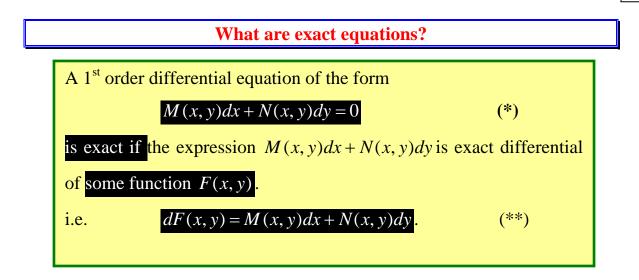
Section 2.4 Exact equations

Learning Outcomes

After completing this section, you will inshaAllah be able to

- 1. know what is meant by a exact differential equations
- 2. check whether or not a 1^{st} order ODE is exact
- 3. solve exact differential equations
- 4. use integrating factors to convert some non-exact equations to exact differential equations



Q. How do we check if an equation is exact?

- Suppose that Eq. (*) is exact. The from Eq. (**) $\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = Mdx + Ndy$
- Hence

$$\circ \qquad M(x,y) = \frac{\partial F}{\partial x} \qquad \Rightarrow \qquad \frac{\partial M}{\partial y} = \frac{\partial^2 F}{\partial x \partial y} \qquad (2)$$

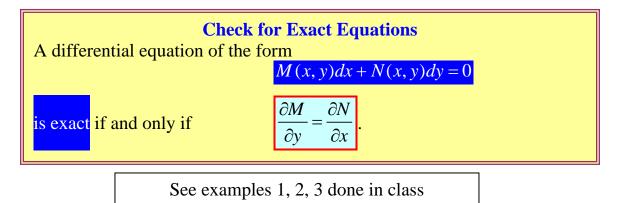
$$\circ \text{ and } \qquad N(x,y) = \frac{\partial F}{\partial y} \qquad \Rightarrow \qquad \frac{\partial N}{\partial x} = \frac{\partial^2 F}{\partial y \partial x} \qquad (2')$$

First (2) (2) we are that if an equation of the

From Eqs. (2), (2') we see that if an equation of the form 2M

$$M(x, y)dx + N(x, y)dy = 0$$
 is exact then $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

It can be proved that the converse is also true.



An important calculation procedure to study exact equations

The method of solving exact equations is based on a special computational procedure

• Given two functions M(x, y) and N(x, y)

• How to find a function
$$F(x, y)$$
 that satisfies

$$\frac{\partial F}{\partial x} = M(x, y) \text{ and } \frac{\partial F}{\partial y} = N(x, y)$$

• We learn with the help of example

See example 4 done in class

Method of solving exact ODE'sMain IdeaGiven an exact ODE M(x, y)dx + N(x, y)dy = 0.o i.e. Mdx + Ndy = dF(x, y) for some function F(x, y)o or $Mdx + Ndy = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$ for some function F(x, y)

• Main task is to find F(x, y) so that ODE becomes dF(x, y) = 0

• This implies
$$F(x, y) = C$$
 is solution.

• To find F(x, y) satisfying $Mdx + Ndy = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$. To find F(x, y) satisfying $\frac{\partial F}{\partial x} = M(x, y)$ and $\frac{\partial F}{\partial y} = N(x, y)$

How to find F(x, y)

Method

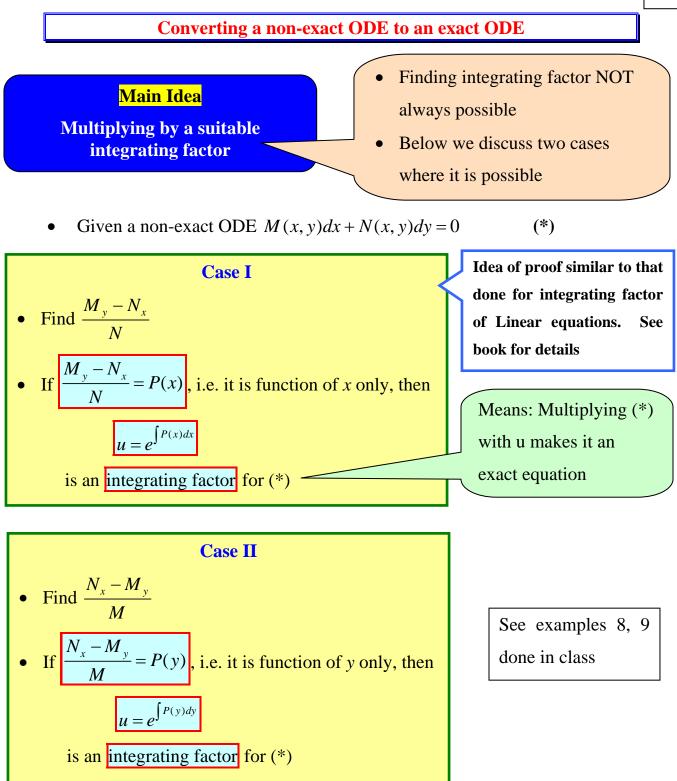
- 1. Write the ODE in the form M(x, y)dx + N(x, y)dy = 0 (*)
- 2. Check exactness: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. If exact proceed as follows.
- 3. Get F(x, y) by solving

$$\frac{\partial F}{\partial x} = M(x, y) \text{ and } \frac{\partial F}{\partial y} = N(x, y)$$

4. Write the general solution by setting F(x, y) = C.

Because Eq. (*) becomes dF(x, y) = 0.

See examples 5, 6, 7 done in class



End of 2.4 Do Qs: 1-39

2.45