

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematical Sciences**  
**Semester II, 2005-2006 (052)**  
**MATH 101 — Final Exam**

Code 001

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**Q1.**  $\lim_{x \rightarrow -\infty} \frac{x^3 - x^2 + 1}{2 - x - x^2} =$

- (a) 0
- (b) 1
- (c) -1
- (d)  $-\infty$
- (e)  $+\infty$

**Q2.** The area of the triangle formed by the tangent line to the curve  $xy = 1$  at  $x = -3$  and the coordinate axes is equal to

- (a)  $5/3$
- (b) 2
- (c)  $-1/9$
- (d)  $7/3$
- (e) 3

**Q3.** The derivative  $\frac{dy}{dx}$  of  $y = \cosh^{-1} \sqrt{x^2 + 1}$  is equal to

- (a)  $\frac{1}{\sqrt{x^2+1}}$
- (b)  $\frac{1}{x\sqrt{x^2+1}}$
- (c)  $\frac{x}{|x|\sqrt{x^2+1}}$
- (d)  $\frac{-1}{\sqrt{x^2+1}}$
- (e)  $\frac{1}{\sqrt{1-x^2}}$

**Q4.** The linear approximation of the function  $f(x) = (1+x)^{1/5}$  at 0 is

- (a)  $\frac{x}{5} + 1$
- (b)  $\frac{x}{5} - 1$
- (c)  $-\frac{x}{5} - 1$
- (d)  $-\frac{x}{5} + 1$
- (e)  $x + \frac{1}{5}$

**Q5.** If the derivative of a function  $f$  is differentiable for all  $x$ , then

$$\lim_{x \rightarrow 1} \frac{\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h/2} - 2f'(x)}{\frac{x}{2} - \frac{1}{2}} =$$

- (a)  $-f'(1)$
- (b)  $2f''(\frac{x}{2})$
- (c)  $\frac{f''(1)}{4}$
- (d)  $-\frac{1}{2}f'(\frac{x}{2})$
- (e)  $-4f''(1)$

**Q6.** The set of  $x$ -coordinates of the points at which the function  $f(x) = \frac{x-1}{x^4-2x^3-x+2}$  has removable discontinuity is:

- (a)  $\{1\}$
- (b)  $\{2\}$
- (c)  $\{1, 2\}$
- (d)  $\{-2, -1\}$
- (e)  $\{-2, 1\}$

**Q7.** Regarding  $y$  as the dependent variable and  $x$  as the independent variable in the equation  $x^y = y^x$ , the derivative  $\frac{dy}{dx}$  is equal to

- (a)  $\frac{yx^{y-1}}{xy^{x-1}}$
- (b)  $\frac{xy^{x-1}}{yx^{y-1}}$
- (c)  $\frac{x \ln y - y}{y \ln x - x}$
- (d)  $\frac{y^2 - xy \ln y}{x^2 - xy \ln x}$
- (e)  $\frac{\ln y - \frac{y}{x}}{\ln x}$

**Q8.** The function  $f(x) = -\frac{1}{4}x^4 + 2x^2 + 3$  has exactly

- (a) one local maximum and one local minimum
- (b) two local minimums and one local maximum
- (c) one local minimum and two local maximums
- (d) one local minimum and no local maximums
- (e) one local maximum and no local minimum

**Q9.** Using Newton's method with initial approximation  $x_1 := -1$ , the second approximation  $x_2$  of the root of the equation  $5x^3 - 3x^2 + x + 2 = 0$  is:

- (a)  $-\frac{15}{22}$
- (b)  $-\frac{1}{2}$
- (c)  $-\frac{10}{3}$
- (d)  $-\frac{1}{4}$
- (e)  $-\frac{1}{6}$

**Q10.**  $\lim_{x \rightarrow 1^-} \frac{1-x^2}{x^3-1} =$

(a)  $\frac{1}{3}$

(b)  $-\frac{1}{3}$

(c)  $\frac{2}{3}$

(d)  $\frac{-2}{3}$

(e)  $\infty$

**Q11.** If the normal line to the curve  $y = f(x)$  at the point  $(-1, 3)$  passes through the point  $(0, 5)$ , then  $f'(-1) =$

(a)  $-2$

(b)  $3/2$

(c)  $1/2$

(d)  $-1/2$

(e)  $-1$

**Q12.** The largest possible  $\delta$  such that

$$|x - 1| < \delta \Rightarrow |x^2 - 1| < \frac{1}{2}$$

is:

(a)  $\sqrt{\frac{3}{2}} - 1$

(b)  $1 - \frac{1}{\sqrt{2}}$

(c)  $\sqrt{\frac{3}{2}}$

(d)  $\frac{1}{\sqrt{2}}$

(e)  $1$

**Q13.** If  $f(x) = \frac{\sin x \sec x}{1+x \tan x}$ , then  $f'(0) =$

- (a)  $-2$
- (b)  $-1$
- (c)  $0$
- (d)  $1$
- (e)  $2$

**Q14.**  $D^{1000}(xe^{-x})$  is equal to

- (a)  $(x - 1001)e^{-x}$
- (b)  $(1000 - x)e^{-x}$
- (c)  $(999 - x)e^{-x}$
- (d)  $(x - 1000)e^{-x}$
- (e)  $(1001 - x)e^{-x}$

