Section 2.5 *Continuity*

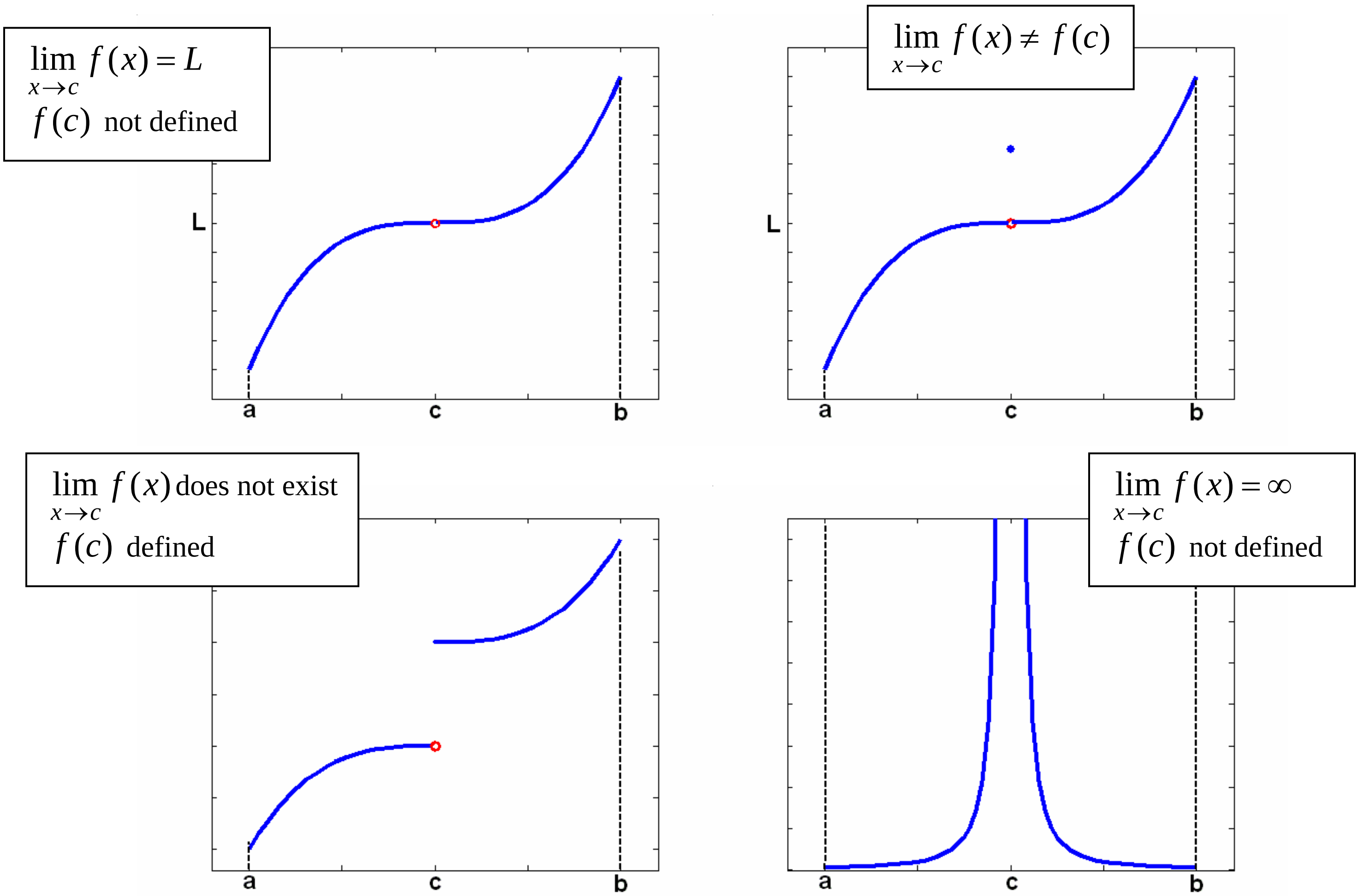
Learning outcomes

After completing this section, you will inshaAllah be able to

1. get an idea about the meaning of a continuous function
2. check whether a function is continuous or discontinuous at a point
3. use basic properties of continuous functions
4. know important examples of continuous functions
5. explain difference between different types of discontinuities
    a. removable discontinuity
    b. jump discontinuity
    c. infinite discontinuity
6. explain and apply intermediate value theorem

Meaning of continuous function

- The following graphs have gaps. Let's see what is happening in these graphs.



Continuity $\approx$ no gap(s) in the graph.
Clearly: To have continuity at x=c, none of above should happen

A function f(x) is continuous at x=a if $\lim_{x \to a} f(x) = f(a)$

i.e.

- $f(a)$ is defined
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a} f(x) = f(a)$

Computations related to continuity

To show f(x) is continuous at x=a we must show

1. $f(a)$ is defined
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x) = f(a)$

- left limit = right limit
- answer is a finite number

See examples 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 done in class

While doing questions we also keep using the following important facts, as needed.

Basic Properties

If f, g are continuous at point 'a' then

1. $f \pm g$
2. $f \cdot g$
3. cf (c constant)
4. $\dfrac{f}{g}$

are also continuous at 'a'.

The composition $f \circ g$ of continuous functions f, g is also continuous

Important examples

- all polynomials are continuous
- rational functions, root functions, trigonometric, inverse trigonometric, logarithmic, exponential functions are continuous at points in their domain
- e.g. This means rational functions are discontinuous only at points where denominator is zero

What are different types of discontinuities that can occur?

- We learn the different types of discontinuities with the help of examples.

