Section 11.9 *Representations of functions as power series*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand what is meant by representation of a function as power series
2. learn different methods of writing power series representation of a function using a known power series
3. use power series of $f(x) = \dfrac{1}{1-x}$ to make new power series representations

Basis of application of power series

is

representation of functions by power series

Meaning of representation of a function by power series

- We look at an example to understand.
- Recall from geometric series that

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \qquad \text{for } |r|<1.$$

- Putting $a=1$ and $r=x$ implies

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad \text{for } |x|<1.$$

The expression

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \textbf{for } |x|<1$$

gives the power series representation of $f(x) = \dfrac{1}{1-x}$**.**

The interval of convergence is $|x|<1$**.**

Methods for finding power series representation of a function

(using known power series)

- Here we will only make new power series representations using

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \text{for } |x| < 1$$

- But the methods work in general.

Method 1

By substitution

See example 1 done in class

Method 2

By algebraic operations on known series

Method 3

Combination of Methods 1 & 2

See example 2 done in class

Method 4

By differentiating or integrating kmown series

See next page for explanation

Finding power series representation of functions

(by differentiating & integrating known power series representations)

Let $f(x) = \sum_{n=0}^{\infty} c_n (x - x_0)^n$ with radius of convergence R.

Then

$$f'(x) = \sum_{k=0}^{\infty} \frac{d}{dx}\left(c_k (x - x_0)^k\right)$$

and

$$\int f(x)dx = \sum_{k=0}^{\infty} \int \left(c_k (x - x_0)^k\right) dx.$$

Both have radius of convergence R.

See examples 3, 4 done in class

Note that radius of convergence stays the same but the interval of convergence may not be the same. There may be a difference at the end points of interval.

End of 11.9