**Q15.** The equation  $x^{201} + 804x = 9 - \frac{402}{101}x^{101}$  has

- (a) exactly one real solution
- (b) no real solutions
- (c) exactly 201 real solutions
- (d) three real solutions
- (e) one real solution in  $(-\infty, 0)$  and another one in  $(0, +\infty)$

**Q16.** A point of the curve of  $y = \ln(\ln x)$  at which the tangent has slope  $\frac{1}{e}$  is

- (a)  $(\frac{1}{e}, 0)$
- (b)  $(e^e, 1)$
- (c)  $(e, 0)$
- (d) Does not have
- (e)  $(e^2, \ln 2)$

**Q17.** The function  $f(x) = \frac{x^2+5}{x+2}$

- (a) changes its concavity from concave down to concave up and has no inflection points
- (b) changes its concavity from concave down to concave up and has an inflection point at  $-2$
- (c) changes its concavity from concave up to concave down and has no inflection points
- (d) changes its concavity from concave up to concave down and has an inflection point at  $-2$
- (e)  $f$  is always concave up

**Q18.** A sketch of the curve of  $f(x) = x^2e^{-x^2}$  would be

**Q19.** Which one of the following curves corresponds to the function

$$f(x) = \frac{x^5}{(x^2 - 1)^2}.$$



**Q20.** The cost of a printer is 625 Riyals, and its value is depreciating (i.e. decreasing) with time according to the formula

$$\frac{dV}{dt} = -800(2t + 1)^{-2},$$

where  $V$  Riyals is its value  $t$  years after its purchase. What is the value of the printer three years after its purchase (rounded to two decimals)?

- (a) 280.33 Riyals
- (b) 292.04 Riyals
- (c) 272.74 Riyals
- (d) 282.14 Riyals
- (e) 290.04 Riyals

**Q21.** The set of vertical asymptotes for the graph of

$$f(x) = \frac{x^4 - 1}{x^3 - x}$$

is:

- (a)  $\{x = -1, x = 0, x = 1\}$
- (b)  $\{x = -1, x = 0\}$
- (c)  $\{x = 0, x = 1\}$
- (d)  $\{x = -1, x = 1\}$
- (e)  $\{x = 0\}$

**Q22.** The derivative  $\frac{dy}{dx}$  for  $y = \sqrt{x + \coth \sqrt{1+x^2}}$  is:

- (a)  $\frac{\sqrt{1+x^2} + \operatorname{csch}^2(\sqrt{1+x^2})}{2\sqrt{1+x^2}\sqrt{x + \coth(\sqrt{1+x^2})}}$
- (b)  $\frac{1 - \operatorname{csch}^2(\sqrt{1+x^2})}{2\sqrt{x + \coth(\sqrt{1+x^2})}}$
- (c)  $\frac{-x \cosh^2(\sqrt{1+x^2})}{2\sqrt{x + \coth(\sqrt{1+x^2})}}$
- (d)  $\frac{1}{2\sqrt{x + \coth(\sqrt{1+x^2})}}$
- (e)  $\frac{\sqrt{1+x^2} - x \operatorname{csch}^2(\sqrt{1+x^2})}{2\sqrt{1+x^2}\sqrt{x + \coth(\sqrt{1+x^2})}}$

**Q23.** The equation of the line through the point  $(2, 3)$  that forms with the positive  $x$ -axis and the positive  $y$ -axis a triangle of non-zero least area is:

(a)  $y = -\frac{2}{3}x + \frac{13}{3}$

(b)  $y = -3x + 9$

(c)  $y = -\frac{3}{2}x + 6$

(d)  $y = -x + 5$

(e)  $y = 2x - 1$

**Q24.** Which one of the following statements is always true?

(a) If  $f$  is continuous on  $[-1, 1]$  and  $f(-1) = 4$  and  $f(1) = 3$ , then there exists a number  $r$  such that  $|r| < 1$  and  $f(r) = \pi$

(b) If the line  $x = 1$  is a vertical asymptote of  $y = f(x)$ , then  $f$  is not defined at 1

(c) If  $f$  is continuous at  $a$ , then  $f$  is differentiable at  $a$

(d) If  $f(x) > 1$  for all  $x$  and  $\lim_{x \rightarrow 0} f(x)$  exists, then  $\lim_{x \rightarrow 0} f(x) > 1$

(e) If  $\lim_{x \rightarrow 6} f(x)g(x)$  exists, then it is equal to  $f(6)g(6)$

**Q25.** Water is leaking out (escaping) of an inverted conical (cone shape) tank at a rate of  $10^4 \text{ cm}^3/\text{min}$ . At the same time water is being pumped into the tank at a constant rate. The height of the tank is  $H = 8\text{ m}$  and its diameter at the top is  $D = 6\text{ m}$ . If the water level is rising at the rate of  $\frac{64}{81\pi} \text{ cm}/\text{min}$  when the height  $h$  of the water is  $h = 3\text{ m}$ , the rate at which the water is being pumped into the tank is

(a)  $20000 \text{ cm}^3/\text{min}$

(b)  $10000 \text{ cm}^3/\text{min}$

(c)  $15000 \text{ cm}^3/\text{min}$

(d)  $10000 \cdot \frac{64}{81\pi} \text{ cm}^3/\text{min}$

(e)  $(10000 + \frac{64000}{81\pi}) \text{ cm}^3/\text{min}$