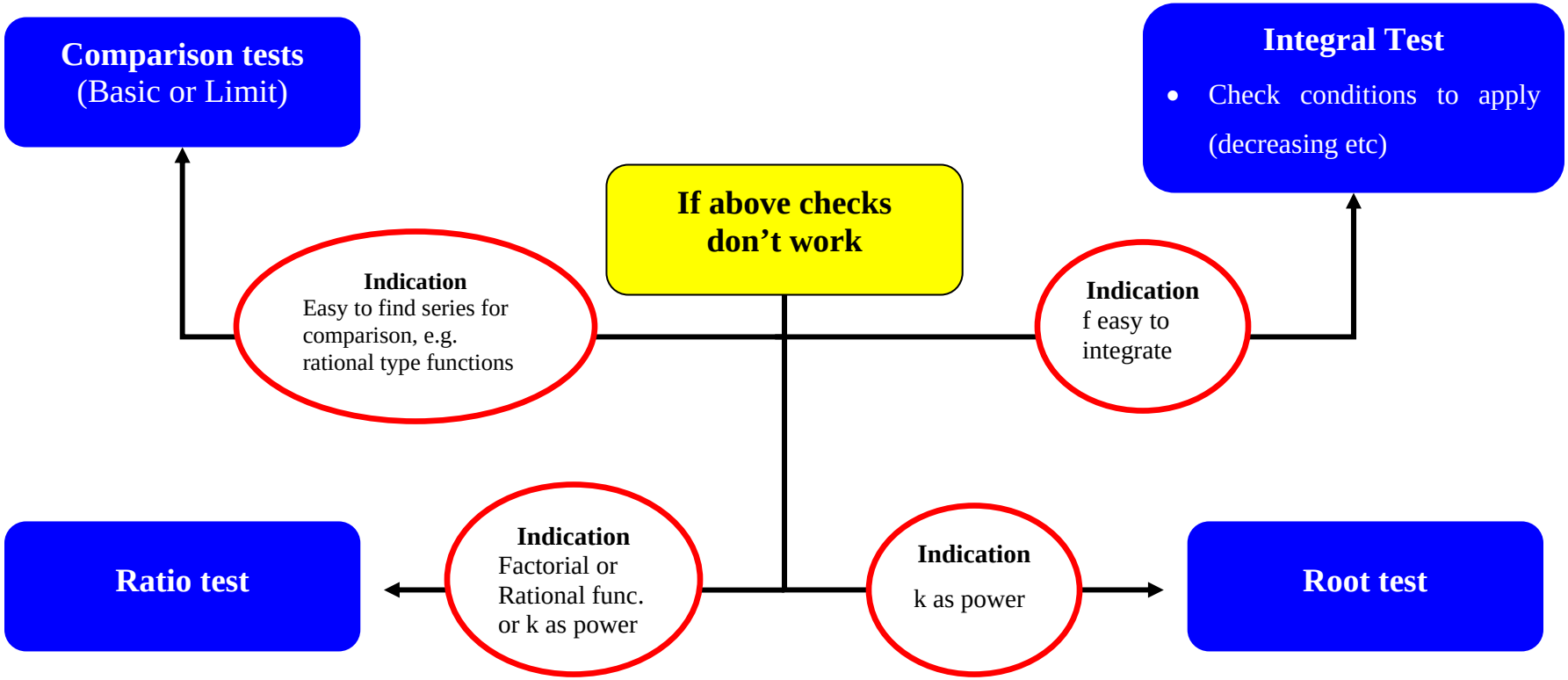
- It is a guideline.
- Not hard & fast rule

Strategy for testing positive term series

Before understanding the strategy

- Recall the **basic series**
  - Geometric series
  - p-series
- Revise all **convergence/divergence tests**
  - Divergence test
  - Integral test
  - Comparison tests (basic & limit)
  - Ratio test
  - Root test


Look at basic important series
Try Divergence test


Comparison tests (Basic or Limit)
Integral Test
Check conditions to apply (decreasing etc)
If above checks don't work
Indication Easy to find series for comparison, e.g. rational type functions
Indication f easy to integrate
Ratio test
Indication Factorial or Rational func. or k as power
Indication k as power
Root test

We learn with the help of examples

1. $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$

Divergence Test

Hint/Indicator

- $\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n^2 - 1}{n^2 + n} = 1 \neq 0$

2. $\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$

Limit Comparison Test

Hint/Indicator

- a_n of the form $\frac{\text{one expression}}{\text{another expression}}$
- b_n easy to choose
- $b_n = \frac{n}{n^2} = \frac{1}{n}$
- Nature of $\sum b_n$ known

