

Part 1 of Section 6.2***Disk/washer method
for
finding volumes of solids of revolution*****Learning outcomes**

After completing this section, you will inshaAllah be able to

1. find **volumes** of solids of revolutions by using
 - a. **disks** perpendicular to X-axis
 - b. **disks** perpendicular to Y-axis
 - c. **washers** perpendicular to X-axis
 - d. **washers** perpendicular to Y-axis

Solid of revolution

Generated by revolving a region
or area about a line

One method to find their volume
is **“Disk/washer method”**, which
we will do in this section.

Volume by disks perpendicular to X-axis

If $y = f(x)$ is revolved about X-axis from $x = a$ to $x = b$ then
volume of the solid is

$$V = \int_a^b \pi [f(x)]^2 dx$$

See explanation
given in class

See example 1 done in class

Volume by disks perpendicular to Y-axis

If $x = g(y)$ is revolved about Y-axis from $y = c$ to $y = d$ then
volume of the solid is

$$V = \int_c^d \pi [g(y)]^2 dy$$

See explanation
given in class

See example 2 done in class

Volume by washers perpendicular to X-axis

If the region between the outer curve $y = f(x)$ and inner curve $y = g(x)$ is revolved about X-axis from $x = a$ to $x = b$ then volume of the solid is

$$V = \int_a^b \pi \left([f(x)]^2 - [g(x)]^2 \right) dx$$

See explanation given in class

It is important that the outer and inner curve stay the same throughout the interval otherwise we need to split the integral into more integrals. see Example 4 to understand this point.

See examples 3, 4 done in class

Volume by washers perpendicular to Y-axis

If the region between the outer curve $x = f(y)$ and inner curve $x = g(y)$ is revolved about Y-axis from $x = a$ to $x = b$ then volume of the solid is

$$V = \int_c^d \pi \left([f(y)]^2 - [g(y)]^2 \right) dy$$

See explanation given in class

See example 5 done in class

Revolution about any line

The above ideas of finding volume can be extended to handle more complicated situations like “revolution about a line”

We learn through examples

See example 6, 7 and exercise 8 discussed in class

End of Part 1 of Section 6.2