Section 11.8 *Power series*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand what is meant by power series
2. understand what is meant by interval & radius of convergence of a power series
3. find interval & radius of convergence of a power series

Power series in x

- Roughly: a power series = a series containing powers of a variable

Let x be a variable. A power series in x is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \cdots\cdots + c_n x^n + \cdots$$

Example

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots + x^n + \cdots$$

For $x = \frac{1}{2}$ it converges

For $x = 2$ it diverges

Observation

- A power series may converge for some values of x and diverge for others
- Every power series converges for x=0.

Main task

- To find all the values of x for which a power series converges.

Convergence of power series

Given a power series in x. Then either

1) it converges only for $x = 0$.

OR

2) it converges for all values of x

OR

3) it converges for x in an open interval $(-R, R)$ and diverges if $x < -R$, $x > R$

- At the end points $x = -R$ or $x = R$, the series may converge or diverge. (we need to investigate at the end points to exactly know about the convergence/divergence)

Interval of convergence

=

all values for which series converges

Radius of convergence

=

half of length of interval of convergence

Main tool for finding interval of convergence

=

Ratio test

See Section 11.6

See examples 1, 2, 3 done in class

Power series in $x - x_0$

Let x be a variable. A power series in $x - x_0$ is a series of the form

$$\sum_{n=0}^{\infty} c_n (x - x_0)^n = c_0 + c_1 (x - x_0) + c_2 (x - x_0)^2 + \cdots\cdots + c_n (x - x_0)^n + \cdots$$

Convergence of power series in $x - x_0$

Given a power series in $x - x_0$. Then either

4) it converges only for $x = x_0$.

OR

5) it converges for all values of x

OR

6) it converges for x in an open interval $(x_0 - R, x_0 + R)$ and diverges if $x < x_0 - R$, $x > x_0 + R$

- At the end points $x = x_0 - R$ or $x = x_0 + R$, the series may converge or diverge. (we need to investigate separately at the end points)

Interval of convergence
=
all values for which series converges

Radius of convergence
=
half of length of interval of convergence

Main tool for finding interval of convergence

=

Ratio test

See Section 11.6

See example 4 done in class

End of 11.8