Section 5.5 *The substitution rule*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. know proper form of basic integration formulas
2. learn how to integrate by making substitutions and using basic integration formulas (proper form)
3. use substitution to calculate definite integrals

Fundamental integration formulas: (Proper form)

$\int du = u + C$, $\int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$

$\int \cos u\,du = \sin u + C$, $\int \sin u\,du = -\cos u + C$

$\int \sec^2 u\,du = \tan u + C$, $\int \csc^2 u\,du = -\cot u + C$

$\int \sec u \tan u\,dx = \sec u + C$, $\int \csc u \cot u\,du = -\csc u + C$

$\int e^u du = e^u + C$, $\int b^u du = \frac{b^u}{\ln b} + C \quad (0 < b,\ b \neq 1)$

$\int \frac{1}{u} du = \ln|u| + C$

$\int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1} u + C$, $\int \frac{1}{1+u^2} du = \tan^{-1} u + C$

$\int \frac{1}{u\sqrt{u^2-1}} du = \sec^{-1} u + C$

General form of above three formulas

$\int \frac{1}{\sqrt{a^2-u^2}} du = \sin^{-1}\frac{u}{a} + C$, $\int \frac{1}{a^2+u^2} du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

$\int \frac{1}{u\sqrt{u^2-a^2}} du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$

Tricks to solve

- Identify which formula to use. **[very important step]**
- Proper use of formula or substitution. [will learn in this section]
- Simplification or use of trigonometry or use of fundamental properties to bring in a form where formulas can be applied.
- Special techniques to be covered in Chapter 7.

Substitution rule for indefinite and definite integrals

We will learn the methods of this section through examples

How to handle limits when we use substitutions for definite integrals

Method 1

Substitution

without

changing the limits

Method 2

Substitution

and

changing the limits

We will prefer Method 2

Remember to use properties of definite integrals (whenever possible)

See the examples done in class

End of Section 5.5