1. (a) (2 points) Verify that $e^y = y - x^2 + C$ is an implicit solution of the differential equation

$$\frac{dy}{dx} = \frac{2x}{1 - e^y}.$$

(b) (2 points) Use the implicit solution given in part (a) to solve the initial-value problem (IVP)

$$\frac{dy}{dx} = \frac{2x}{1 - e^y}, \quad y(1) = 1.$$

(c) (2 points) Tell whether the IVP given in part (b) has a unique solution. Justify your answer.

2. (7 points) Solve: $\sin x \dfrac{dy}{dx} - y \cos x = \sin^2 x$, where $0 < x < \frac{\pi}{2}$.

3. (7 points) Solve: $e^{x^3+y^2}\,dx + \dfrac{y}{x^2}\,dy = 0.$

4. (7 points) Solve: $(ye^x + \sin x)dx + (2y + e^x + \cos y)dy = 0$.

5. (6 points)

(a) Use an appropriate substitution to reduce the following differential equation

$$\frac{dy}{dx} = \frac{2y - x}{x + 3y}$$

to a separable equation.

(b) Is it possible to write the separable equation obtained in (a) as a linear differential equation? Justify your answer.

6. (7 points) According to Newton's Law of cooling/warming, the rate of change of temperature $T(t)$ of an object at any time t is proportional to the difference between T and the surrounding temperature T_m. Let k be the constant of proportionality.

   (a) Write the differential equation that models this phenomenon.

   (b) Solve the differential equation found in (a) and write its general solution as $T(t) = T_m + ce^{kt}$.

   (c) An object of temperature 10°C is left in a room of temperature 30°C. After 2 minutes the object temperature is 15°C. How long will it take for the object to reach 25°C?