3. $\sum_{n=1}^{\infty}\left(\frac{3n}{1+8n}\right)^{n}$

Root Test

Hint/Indicator

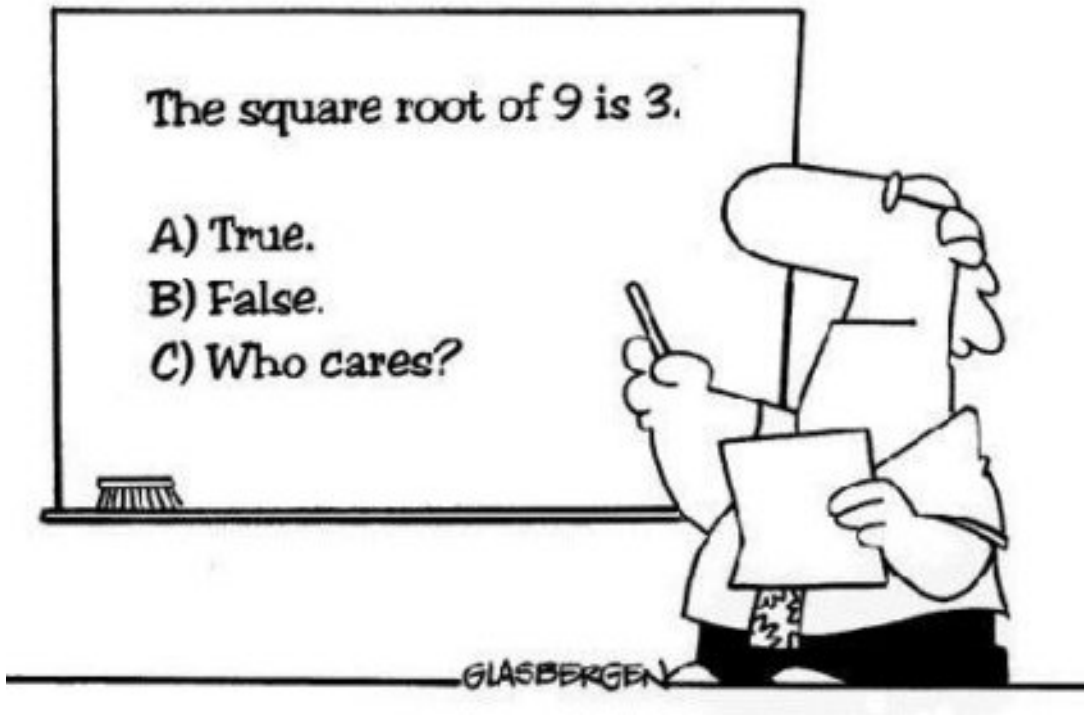
- n as a power

4. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Integral Test

Hint/Indicator

- $f(x) = \frac{1}{x\sqrt{\ln x}}$ is easy to integrate
- Use $u = \sqrt{\ln x}$ to evaluate $\int_{2}^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx$


The square root of 9 is 3.
A) True.
B) False.
C) Who cares?
GLASBERGEN

5. $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

Ratio Test

Hint/Indicator

- Factorial in the question

6. $\sum_{n=1}^{\infty} \frac{1}{n + n\cos^2 n}$

Basic Comparison Test

Hint/Indicator

- a_n of the form $\frac{\text{one expression}}{\text{another expression}}$
- a typical case of choosing b_n and comparing as explained below

$$0 \le \cos^2 n \le 1 \quad \Rightarrow \quad 0 \le n\cos^2 n \le n \quad \Rightarrow \quad n + n\cos^2 n \le 2n$$

