Section 2.7 *Derivatives and rates of change*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. use a special type of limit to find slopes of tangent lines
2. use a special type of limit to find rate of change of a function
3. explain the definition of derivative of a function at a point
4. use a special type of limit to find derivative at a point

Slope of tangent line to a curve y=f(x)

- Given a curve $y = f(x)$

The slope of tangent line at $(a, f(a))$ is given by

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

See class explanation

See example 1 done in class

If we take $x = a + h$ then the above definition becomes

The slope of tangent line at $(a, f(a))$ is given by

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

See class explanation

See example 2 done in class

Instantaneous rate of change

- Given a function $y = f(x)$

Average rate of change of $f(x)$ over $[a, a+h]$

$$y_{av} = \frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(a+h) - f(a)}{h}$$

Instantaneous rate of change in $f(x)$ at $x = a$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

See class explanation

Every day example

$S=f(t)$: position of object at time t

Average velocity v_{av} over interval $[a, a+h]$

= average rate of change of displacement over interval

$$\Rightarrow \quad v_{av} = \frac{f(a+h) - f(a)}{h}$$

See example 3 done in class

Velocity at time t =

Instantaneous rate of change of displacement at time t

$$\Rightarrow \quad v = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

An important remark

- Given a function $y = f(x)$.

- We have seen above that limits of the form

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

are very special and can be used to answer important questions.

- For example, the rate of change of *f(x)* at a point *'x'* is given by

$$m = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

- Therefore such limits become very important.
- The study of important limit of above type leads to the concept of derivative, which we study next.

Derivative of a function

The derivative of *f(x)* at a point '*a*' is defined by

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h},$$

if the limit exists.

If we take $x=a+h$ then the above definition becomes

The derivative of *f(x)* at a point '*a*' is defined by

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

if the limit exists.

See examples 4, 5 done in class

End of 2.7