Section 5.4 *Indefinite integrals and the net change theorem*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. recall what is meant by anti-derivative of a function
2. know what is meant by integration and indefinite integrals
3. get an idea about different important methods for integration
4. learn how to integrate using basic integration formulas and basic properties of integration
5. know and apply the net change theorem

Antiderivative

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$

Just a process which is reverse of differentiation

Example Guess few antiderivatives of the following and give reason for your answer.

1. $f(x) = x^2$
2. $f(x) = \cos x$

Do we get unique antiderivatives?

$F(x)$ is antiderivative of $f(x)$ on I

$$\Leftrightarrow$$

$F(x) + C$ is antiderivative of $f(x)$ on I

What is integration

A process of finding antiderivative

- Standard Notation:

$$\int f(x)dx = F(x) + C$$

indefinite integral of f w.r.t x

An antiderivative of f i.e. $\frac{d}{dx}\left(F(x)\right) = f(x)$

Computations of integration

- Basic trick: Educated guess work (as inverse process of differentiation)
- Proper computations based on
  - Formulas
  - basic properties
  - calculation techniques

Developed as inverse of differentiation

Our aim in this course, regarding integration

- techniques of integration
- application of integration

Here we begin by learning to perform integration using basic formulas and properties

Fundamental integration formulas:

$$\int dx = x + C, \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \cos x\,dx = \sin x + C, \qquad \int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C, \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C, \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int e^x dx = e^x + C, \qquad \int b^x dx = \frac{b^x}{\ln b} + C \quad (0 < b,\ b \neq 1)$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \qquad \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

Fundamental properties:

- $\int cf(x)dx = c\int f(x)dx$
- $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

Tricks to solve

- Identify which formula to use.
- Proper use of formula (will learn more about it in Ex. 5.5).
- Simplification or use of trigonometry or use of fundamental properties to bring in a form where formulas can be applied.
- Special techniques to be covered in Chapter 7.

See the examples done in class

The net change theorem and its applications

Clearly

$$\int_a^b F'(x)dx = F(b) - F(a)$$

Net Change Theorem

The integral of the rate of change of F(x) is the net change in F(x) from x=a to x=b.

Different Real Life Applications

- If $V'(t)$ is rate of change of volume then

$$\int_{t_1}^{t_2} V'(t)dt = V(t_2) - V(t_1)$$

Net change in volume

- If $\frac{dn}{dt}$ is rate of change of population then

$$\int_{t_1}^{t_2} \frac{dn}{dt}dt = n(t_2) - n(t_1)$$

Net change in population

- If $\rho(x) = m'(x)$ is density then

$$\int_a^b \rho(x)dx = m(b) - m(a)$$

Net change in mass

- See questions in Ex. 5.4 for above applications like Q. 57, 58.
- Here (below) we focus on velocity related applications

The net change theorem and its applications (contd.)

- If $v(t) = s'(t)$ is velocity (i.e. rate of change of displacement) then

$$\int_{t_1}^{t_2} v(t)dt = s(t_2) - s(t_1)$$

Net change in displacement

- If $a(t) = v'(t)$ is acceleration (i.e. rate of change of velocity) then

$$\int_{t_1}^{t_2} a(t)dt = v(t_2) - v(t_1)$$

Net change in velocity

See the example done in class

How to compute total distance traveled, if velocity is known

See class explanation

- Identify the intervals where $v(t) \geq 0$ and $v(t) \leq 0$.
- Find $\int_{t_1}^{t_2} |v(t)| dt$ (by breaking into more integrals)

See the example done in class

End of Section 5.4