

Section 1.2 *Initial value problems*

Learning Outcomes

After completing this section, you will inshaAllah be able to

1. know what is meant by **initial value problems (IVPs)** & their solutions
2. understand the facts about **existence & uniqueness of solutions of IVPs**
3. know about the **interval of existence** of solution of IVPs

Initial value problems

An initial value problem consists of

- a differential equation and
- appropriate number of initial condition(s)

Number of initial conditions
=
Order of ODE

Examples

1.
$$2x \frac{dy}{dx} + 4y = 3$$

$$y(1) = -4$$

2.
$$\frac{d^2y}{dx^2} - y = 0$$

$$y(0) = 0, y'(0) = 1$$

Solution of IVP is a function that satisfies both

- ODE and
- initial conditions

See Examples 1, 2 done in class

Existence of unique solution of IVPs of 1st order ODEs

If $f(x, y)$ and $\frac{\partial f}{\partial y}$ are continuous near (x_0, y_0)

then the IVP

$$\begin{aligned} \frac{dy}{dx} &= f(x, y) \\ y(x_0) &= y_0 \end{aligned}$$

has a **unique solution**.

See Examples 3, 4
done in class

Interval of existence for solution $y(x)$ of IVPs

This is the interval

- that contains initial point x_0
- and
- the function $y(x)$ is defined and differentiable on it.

See Example 5 done in class

*End of 1.2
Do Qs. 1-27*