

**Help file for solution of Example 5 (done in class), for Section 8.2**

**Find the eigenvalues of**  $A = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

**Solution**

$$\begin{aligned} |A - \lambda I| = 0 &\Rightarrow \begin{vmatrix} 3 - \lambda & -1 & -1 \\ 1 & 1 - \lambda & -1 \\ 1 & -1 & 1 - \lambda \end{vmatrix} = 0 \\ \Rightarrow (3 - \lambda) \begin{vmatrix} 1 - \lambda & -1 \\ -1 & 1 - \lambda \end{vmatrix} - (-1) \begin{vmatrix} 1 & -1 \\ 1 & 1 - \lambda \end{vmatrix} + (-1) \begin{vmatrix} 1 & 1 - \lambda \\ 1 & -1 \end{vmatrix} = 0 \end{aligned}$$

Expanding by  
1<sup>st</sup> row

$$\Rightarrow (3 - \lambda)\{(1 - \lambda)^2 - 1\} + 1\{1 - \lambda + 1\} - 1\{-1 - 1 + \lambda\} = 0$$

$$\Rightarrow (3 - \lambda)\{\lambda^2 - 2\lambda + 1 - 1\} + \{2 - \lambda\} - \{\lambda - 2\} = 0$$

$$\Rightarrow (3 - \lambda)\{\lambda(\lambda - 2)\} - \{\lambda - 2\} - \{\lambda - 2\} = 0$$

$$\Rightarrow (3 - \lambda)\{\lambda(\lambda - 2)\} - 2\{\lambda - 2\} = 0$$

$$\Rightarrow (\lambda - 2)[(3 - \lambda)\lambda - 2] = 0$$

$$\Rightarrow (\lambda - 2)[3\lambda - \lambda^2 - 2] = 0$$

$$\Rightarrow (\lambda - 2)[\lambda^2 - 3\lambda + 2] = 0$$

$$\Rightarrow (\lambda - 2)(\lambda - 2)(\lambda - 1) = 0$$