Section 5.3 *The fundamental theorem of calculus*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. know what is fundamental theorem of calculus
2. evaluate definite integrals using Part 2 of fundamental theorem of calculus
3. apply Part 1 of fundamental theorem (to solve problems)
4. understand (and work with) functions defined as integral

Fundamental theorem of calculus

- Until now we have calculated definite integrals by ways which are restricted and not practical.
- Here we look for an efficient method of calculating definite integrals
- The fundamental theorem is very useful because it
  - relates $\int_a^b f(x)dx$ with integration (or differentiation)
  - gives efficient way of calculating definite integrals

The fundamental theorem

Part 1

Existence of anti-derivative

If f is continuous on $[a,b]$, then the function F defined by

$$F(x) = \int_a^x f(t)dt$$

is differentiable and $F'(x) = f(x)$

(i.e. F is anti-derivative of f)

Part 2

Evaluation of definite integral

If f is continuous on $[a,b]$, and F is any antiderivative of f then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Evaluating definite integrals using fundamental theorem (Part 2)

Best Strategy

A combination of properties (done in Sec. 5.2) and fundamental theorem

See examples 1, 2, 3 done in class

Do exercises given in class

Applying Part 1 of Fundamental Theorem

Main Point

To bring in proper form

before using formula

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

See examples 4, 5, 6, 7, 8 done in class

Do exercises given in class

Functions defined as integral

How to work with such functions?

Main tool

Fundamental theorem Part 1

- Is $f(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ a function.

  - Yes. See class explanation.
  - Called functions defined by integrals

- Why do we need to study such functions?
  - See class explanation.

- How to differentiate such functions?
  - Use “Fundamental Theorem Part 1”

Standard Notation

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

See examples 9, 10 done in class

Do exercise given in class

End of Section 5.3