Section 8.2 *Area of a surface of revolution*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. find surface area of a surface of revolution

First recall some formulas

Surface area of a cone with base radius r and slant height l is

$S = \pi r l$

What is surface area of frustum of a cone with one base of radius r_1, other base of radius r_2 and slant height l

$$S = \pi(r_1 + r_2)l = 2\pi\left(\frac{r_1 + r_2}{2}\right)l$$

$\Rightarrow$

$S = 2\pi(\text{average radius})(\text{slant height})$

See figure and explanation given in class

Formulas for finding surface area

(revolution about X-axis)

The surface area of the surface obtained by revolving $y = f(x) \geq 0$ (from $x = a$ to $x = b$) about X-axis is

$$S = \int_a^b 2\pi f(x)\sqrt{1 + \left[f'(x) \right]^2}\, dx$$

or

$$S = \int_a^b 2\pi y \sqrt{1 + \left[\frac{dy}{dx} \right]^2}\, dx$$

See class explanation about derivation of formula

See examples 1, 2 done in class

The surface area of the surface obtained by revolving $x = f(y)$ (from $y = c$ to $y = d$) about X-axis is

$$S = \int_c^d 2\pi y \sqrt{1 + \left[\frac{dx}{dy} \right]^2}\, dy$$

See class explanation

See example 3 done in class

- Note $\sqrt{1 + \left[\frac{dy}{dx} \right]^2}\, dx = \sqrt{1 + \left[\frac{dx}{dy} \right]^2}\, dy$
- See class explanation

Formulas for finding surface area

(revolution about Y-axis)

The surface area of the surface obtained by revolving $x = f(y) \geq 0$ (from $y = c$ to $y = d$) about Y-axis is

$$S = \int_c^d 2\pi f(y)\sqrt{1 + \left[f'(y) \right]^2}\, dy$$

or

$$S = \int_c^d 2\pi x\sqrt{1 + \left[\frac{dx}{dy} \right]^2}\, dy$$

Derivation of formula similar to above

See example 4 done in class

The surface area of the surface obtained by revolving $y = f(x)$ (from $x = a$ to $x = b$) about Y-axis is

$$S = \int_a^b 2\pi x\sqrt{1 + \left[\frac{dy}{dx} \right]^2}\, dx$$

See class explanation

See example 5 done in class

- Note $\sqrt{1 + \left[\frac{dx}{dy} \right]^2}\, dy = \sqrt{1 + \left[\frac{dy}{dx} \right]^2}\, dx$
- See class explanation

End of Section 8.2