Section 11.4 *The comparison tests*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. apply comparison tests to determine convergence/divergence of a positive term series

We have done integral test

- nice but not always easy to apply {because of involvement of integration}
  - e.g. $\sum_{n=1}^{\infty} \frac{1}{3^n + n}$ can't be solved by integral test.

Here we do some tests

- which are particularly easy to study convergence of series like $\sum_{n=1}^{\infty} \frac{1}{3^n + n}$

Comparison test

- Given the series $\sum a_n$, $\sum b_n$ with positive terms.
- Suppose $a_n \le b_n$.
  - ❖ **If bigger converges then the smaller converges.**
  - ❖ **If smaller diverges then the bigger diverges.**

Be very careful in using this test

- the convergence of smaller series does not say anything about the bigger series; the bigger may converge or diverge
- Similarly, if the bigger series diverges we cannot say anything about the smaller series; it may converge or diverge.

Important Question:

- Choice of series for comparison

Answer/trick

- Discard all but leading terms (terms with maximum contribution) from numerator & denominator

See class explanation to understand leading term

See examples 1, 2, 3 done in class

Limit comparison test

- Question: Can we use comparison test to determine convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^n - n}$
- Answer: No. {Why? See class explanation}
- This shows a shortcoming of comparison test.
- Such series can be studied by the limit comparison test explained below.

- Given the series $\sum a_n$, $\sum b_n$ with positive terms.
- Suppose $c = \lim\limits_{n\to\infty}\dfrac{a_n}{b_n}$.

  If c is finite and $c > 0$ then either both series converge or both series diverge.

Choice of $\sum b_n$ similar to above

See examples 4, 5 done in class

End of 11.4