Section 2.8 *The derivative as a function*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. explain the definition of derivative of a function
2. find derivatives using definition

- In the last section we saw that the derivative of *f(x)* at a point *'a'* is given by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- Now we study the concept of derivative of *f(x)* at any point *x* as a a function.

Derivative of a function

The derivative of $f(x)$ is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

if the limit exists.

- Another notation: $\frac{dy}{dx}$
- $f'(a)$ means derivative at the point a

See examples 1, 2, 3 done in class

Geometric interpretation of derivative

See class explanation

Checking differentiability graphically

See class explanation

See example 4 done in class

Left/Right derivative of a function
Existence of a derivative

The left derivative of $f(x)$ is defined by

$$f'_-(a) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$

The right derivative of $f(x)$ is defined by

$$f'_+(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

See explanation given above for geometric interpretation of derivative

The derivative $f'(a)$ exists if the left and right derivatives are same.

i.e. the function will surely be not differentiable at a point where it is discontinuous

Also note the fact that "if a function is differentiable at a point then it is continuous at that point"

- The next example uses all of these ideas.

See example 5 done in class

End of 2.8