$$\Rightarrow \quad \frac{1}{n + n\cos^2 n} \ge \frac{1}{2n}$$

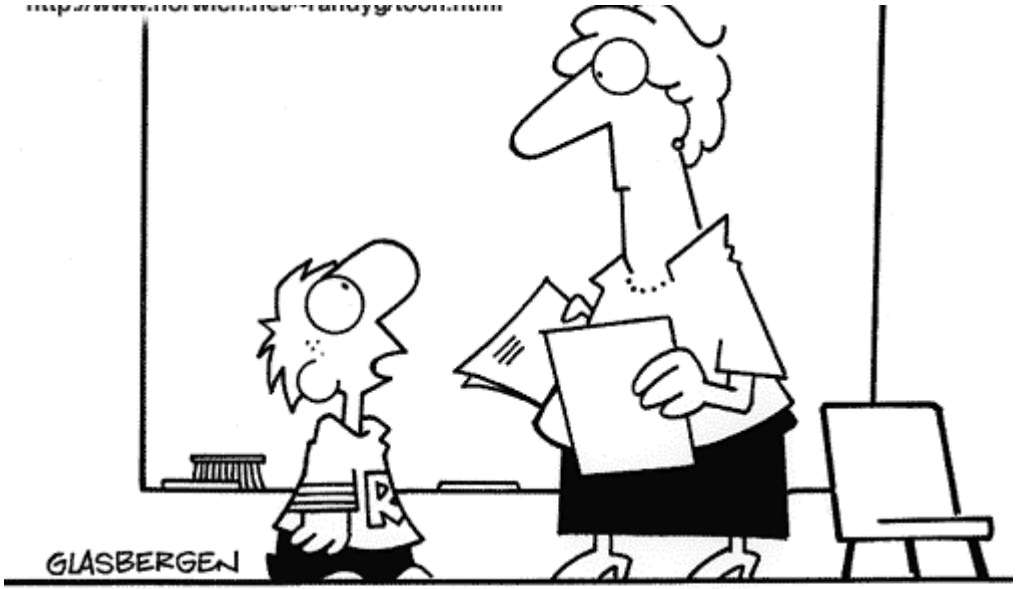
- Choose $b_n = \frac{1}{2n}$

7. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

Integral Test

Hint/Indicator

- $f(x) = x^2 e^{-x^3}$ is easy to integrate
- Use $u = -x^3$ to evaluate $\int_1^{\infty} x^2 e^{-x^3} dx$



"I couldn't do my homework because my computer has a virus and so do all my pencils and pens."

8. $\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$

Ratio Test

Hint/Indicator

- a_n of the form $\frac{\text{one expression}}{\text{another expression}}$
- n in the power

Interesting Question

Why don't we try Comparison Test?

9. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

Limit Comparison Test

Hint/Indicator

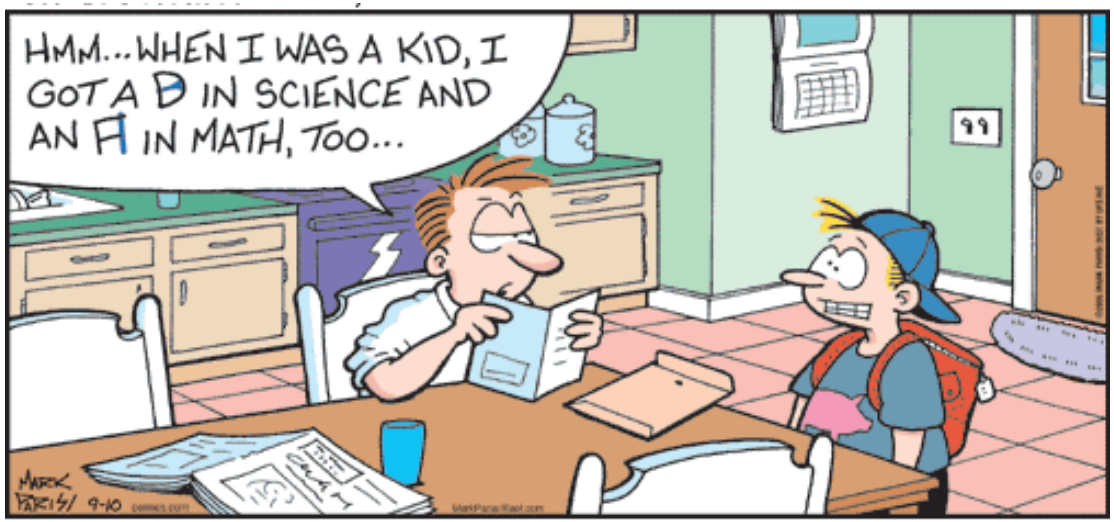
- a_n of the form $\frac{\text{one expression}}{\text{another expression}}$
- b_n easy to choose
- $b_n = \frac{n}{n^3} = \frac{1}{n^2}$
- Nature of $\sum b_n$ known

10. $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$

Root Test

Hint/Indicator

- $a_n = \frac{(2n)^n}{n^{2n}} = \left(\frac{2n}{n^2}\right)^n$
- n as a power


HMM... WHEN I WAS A KID, I GOT A B IN SCIENCE AND AN F IN MATH, TOO...
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MARK PARISI 9-10