

Learning outcomes

After completing this section, you will inshaAllah be able to

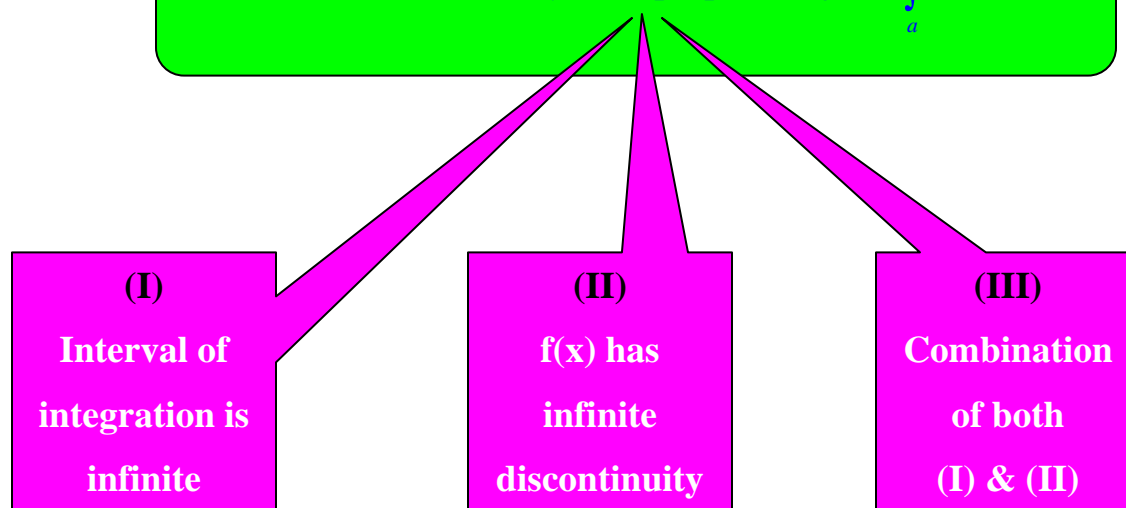
1. know what is meant by an improper integral
2. learn methods to evaluate different kind of improper integrals

Until now we have considered integrals $\int_a^b f(x)dx$ such that

- $[a,b]$ is finite and
- $f(x)$ is continuous on $[a,b]$

Such integrals are called proper integrals

Here we extend our study to improper integrals $\int_a^b f(x)dx$



Case (I): Integrals over infinite intervals

(Ia): $I = \int_a^{\infty} f(x)dx$ or $I = \int_{-\infty}^b f(x)dx$

Use

- $\int_a^{\infty} f(x)dx = \lim_{l \rightarrow \infty} \int_a^l f(x)dx$

OR

- $\int_{-\infty}^b f(x)dx = \lim_{l \rightarrow -\infty} \int_l^b f(x)dx$

- The integral **converges** if the limit exists
- And the limit is the value of integral.

- The integral **diverges** if the limit doesn't exist
- And the integral has no value.

See examples 1, 2, 3, 4 done in class

(Ib): $I = \int_{-\infty}^{\infty} f(x)dx$

Use

- $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^{\infty} f(x)dx$

('c' any number) and use (Ia)

- The integral **converges** if both the integrals converge

- Otherwise **diverges** and has no value

See examples 5, 6 done in class

Case (II): Integrals whose integrands have infinite discontinuity

(IIa): Discontinuity at end points

- If f has infinite discontinuity at $x = a$ then

$$\int_a^b f(x) dx = \lim_{l \rightarrow a^+} \int_l^b f(x) dx$$

- If f has infinite discontinuity at $x = b$ then

$$\int_a^b f(x) dx = \lim_{l \rightarrow b^-} \int_a^l f(x) dx$$

- The integral **converges** if the limit exists
- And the limit is the value of integral.

- The integral **diverges** if the limit doesn't exist
- And the integral has no value.

See examples 7, 8 done in class

(IIb): Discontinuity at interior points

- If f has infinite discontinuity at $c \in (a, b)$ then use

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

and use (IIa)

- The integral **converges** if both the integrals converge

- Otherwise **diverges** and has no value

See examples 9 done in class

Case (II): Combination of Cases I & II

Combined use of methods I & II done above

See examples 10 done in class

End of Section 7.8