Appendix II *Introduction to Matrices*

Part-1 Basic definitions & theory of matrices

Learning Outcomes

After completing this sub-section, you will inshaAllah be able to

1. recall the basic notions about matrices
2. recall basic important matrices
3. perform matrix addition and multiplication
4. calculate determinant of a square matrix
5. explain the concept of matrix inverse
6. differentiate & integrate matrices with variable entries

Review of elementary stuff about matrices

Matrix: $m \times n$ matrix is a collection of numbers or functions in m rows and n columns

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{bmatrix}$$

You are required to review the following basic notions from your prep-year notes

- Size of a matrix ($m \times n$?)
- Equal matrices
- Zero matrix
- Square matrix
- Diagonal matrix
- Identity matrix
- Row vector ($1 \times n$ matrix)
- Column vector ($n \times 1$ matrix)
- Addition of matrices (when is it possible and how to do it)
- Multiplication of a matrix by a number

Will be discussed in class,
if really needed.

Matrix operations

1) Matrix addition [self review]

2) Multiplication of a matrix by a number [self review]

3) Multiplication of matrices

AB defined only if
the number of columns of A = number of rows of B.

How: The $(i, j)th$ element of AB is obtained by multiplying the corresponding elements of row 'i' of A and column 'j' of B and adding the product.

$$\overset{A}{\begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ a_{41} & a_{42} & a_{43} \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}} \overset{B}{\begin{bmatrix} \bullet & \bullet & b_{13} & \bullet & \bullet \\ \bullet & \bullet & b_{23} & \bullet & \bullet \\ \bullet & \bullet & b_{33} & \bullet & \bullet \end{bmatrix}} = \overset{C}{\begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & c_{43} & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix}}$$

$$c_{43} = a_{41}b_{13} + a_{42}b_{23} + a_{43}b_{33}$$

Example: Find the missing entry in the following

$$\begin{bmatrix} 2 & 1 & -3 \\ 5 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & 2 \\ 3 & 1 & -2 & 4 \\ 0 & -1 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 5 & ? & -11 & 5 \\ 11 & -2 & 31 & 22 \end{bmatrix}$$

Note:
$size(A) = 2\times 3$, $size(B) = 3\times 4$,
$size(C) = 2\times 4$ (Why?)

See Example 1 done in class

Multiplicative identity matrix 'I'

A square matrix I such that

$$AI = IA = A$$

for any matrix A

Example 3×3 identity matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Finding determinant of a square matrix

- 1×1 matrix $\quad A = [a]$; $\quad \det A = a$
- 2×2 matrix $\quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$; $\quad \det A = a_{11}a_{22} - a_{12}a_{21}$

Example:

- $A = \begin{bmatrix} 5 & 4 \\ 1 & 4 \end{bmatrix}$; $\quad \det A = 16$
- $A = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$; $\quad \det A = 4$

Determinants of higher order matrices

- Calculated as row or column expansion
- We understand with the help of examples.

See Example 2, 3 done in class

Non-Singular matrix

a matrix with non-zero determinant

Inverse of a square matrix

A square matrix A is called invertible if there exists a matrix B such that

$$AB = BA = I.$$

B is called inverse of A and is denoted as A^{-1}.

only non-singular matrices have inverse

- We will find inverse using row operations.
- See part 2 of Appendix.

Differentiation & integration of matrices whose entries are functions

- While studying systems of differential equations we will be dealing with matrices whose entries are variables or functions.
- Also our solutions will be in the form of matrices and vectors.
- Hence we need to learn how to differentiate & integrate such matrices.

To differentiate or integrate a matrix whose entries are functions

just

differentiate or integrate each entry

See Examples 4, 5 done in class

End of Part-1 of Appendix
Do Qs. 1-30