

1. (8 points) Find the general solution of the differential equation

$$(1 - x^2)y'' + 2xy' - 2y = 0 \quad (x > 1)$$

given that $y_1 = x$ is a solution of the differential equation.

2. (8 points) Find the general solution of the differential equation

$$y^{(4)} - 2y''' + 5y'' = 0.$$

3. (9 points) Consider the differential equation

$$x^4 y'' + x^3 y' - 4x^2 y = 1 \quad (x > 0).$$

- (a) Given that $y_1 = x^2$ and $y_2 = x^{-2}$ are solutions of the associated homogeneous equation. Show that they form a fundamental set of solutions of the associated homogeneous equation.
- (b) Find a particular solution of the given non-homogeneous equation.

4. (7 points) Find the general solution of the differential equation

$$x^2y'' + 11xy' + 41y = 0 \quad (x > 0).$$

5. (8 points) Find the recurrence relation for the coefficients of power series solutions of $y'' + 2xy' + 2y = 0$ about the ordinary point $x = 0$.

6. (10 points) By substituting $y = \sum_{n=0}^{\infty} c_n x^n$ in a differential equation, we obtain

$$2c_2 - c_0 + 6c_3x + \sum_{k=2}^{\infty} [(k+1)(k-1)c_k - (k+2)(k+1)c_{k+2}]x^k = 0, \quad \text{for all } x.$$

Find the general solution of that differential equation.