<h2 style="color:red; text-align:center">Section 2.2 Separable variables</h2>

We begin our methods for solving 1st order ODE's by considering a class of (generally) non-linear 1st order equations called **"Separable Equations"**.

Learning Outcomes

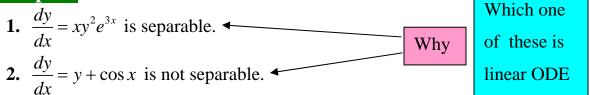After completing this section, you will inshaAllah be able to

1. know what is meant by a separable differential equation
2. solve separable equations

What are separable equations?

A 1st order ODE $\dfrac{dy}{dx} = f(x,y)$ is called **separable** if it can be written in the form

$$\frac{dy}{dx} = g(x) \cdot h(y)$$

or $\qquad\qquad N(y)dy = M(x)dx$

i.e. the x-terms and y-terms can be separated on two sides of the equation

Examples:

1. $\dfrac{dy}{dx} = xy^2 e^{3x}$ is separable.

2. $\dfrac{dy}{dx} = y + \cos x$ is not separable.

Why

Which one of these is linear ODE

We will see different kind of solutions in the examples in this section.

- Explicit solution   {see example 1}

- Implicit solution   {see example 2}

- Constant or singular solution   {see example 4}

Method of solving separable equations

Main idea: Separate and integrate

Question: Given separable $\dfrac{dy}{dx} = f(x, y)$. To find solution $y(x)$.

Step 1: Separate the equation to put in the form
$N(y)dy = M(x)dx$
{See special tricks in example 6 & exercise after it.}

Step 2: Integrate both sides to get a general solution.

Step 3(a): Solve for $y(x)$ to get a general explicit solution.

3(b): If explicit solution is not possible, write the implicit solution.

Step 4: (If given) use initial conditions (e.g. in case of IVP) to get particular solution.

Step 5: Check your solution by taking derivatives & putting back in equation.

Further, be careful about the following:

Even if you have done all the above steps, you may not get correct or all solutions because of the following possible problems.

Caution 1: In case of IVP, be sure that you have chosen the correct solution. {See example 3}.

Caution 2: If there are any singular and constant solutions, they may or may not be contained in general solution {See example 4}. In case they are not, be sure to write these solutions as well.

Caution 3: Be sure that the arbitrary constant is introduced immediately after the integration and not later {See example 5}.

See Examples 1, 2, 3, 4, 5, 6 done in class

End of 2.2
Do Qs. 1-36