Removable discontinuity

See example 12 and explanation provided in class

Infinite discontinuity

See example 13 and explanation provided in class

Jump discontinuity

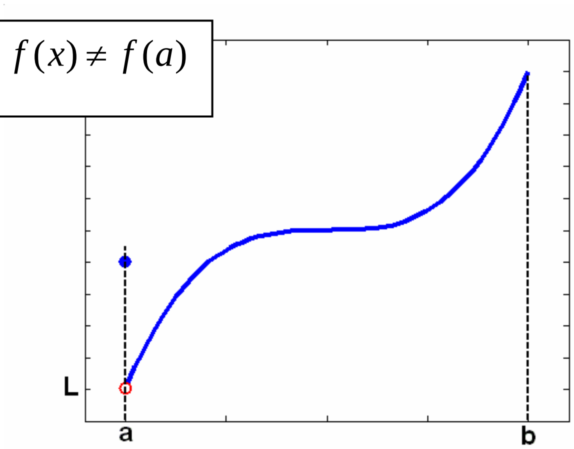
See example 14 and explanation provided in class

See examples 15, 16 done in class

How to check continuity at end points of a closed interval [a,b]

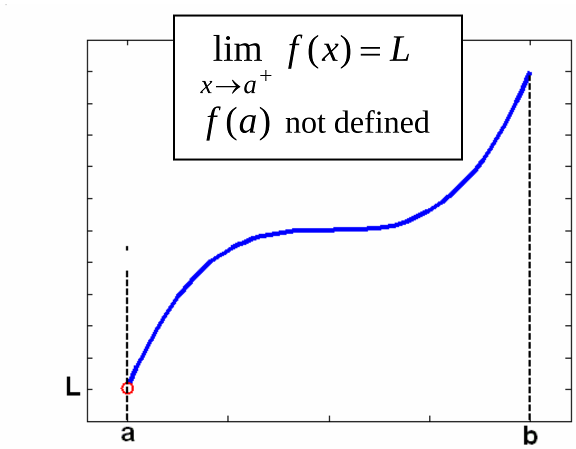
- Look at following graphs for left end point x=a

$\lim_{x\to a^+} f(x) \neq f(a)$



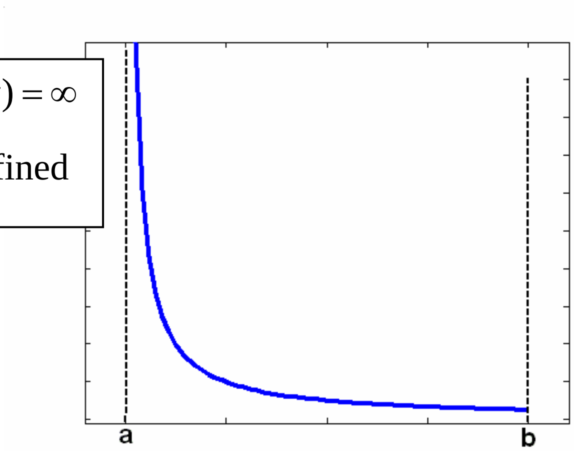
$\lim_{x\to a^+} f(x) = L$

$f(a)$ not defined



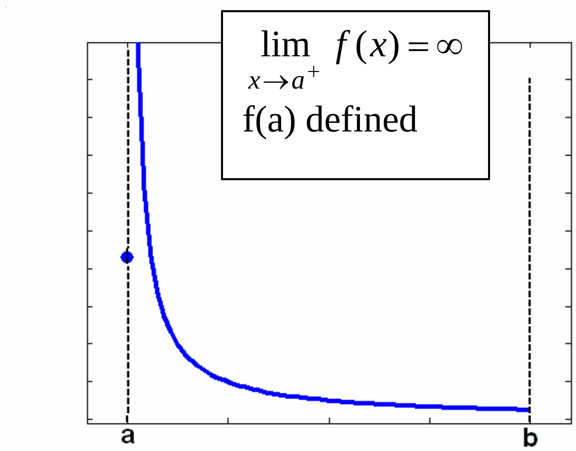
$\lim_{x\to a^+} f(x) = \infty$

f(a) undefined



$\lim_{x\to a^+} f(x) = \infty$

f(a) defined



Continuity $\approx$ no gap(s) in the graph.
Clearly: To have continuity at x=c, none of above should happen

f(x) is continuous at left end point x=a of [a,b] if $\lim_{x\to a^+} f(x) = f(a)$

i.e.

- $f(a)$ is defined
- $\lim_{x\to a^+} f(x)$ exists (i.e finite)
- $\lim_{x\to a^+} f(x) = f(a)$

Similarly

f(x) is continuous at right end point x=b of [a,b] if $\lim_{x\to b^-} f(x) = f(b)$

Continuity on an interval

A function f(x) is continuous on an interval if

it is continuous at every point in the interval

If an end point is included in the interval,

use the idea of

left-continuous or right-continuous

as appropriate.

See explanation on last page

or definitions below

f(x) is continuous from right at x=a if $\lim_{x \to a^+} f(x) = f(a)$

f(x) is continuous from leftt at x=b if $\lim_{x \to b^-} f(x) = f(b)$

See example 17 done in class

Some applications

Limits of composition of continuous functions

$$\lim_{x\to a} f\left(g(x)\right) = f\left(\lim_{x\to a} g(x)\right)$$

Very useful tool for calculating complicated limits

See example 18 done in class

Intermediate value theorem & applications

If $f(x)$ is continuous on $[a,b]$ and

N is any number between $f(a)$ and $f(b)$

then

there is a number c in (a,b) such that $f(c) = N$

See class explanation

See examples 19, 20, 21 done in class

End of 2.5