

**Learning outcomes**

After completing this section, you will inshaAllah be able to

1. compute limits of the form  $\lim_{x \rightarrow \pm\infty} f(x)$
2. explain what are horizontal asymptotes
3. find horizontal asymptotes of a function

## Computing limits $\lim_{x \rightarrow \pm\infty} f(x)$

### Based on following basic limits

- $\lim_{x \rightarrow \infty} k = k$                        $k$  : constant
- $\lim_{x \rightarrow \pm\infty} \frac{k}{x^n} = 0$                       for  $n > 0$
- $\lim_{x \rightarrow \infty} x^n = \infty$                       for  $n > 0$
- $\lim_{x \rightarrow -\infty} x^n = \begin{cases} \infty & n = 2, 4, 6, \dots \\ -\infty & n = 1, 3, 5, \dots \end{cases}$

### Technique for finding $\lim_{x \rightarrow \pm\infty} f(x)$

- Take **highest power common** from numerator & denominator
- **Simplify & use above** basic limits

See examples 1, 2, 3, 4, 5, 6, 7, 8 done in class

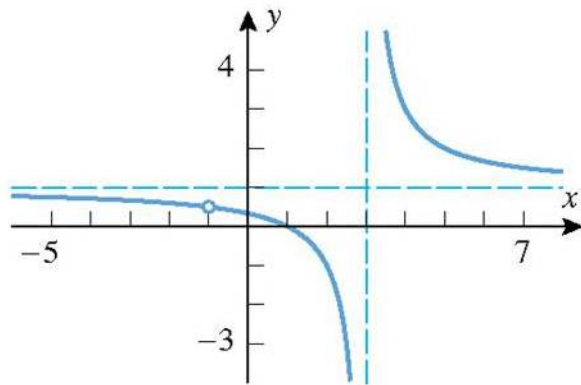
## What to do if $\lim_{x \rightarrow \pm\infty} f(x)$ gives $\infty - \infty$

- We learn with the help of example

See example 9 done in class

## Horizontal Asymptotes

- Look at the following graph.



It runs (very close &) parallel to graph up to  $x = \pm\infty$

What's special about line  $y=1$

What happens to graph when we  $x$  gets near  $\pm\infty$

The graph approaches (gets closer to) the horizontal line  $y=1$

A horizontal line  $y = b$  is called **horizontal asymptote** of graph of  $f(x)$  if

▪  $\lim_{x \rightarrow \infty} f(x) = b$

or

▪  $\lim_{x \rightarrow -\infty} f(x) = b$

See examples 10, 11, 12 done in class